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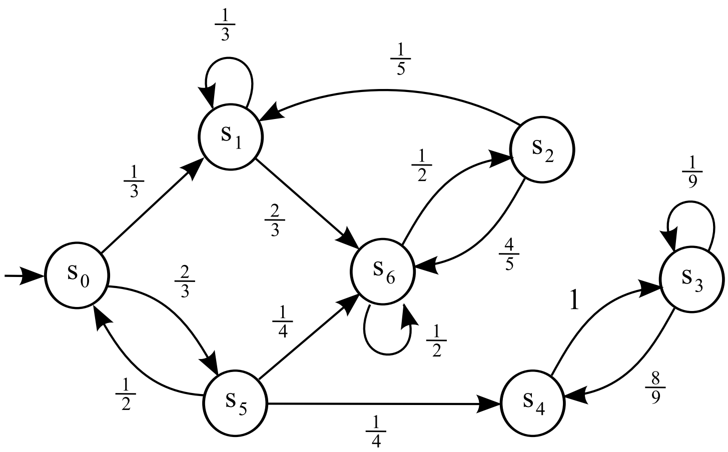
Advanced Model Checking
Summer term 2009

– Series 12 –

Hand in on July 20'th before the exercise class.

Exercise 1 (3 points)

Consider the DTMC below:



Let $A = \{s_3\}$ and $B = \{s_2\}$.

a) Compute the probability measure of the union of the following cylinder sets:
$\text{Cyl}(s_0 s_1)$, $\text{Cyl}(s_0 s_5 s_6)$, $\text{Cyl}(s_0 s_5 s_4 s_3)$, $\text{Cyl}(s_0 s_1 s_6)$

b) Compute the probability, from each state of the Markov chain, of reaching a state in A within 4 steps.

c) Compute the probability, from each state of the Markov chain, of reaching a state in A.

d) What is the probability, from the initial state, of reaching the set of states $A \cup B$?

e) What is the probability, from the initial state, that a state from $A \cup B$ is visited infinitely often?

Exercise 2 (2 points)

Consider a finite DTMC $(S, l_{\text{init}}, \mathbf{P}, L)$ and subsets of states $A, B \subseteq S$. Show that the following two sets of paths are measurable, i.e. contained in the σ-algebra $\Sigma_{Path(l_{\text{init}})}$:

a) the set of paths starting in state l_{init} and remaining forever in states from A;

b) the set of paths starting in state l_{init}, remaining forever in states from A and passing through a state in B after exactly 5 time-steps.

Exercise 3 **(5 points)**

Consider the following simple game between 2 players who, between them, have n coins. Initially, player 1 has m of these coins. In each turn of the game, both players simultaneously toss one of their coins. If the two coins are the same, player 1 keeps both coins; if they differ, player 2 keeps them. The game ends when one player has all the coins and is declared the winner.

a) Draw a DTMC to represent the evolution of this game.

b) What is the probability of the game terminating?

c) Assume we wish to establish if "player 1 has a better chance of winning than player 2". Express this statement using PCTL.

d) Express the following statement in PCTL: "with probability at most 0.1, player 1 will, within 5 more turns, be in a position where he has a chance to win the game in the next turn".