
Errata "Principles of Model Checking" (2008)

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Comments are provided as:

⟨ page number ⟩ ⟨ line number ⟩ ⟨ short quote of the wrong word(s) ⟩ ▷ ⟨ correction ⟩

Chapter 1: System Verification

pp. 1, l. -5, *Pentium II* ▷ Pentium

pp. 5, l. 9, *lines of code lines* ▷ lines of code

pp. 5, l. footnote, *much higher* ▷ as the number of lines of code in the "golden" version of Windows95 is about 15 million, the error rate is in fact lower than normal.

pp. 6, l. 4, *Pentium II* ▷ Pentium

Chapter 2: Modeling Concurrent Systems

pp. 25, l. 11, *heading Example 2.8* ▷ Execution fragments of the Beverage Vending Machine

pp. 27, l. -15, *function λ_y* ▷ The function λ_y has no impact on the transitions (as suggested), but only affects the state labeling.

pp. 31, l. Fig. 2.3, *beer, soda* ▷ *bget* and *sget*, respectively

pp. 31, l. Fig. 2.3, *state with 1 beer, 2 soda* ▷ the grey circle should be a white circle.

pp. 34, l. 2, $\langle \ell, v \rangle$ ▷ $\langle \ell, \eta \rangle$

pp. 42, l. -10, *interlock* ▷ interleave

pp. 46, l. Fig. 2.9, *locations in PG₂* ▷ should be subscripted with 2 (rather than 1)

pp. 48, l. -1, $H = Act_1 \cap Act_2 \triangleright H = (Act_1 \cap Act_2) \setminus \{\tau\}$

pp. 51, l. Fig. 2.12, $T_1 \parallel T_2 \triangleright TS_1 \parallel TS_2$ (this occurs twice)

pp. 51, l. Fig. 2.12, \triangleright All downgoing transitions should be labeled with *request*, and all upgoing ones with *release*

pp. 51, l. -7, *all trains* \triangleright the train

pp. 52, l. 3, *(above)* \triangleright (page 54)

pp. 53, l. -1, *finite set of channels* \triangleright set of channels

pp. 54, l. Fig. 2.16, *the transition labeled approach emanating from state $\langle far, 3, down \rangle$* \triangleright should be removed, and all the states that thus become unreachable

pp. 54, l. Fig. 2.16, *the transition labeled exit emanating from state $\langle in, 1, up \rangle$* \triangleright should be removed, and all the states that thus become unreachable

pp. 55, l. -10, $(Cond(Var) \times) \triangleright Cond(Var) \times$

pp. 62, l. -3, $gen_msg(1) \triangleright snd_msg(1)$

pp. 64, l. 4, *ack* \triangleright message

pp. 65, l. Fig. 2.21, *second do* \triangleright **od**

pp. 66, l. 8, *Staements build* \triangleright Statements built

pp. 71, l. 15, *label in conclusion of inference rule cle* \triangleright it is meant that the value of expression e is transferred; cf. Exercise 2.8, pp. 85

pp. 74, l. 1, $\xi[c := v_2 \dots v_k] \triangleright \xi' = \xi[c := v_2 \dots v_k]$

pp. 74, l. 1, $\xi[c := v_1 \dots v_k v] \triangleright \xi' = \xi[c := v_1 \dots v_k v]$

pp. 76, l. Figure 2.23 (top), $x \triangleright x'$

pp. 79, l. -6,-8, $|dom(c)|^{cp(c)} \triangleright |dom(c)|^{cap(c)}$

pp. 82, l. Exercise 2.2, line 2, P_i is $\triangleright P_i$ is

Chapter 3: Linear-Time Properties

pp. 89, l. 9, *parallel systems* \triangleright reactive systems

pp. 90, l. 1, *Fault Designed Traffic Lights* \triangleright Faulty Traffic Lights

pp. 91, l. 7, *a deadlock occurs when all philosophers* \triangleright a deadlock may occur when all philosophers

pp. 92, l. Fig. 3.2, *request and release* \triangleright req and rel

pp. 92, l. 6, *request*₄ \triangleright *req*_{4,4}; similar to the other request actions

pp. 93, l. -4,-5 and Fig. 3.3, Fig. 3.4, *state available*_i \triangleright *available*_{i,i}

pp. 93, l. -4,-5 and Fig. 3.3, Fig. 3.4, *state available*_{i+1} \triangleright *available*_{i,i+1}

pp. 93, l. 10, *The corresponding is* \triangleright The corresponding condition is

pp. 94, l. Fig. 3.4, *falls* *x*_i \triangleright *x*_i

pp. 96, l. 3, *finite paths* \triangleright finite path fragments

pp. 96, l. 4, *infinite path* \triangleright infinite path fragment

pp. 100, l. 9, *(over AP)* \triangleright *(over* 2^{AP} *)*

pp. 101, l. -3, *red*₁ *green*₂ \triangleright *red*₁, *green*₂

pp. 103, l. 11, *lwait*_i \triangleright *wait*_i

pp. 103, l. 11, $\exists k \geq j. \text{wait}_i \in A_k \triangleright \exists k > j. \text{crit}_i \in A_k$

pp. 111, l. Theorem 3.21, $M = \sum_{s \in S} |\text{Post}(s)| \triangleright M = \sum_{s \in \text{Reach}(TS)} |\text{Post}(s)|$

pp. 111, l. 22, *The time needed to check s* $\models \Phi$ *is linear in the length of* Φ \triangleright Add: This implicitly assumes that $a \in L(s)$ can be checked in $\mathcal{O}(1)$ time.

pp. 112, l. -2, \triangleright A minimal bad prefix is one such that the first occurrence of Φ is the last symbol in the word.

pp. 115, l. Lemma 3.27, *Proof* \triangleright add the following sentence to the beginning of the proof: First note that for $P = (2^{AP})^\omega$ the claim trivially holds, since *closure*(P) = P and the fact that P is a safety property since \overline{P} is empty. In the remainder of the proof we consider $P \neq (2^{AP})^\omega$.

pp. 118, l. 10,11, $\pi^{m_0} \pi^{m_1} \pi^{m_2} \dots$ of $\pi^0 \pi^1 \pi^2 \dots$ such that $\triangleright \pi^{m_0}, \pi^{m_1}, \pi^{m_2}, \dots$ of $\pi^0, \pi^1, \pi^2, \dots$ such that

pp. 124, l. -3, *By definition* \triangleright By Lemma 3.27

pp. 130, l. 3, *without being taken beyond* \triangleright without being taken infinitely often beyond

pp. 131, l. 17, *assignment* $x = -1 \triangleright$ assignment $x := -1$

pp. 132, l. 2, *an execution fragment ... but not strongly A-fair.* \triangleright an execution fragment that visits infinitely many states in which no *A*-action is enabled is weakly *A*-fair (as the premise of weak *A*-fairness does not hold) but may not be strongly *A*-fair.

pp. 134, l. 10, *any finite trace is fair by default* \triangleright any finite trace is strongly or weakly fair by default

pp. 136, l. -5, *strong fairness property* \triangleright fairness property

pp. 138, l. 4, *It forces synchronization actions to happen infinitely often.* \triangleright It forces synchronization actions to happen infinitely often provided they are enabled infinitely often.

pp. 138, l. -14, *This requires that ... is enabled.* \triangleright This requires that infinitely often a synchronization takes place when such synchronization is infinitely often enabled.

pp. 141, l. 5, *the set of properties that has* \triangleright the property that has

pp. 145, l. Exercise 3.5(g), *between zero and two* \triangleright between zero and non-zero

Chapter 4: Regular Properties

pp. 157, l. -11, $w = A_1 \dots A_n \in \Sigma \triangleright w = A_1 \dots A_n \in \Sigma^*$

pp. 157, l. -10, *starts in Q_0* \triangleright starts in state Q_0

pp. 157, l. -4, $Q_0 \triangleright \{Q_0\}$

pp. 158, l. -14, *NFAs can be much more efficient.* \triangleright NFAs can be much smaller.

pp. 161, l. -9, (2) ... *for all $1 \leq i < n$* \triangleright ... for all $0 \leq i < n$. (Note: the invariant false has minimal bad prefix ε .)

pp. 161, l. -8, $1 \leq i < n \triangleright 0 \leq i < n$

pp. 163, l. Example 4.15, *Minimal bad prefixes for this safety property constitute the language $\{pay^n drink^{n+1} \mid n \geq 0\}$* \triangleright Bad prefixes for this safety property constitute the language $\{\sigma \in (2^{\{pay, drink\}})^\omega \mid w(\sigma, drink) > w(\sigma, pay)\}$ where $w(\sigma, a)$ denotes the number of occurrences of a in σ .

pp. 164, l. 5, 6, *two NFAs intersect.* \triangleright the languages of two NFAs intersect.

pp. 164, l. -8, *path fragment π* \triangleright initial path fragment π

pp. 164, l. -6, *TS $\otimes \mathcal{A}$ which has an initial state* \triangleright $TS \otimes \mathcal{A}$ such that there exists an initial state

pp. 167, l. 7, 11, -4, $P_{inv(A)} \triangleright P_{inv(\mathcal{A})}$

pp. 167, l. -2, $q_1, \dots, q_n \notin F \triangleright$ Note: this condition is not necessary.

pp. 168, l. 1, $0 \leq i \leq n \triangleright 0 < i \leq n$

pp. 171, l. 8, *single word* \triangleright a set containing a single word

pp. 177, l. -7, *Example 4.13 on page 161* \triangleright Example 4.14 on page 162

pp. 183, l. -3, -1, $\mathcal{L}_{q_1 q_3} = \dots \triangleright \mathcal{L}_{q_1 q_3} = C^* AB(B + BC^* AB)^*$

pp. 196, l. Example 4.57, *page 193* \triangleright page 194

pp. 200, l. -7, $\bigwedge_{q \in Q} \triangleright \bigwedge_{q \in F}$

pp. 202, l. Fig. 4.22, \triangleright The two states should be labeled s_0 and s_1 , respectively

pp. 203, l. 4, \overline{P} = "eventually forever \neg green \triangleright P = infinitely often green
 pp. 206, l. Proof:, $TS = (S, Act, \rightarrow, I, AP) \triangleright TS = (S, Act, \rightarrow, I, AP, L)$
 pp. 207, l. -4, We now DFS-based cycle checks ... checking \triangleright We now present a DFS-based algorithm for persistence checking that searches backwards edges to check for cycles.
 pp. 212, l. 6, ignores T \triangleright does not revisit the states in T
 pp. 218, l. 10, *Regula r* \triangleright Regular

Chapter 5: Linear Temporal Logic

pp. 230, l. 5, *eventually in the future* \triangleright now or eventually in the future
 pp. 236, l. Figure 5.2, \triangleright It is assumed that $\sigma = A_0A_1A_2\dots$
 pp. 240, l. -10, $\delta_{r_2} = \neg r_1 \triangleright \delta_{r_2} = \neg r_2$
 pp. 241, l. Fig. 5.6, \triangleright Note that the inputs of the r registers are on the right, and their outputs on the left.
 pp. 267, l. 7, *as soon as* \triangleright before
 pp. 270, l. Fig. 5.15, \triangleright The bottom cell should be white and not gray.
 pp. 276, l. -11, $\psi \in B$ if and only if ... \triangleright $\psi \in B$ if and only if ...
 pp. 281, l. 1-5, For $B_0B_1B_2\dots$ a sequence ... we have for all $\psi \in cl(\varphi)$: $\psi \in B_0 \Leftrightarrow A_0A_1A_2\dots \models \psi$ \triangleright For all $\psi \in cl(\varphi)$ and $B_0B_1B_2\dots$ a sequence ... we have: $\psi \in B_0 \Leftrightarrow A_0A_1A_2\dots \models \psi$
 pp. 283, l. 10, $\neq \bigcirc \psi \in B$ if and ... \triangleright $\neg \bigcirc \psi \in B$ if and ...
 pp. 283, l. 17, and $\varphi = \bigcirc a \in B_1, B_2 \triangleright$ and $\varphi = a \in B_1, B_2$
 pp. 284, l. -14, $B_3 B_3 B_1 B_4^\omega \triangleright B_3 B_3 B_1 B_5^\omega$
 pp. 287, l. -5, $|\neg(fair \rightarrow \varphi)| = |fair| + |\varphi| \triangleright |\neg(fair \rightarrow \varphi)| = |\neg(\neg fair \vee \varphi)| = |fair| + |\varphi| + 3$
 pp. 289, l. 11, a new vertex b to G \triangleright a new vertex b to TS
 pp. 292, l. Figure 5.23, \triangleright the self-loop at state $P(n)$ should be omitted
 pp. 292, l. -1, $\bigcirc^{2i-1}(q, A, i) \rightarrow \triangleright$ begin \wedge $\bigcirc^{2i-1}(q, A, i) \rightarrow$
 pp. 294, l. -6, $\mathcal{G}_\varphi \triangleright \mathcal{G}_\varphi$
 pp. 297, l. 7, *Membership to* \triangleright Membership in
 pp. 303, l. Exercise 5.7(b), $W \triangleright Y$ (to avoid confusion with unless)

Chapter 6: Computation Tree Logic

pp. 320, l. -4, *state formula* \triangleright State formula

pp. 327, l. -12, *since* $\exists(\varphi \cup \psi \vee \Box \varphi) \triangleright$ *since* $\forall(\varphi \cup \psi \vee \Box \varphi)$

pp. 333, l. 10, $\neg \exists \Diamond \neg \Phi = \neg \exists(\text{true} \cup \Phi) \triangleright \neg \exists \Diamond \neg \Phi \equiv \neg \exists(\text{true} \cup \neg \Phi)$

pp. 338, l. -5 and -6, \triangleright transitions to s'_{n-1} are non-existing for $n=0$

pp. 342, l. Algorithm 13, and -8 and -4, *maximal genuine* \triangleright maximal proper

pp. 343, l. 4, *subformula of* $\Psi \triangleright$ subformula of Ψ'

pp. 345, l. -2, *Sat*($\exists(\Phi \cup \Psi)$) \triangleright *Sat*($\exists(\Phi \cup \Psi)$)

pp. 345, l. proof of (g)(ii), *Let* $\pi = s_0s_1s_2\dots$ *be a path starting in* $s=s_0$. \triangleright Delete.

pp. 349, l. -9, $(a = c) \wedge (a \neq b) \triangleright (a \leftrightarrow c) \wedge (a \not\leftrightarrow b)$

pp. 351, l. Algorithm 15, \triangleright comments in the first two lines of algorithm need to be swapped while replacing E by T and T by E

pp. 354, l. Example 6.28, *see the gray states* \triangleright Delete.

pp. 354, l. Example 6.28, *Figure 6.13(b), Figure 6.13(c)* \triangleright *Figure 6.13(c), Figure 6.13(d)*

pp. 358, l. 11, \triangleright Note that the length of $\Phi_n \in \mathcal{O}(n!)$

pp. 371, l. -6, *ifstatement* \triangleright if statement

pp. 372, l. Algorithm 19, line 4, $C \cap \text{Sat}(b_j) \neq \emptyset \triangleright C \cap \text{Sat}(b_i) \neq \emptyset$

pp. 378, l. -6, *Eaxmple* \triangleright Example

pp. 383, l. 9 and 10, $\dots z_m \triangleright \dots, z_m$

pp. 386, l. 6, $y_1 \vee y_2 \triangleright y_2 \vee y_1$

pp. 386, l. 6, $y_1 \wedge y_2 \triangleright y_2 \wedge y_1$

pp. 386, l. 13 and 15 (twice), $s\{\bar{y} \leftarrow \bar{z}\} \triangleright s\{\bar{z} \leftarrow \bar{y}\}$

pp. 386, l. 15–17, $f\{\bar{z} \leftarrow \bar{y}\} \triangleright f\{\bar{y} \leftarrow \bar{z}\}$

pp. 387, l. 18, $t\{\bar{x}/\bar{x}'\} \triangleright t\{\bar{x}' \leftarrow \bar{x}\}$

pp. 388, l. 7, $x' \triangleright x'_1$

pp. 388, l. 7, $\bigwedge_{j < i \leq n} (x_j \leftrightarrow x'_j) \triangleright (\neg x_1 \rightarrow x'_1) \wedge \bigwedge_{i < j \leq n} (x_j \leftrightarrow x'_j)$

pp. 388, l. 14–17, \triangleright x and x' should be swapped

pp. 388, l. Example 6.58 (four times), $\{x \leftarrow x'\} \triangleright \{x' \leftarrow x\}$

pp. 390, l. 8, $\exists s' \in S s.t. s' \in Post(s) \triangleright \exists s' \in S. s' \in Post(s)$

pp. 390, l. Algorithm 20, line 4, $f_{j+1}(\bar{x}) := f_{j+1}(\bar{x}) \vee \dots \triangleright f_{j+1}(\bar{x}) := f_j(\bar{x}) \vee \dots$

pp. 391, l. Algorithm 21, line 4, $f_{j+1}(\bar{x}) := f_{j+1}(\bar{x}) \wedge \dots \triangleright f_{j+1}(\bar{x}) := f_j(\bar{x}) \wedge \dots$

pp. 393, l. Figure 6.21 (right), *solid line between z_3 and 0* \triangleright dashed line between z_3 and 0

pp. 396, l. -15, *The semantics* \triangleright The semantics of

pp. 398, l. 9, *left subtree* \triangleright right subtree

pp. 393, l. Figure 6.21, right, *solid line z_3 between 0* \triangleright dashed line z_3 between 0

pp. 405, l. 2, $z_m = a_m, z_m = b_m, \dots, z_i = a_i, z_i = b_i \triangleright z_m = a_m, y_m = b_m, \dots, z_i = a_i, y_i = b_i$

pp. 405, l. 3, $z_m = a_m, z_m = b_m, \dots, z_{i+1} = a_{i+1}, z_{i+1} = b_{i+1}, z_i = a_i \triangleright z_m = a_m, y_m = b_m, \dots, z_{i+1} = a_{i+1}, y_{i+1} = a_{i+1}, z_i = a_i$

pp. 405, l. -4, *As $f \bar{b}$, $\bar{c} \in \{0, 1\}^m$* \triangleright *As \bar{b} , $\bar{c} \in \{0, 1\}^m$*

pp. 409, l. -12, $info(v) = \langle var(v), succ_0(v), succ_0(v) \rangle \triangleright info(v) = \langle var(v), succ_1(v), succ_0(v) \rangle$

pp. 412, l. 7, $u \triangleright v$

pp. 413, l. 13, $f_2 z_1 = b_1, \dots, z_i = b_i \triangleright f_2|_{z_1=b_1, \dots, z_i=b_i}$

pp. 417, l. heading Algorithm 24, $(v, \{\bar{x} \leftarrow \bar{x}'\}) \triangleright (v, \{\bar{x}' \leftarrow \bar{x}\})$

pp. 417, l. Algorithm 24, line 4, *ist* \triangleright is a

pp. 417, l. Algorithm 24, \triangleright replace z by x

pp. 418, l. -6, $f|_{x=\bar{b}} \triangleright f|_{x=b}$

pp. 469, l. Remark 7.19, line 10, $s_2 \models \varphi$, but $s_1 \not\models \varphi \triangleright s_2 \not\models \neg\varphi$, but $s_1 \models \neg\varphi$

Chapter 7: Equivalences and Abstraction

pp. 454, l. 3, *Sssume* \triangleright Assume

pp. 466, l. 8, $H = Act_1 \cap Act_2 \triangleright H = (Act_1 \cap Act_2) \setminus \{\tau\}$

pp. 498, l. Algorithm 32, line 6+7, \triangleright these lines need to be swapped

pp. 513, l. 9, $\{a\} \emptyset \notin Traces(TS_1) \triangleright \{a\} \emptyset \notin Traces(TS_2)$

pp. 518, l. 8, $\forall \Phi \in \forall CTL^* \triangleright \forall \Phi \in \forall CTL$

pp. 519, l. -10, *fragment of CTL** \triangleright fragment of CTL

pp. 528, l. -9, $s_1 \in Pre(s'_2) \triangleright s_1 \in Pre(s'_1)$

pp. 537, l. -5, $\langle c_2, n_1 \rangle \triangleright \langle n_1, c_2 \rangle$

pp. 539, l. 2, \mathcal{R} on $(S_1 \times S_2) \cup (S_1 \times S_2) \triangleright \mathcal{R}$ on $TS_1 \oplus TS_2$

pp. 542, l. 5, $\langle c_2, n_1 \rangle \triangleright \langle n_1, c_2 \rangle$

pp. 546, l. 13, s_2 is \approx_{TS}^{div} -divergent whereas s_0 and s_1 are not. $\triangleright s_2$ is not \approx_{TS}^{div} -divergent whereas s_0 and s_1 are.

pp. 562, l. 1, and $s_1 \exists \varphi \triangleright$ and $s_1 \models \exists \varphi$

pp. 563, l. 4, $\Phi_B \cup \Phi_C$ is a $CTL_{\setminus \Diamond}$ formula $\triangleright \exists(\Phi_B \cup \Phi_C)$ is a $CTL_{\setminus \Diamond}$ formula

pp. 566, l. 16, $\ell_2 : \langle \text{if } (\text{free} > 0) \text{ then } i := 0; \text{free}-- \text{ fi} \rangle \triangleright \ell_2 : \langle \text{if } (\text{free} > 0) \text{ then } i := 0; \text{free}-- \text{ fi} \rangle ; \text{goto } \ell_0$

pp. 566, l. -3, $\langle \ell_0, \ell'_2, 2, 0, 0 \rangle \rightarrow \langle \ell_0, \ell'_2, 2, 0, 0 \rangle \triangleright \langle \ell_1, \ell'_2, 2, 0, 0 \rangle \rightarrow \langle \ell_1, \ell'_0, 2, 0, 0 \rangle$

pp. 569, l. 7, there are some states in B that cannot reach C by only visiting states in B . For such states, the only possibility is to reach C via some other block $D \neq B, C$. $\triangleright C$ can only be reached via paths that entirely go through B .

pp. 569, l. -5, $B \cap \text{Pre}_\Pi^*(C) \triangleright B \cap \text{Pre}(C)$

pp. 572, l. 11, $t \in \text{Exit}(B) \triangleright t \in \text{Bottom}(B)$

pp. 578, l. item 3., self-loops $[s]_{div} \rightarrow [s]_{div} \triangleright$ self-loops $[s] \rightarrow [s]$

Chapter 9: Timed Automata

pp. 699, l. -3, $\forall \Diamond^{>2} \neg on \triangleright \forall \Diamond^{\leq 2} \neg on$

Chapter 10: Probabilistic Systems

pp. 778, l. 4, $\mathbf{P}'(s, t) = \dots \triangleright$

$$\mathbf{P}'(s, t) = \begin{cases} 1 & \text{if } s = t \text{ and } s \in B \cup S \setminus (C \cup B) \\ 0 & \text{if } s \neq t \text{ and } s \in B \cup S \setminus (C \cup B) \\ \mathbf{P}(s, t) & \text{otherwise.} \end{cases}$$

pp. 857, l. 2, $\sum_{s \in S_? \setminus \{s\}} \mathbf{P}(s, \alpha, t) \cdot x_t \triangleright - \sum_{s \in S_? \setminus \{s\}} \mathbf{P}(s, \alpha, t) \cdot x_t$

pp. 870, l. Lemma 10.119, *any* $s \in S \triangleright$ *any* $s \in T$

pp. 876, l. 11, $U_{\Box\Diamond P} \triangleright U_{\Box\Diamond B}$

pp. 903, l. Exercise 10.14, $\varphi = \Box\Diamond a \triangleright \varphi = \Diamond\Box a$

pp. 903/904, l. Exercise 10.17, *Markov chain* $\mathcal{M} \triangleright$ Markov chain \mathcal{M} where all states are equally labeled

pp. 905, l. Exercise 10.22, \triangleright Compute also the values $y_s = \Pr^{\max}(s \models C \cup B)$ with $C = S \setminus \{s_3\}$ and $B = \{s_6\}$

pp. 905, l. Exercise 10.23, (a), 1. and (b) \triangleright (a), (b), (c)

Appendix

pp. 912, l. footnote, $\sigma = A_1 A_2 A_3 \dots \triangleright \sigma = A_0 A_1 A_2 \dots$

pp. 918, l. 8, *not to* 1 \triangleright not to n

pp. 925, l. 1, *they are composed of simple paths* \triangleright they are composed of paths, at least one of which is simple.