Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps ⟵
- simulation relations

Divergent states

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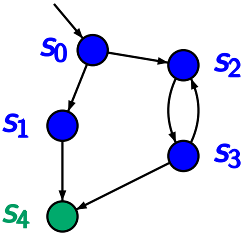

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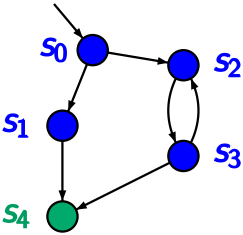

stutter equivalence classes:

$\{s_0, s_1, s_2, s_3\}$ $\{s_4\}$

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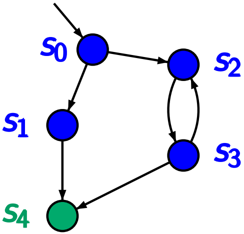

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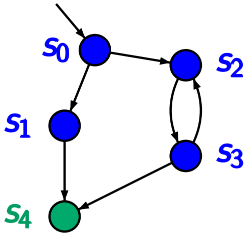

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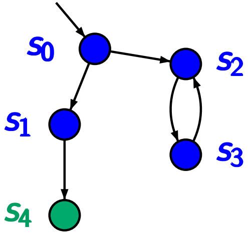

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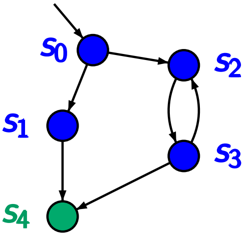

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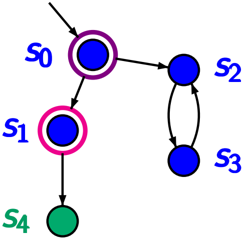

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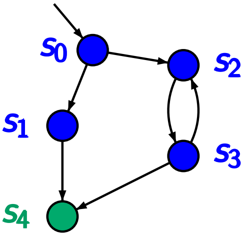

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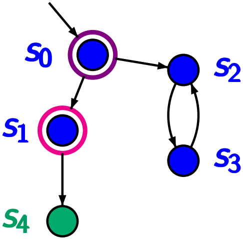

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Divergence-sensitivity

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$\mathcal{T}$ is called *divergence-sensitive* if for all states s_1 and s_2 in $\mathcal{T}$:

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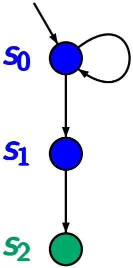

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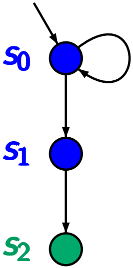

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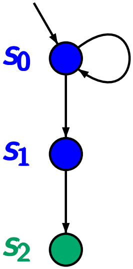

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s_0 is $\approx_{\mathcal{T}}$-divergent,
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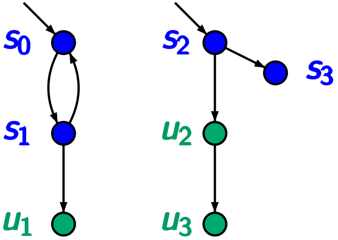
Example: divergence-sensitivity



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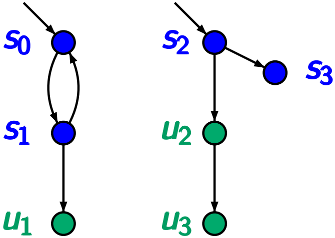

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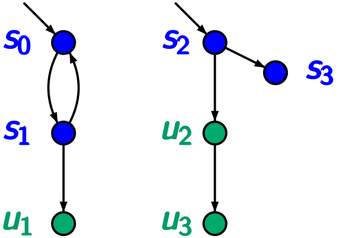

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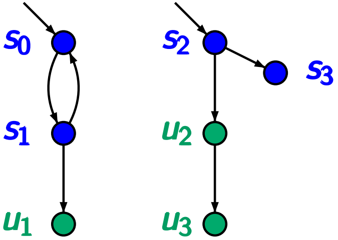

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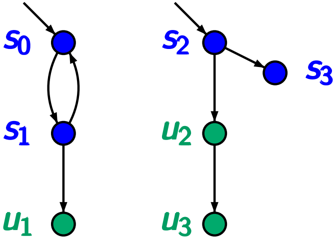

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If $\mathcal{T}$ is finite and divergence-sensitive then for all states s_1, s_2 and $\mathbf{CTL^*}_{\setminus\bigcirc}$ formulas Φ:

$$\text{if } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \models \Phi \text{ then } s_2 \models \Phi$$

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to prove this we show:

stutter bisimulation equivalence with divergence $=$ $\mathbf{CTL}^*_{\setminus\bigcirc}$ equivalence

Let $\mathcal{T}$ be a transition system with state space S and $\mathcal{R}$ an equivalence relation on S.

Divergence-sensitive equivalences

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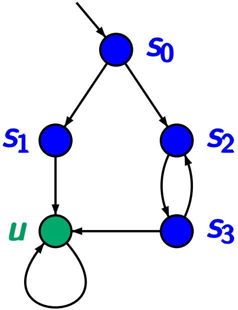

● $\widehat{=} \{a\}$

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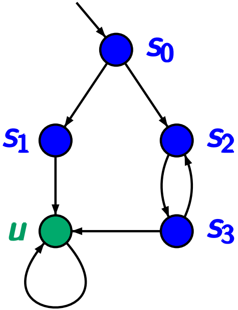

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(quotients of) equivalences:

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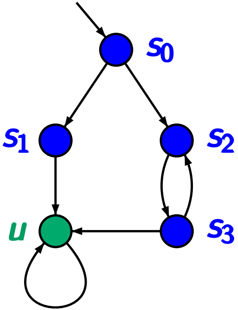

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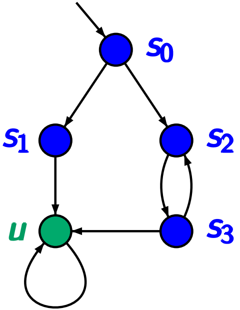

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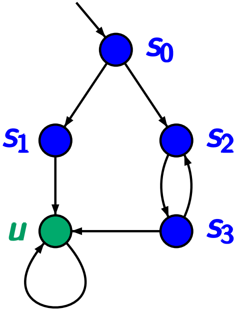

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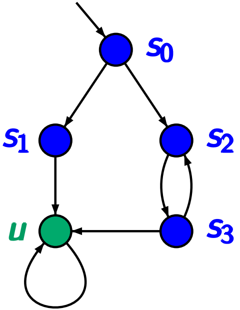

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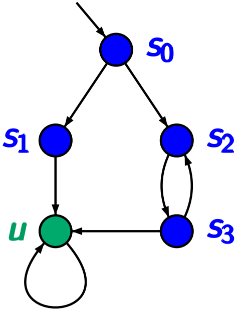

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Stutter bisimulation with divergence

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$s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ iff there exists an equivalence $\mathcal{R}$ on S that is a divergence-sensitive stutter bisimulation for $\mathcal{T}$ with $(s_1, s_2) \in \mathcal{R}$

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$\approx_{\mathcal{T}}^{\text{div}}$ is an equivalence relation on S and the coarsest divergence-sensitive stutter bisimulation for $\mathcal{T}$

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$\approx_{\mathcal{T}}^{\mathsf{div}}$ = coarsest equivalence on the state space S of $\mathcal{T}$ such that for all $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$:

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(2)
$$
\begin{tikzcd}
s_1 \arrow[d] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \\
s_1' & &
\end{tikzcd}
\quad \rightsquigarrow \quad
\begin{tikzcd}
s_1 \arrow[dddd] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \arrow[d] & \\
 & & u_1 \arrow[d] & \approx_{\mathcal{T}}^{\mathsf{div}} \\
 & & \ldots \arrow[d] & \\
 & & u_n \arrow[d] & \\
s_1' & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2' &
\end{tikzcd}
$$

with $s_1' \not\approx_{\mathcal{T}}^{\mathsf{div}} s_1$

Stutter bis. equivalence with divergence

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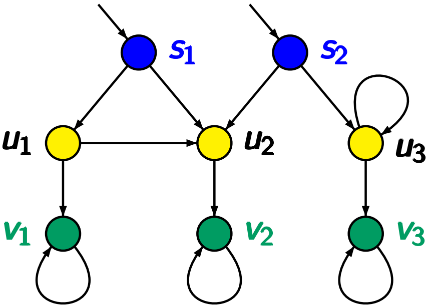
with $s_1' \not\approx^{\mathsf{div}}_{\mathcal{T}} s_1$

(3) s_1 $\approx^{\mathsf{div}}_{\mathcal{T}}$-divergent iff s_2 $\approx^{\mathsf{div}}_{\mathcal{T}}$-divergent

Example: $\approx_{\mathcal{T}}$ vs. $\approx_{\mathcal{T}}^{\text{div}}$



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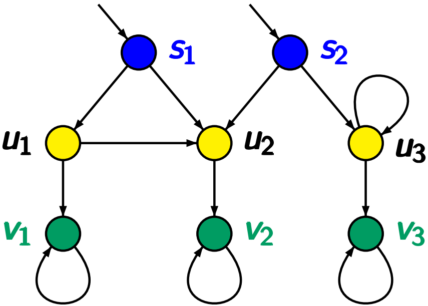

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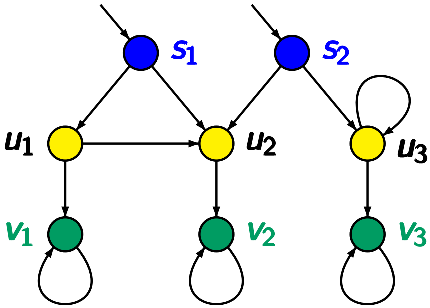

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stutter bisimulation equiv. classes with divergence:

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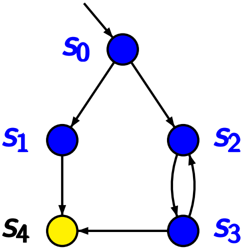
Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathsf{div}}$



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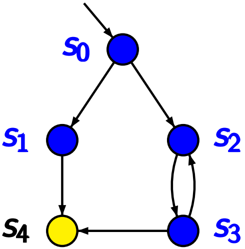
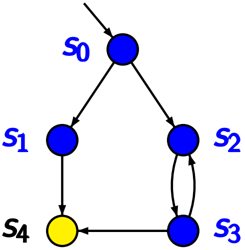


stutter bis. equivalence classes:

$$S/\approx_{\mathcal{T}} = \{\, \{s_0, s_1, s_2, s_3\}, \{s_4\} \,\}$$

Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathsf{div}}$



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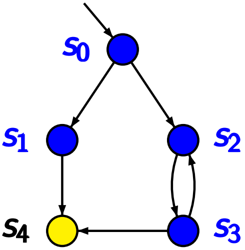
Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathbf{div}}$



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stutter bis. equiv. classes with **div.**:

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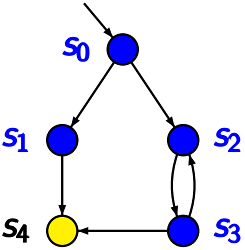
Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\text{div}}$



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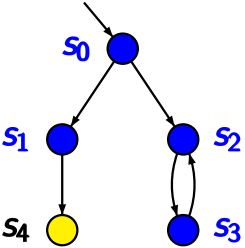
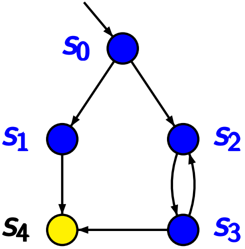


Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathsf{div}}$



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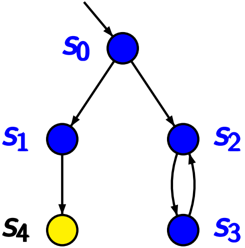

stutter bis. equivalence classes

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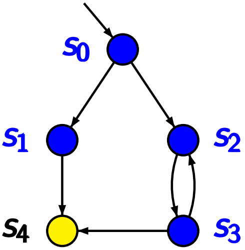

stutter bis. equivalence classes:

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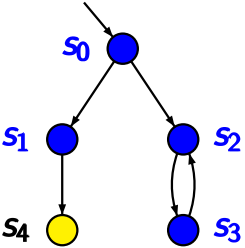

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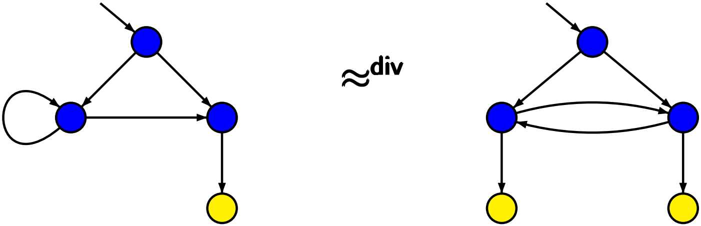
Correct or wrong?



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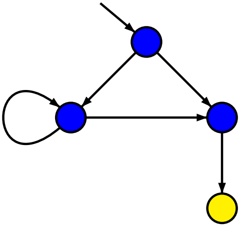
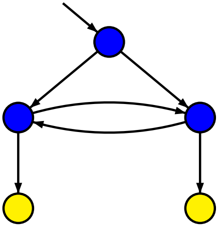
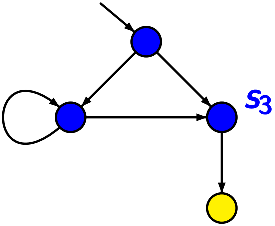


$\approx^{\mathsf{div}}$



wrong



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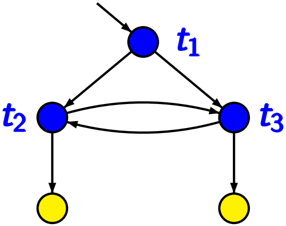
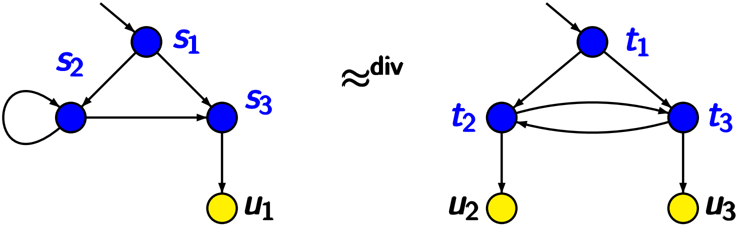


wrong

because s_3 is not divergent,
while t_1, t_2, t_3 are divergent

Correct or wrong?



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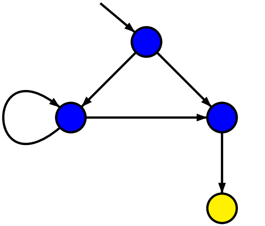
$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes:

$$\big\{\{t_1, t_2, t_3\},\ \{s_1, s_2\},\ \{s_3\},\ \{u_1, u_2, u_3\}\big\}$$

Correct or wrong?



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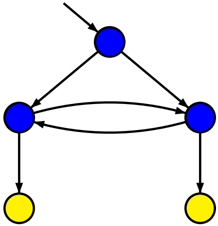

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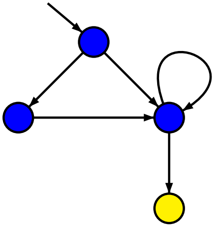

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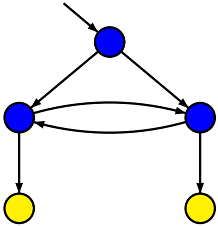

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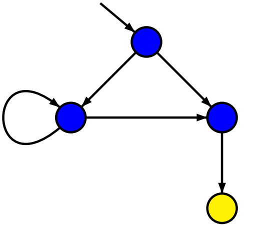

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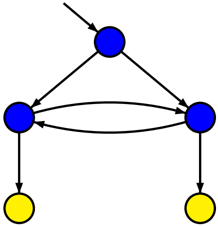

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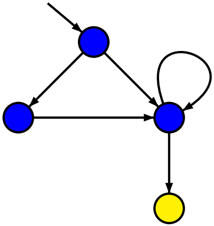

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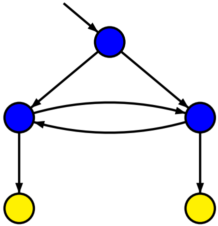

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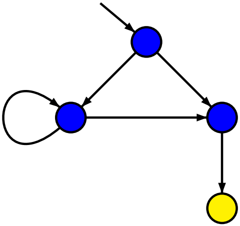

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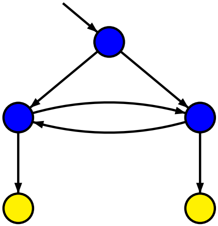
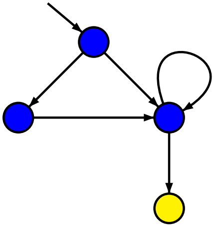


wrong



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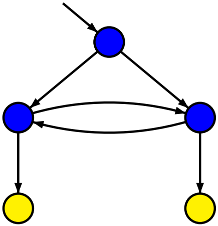
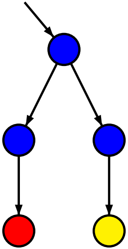


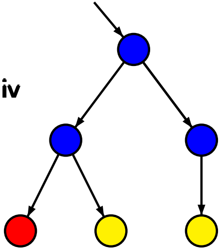
correct

all blue states are $\approx^{\mathsf{div}}$-equivalent and divergent

Correct or wrong?



$\approx^{\mathsf{div}}$



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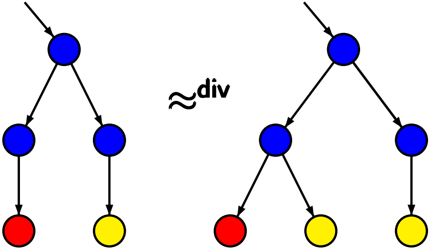

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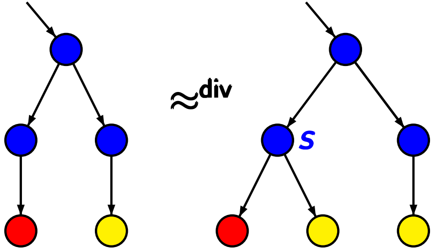
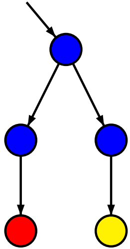


wrong

even not $\approx$-equivalent, since s has no equivalent state

Correct or wrong?



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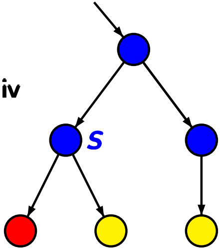

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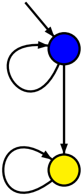

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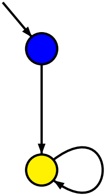

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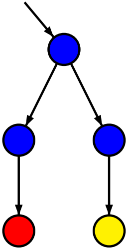

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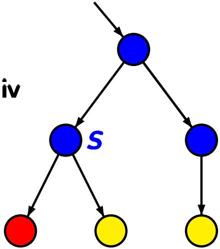

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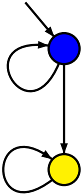

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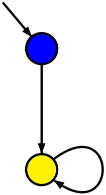

wrong

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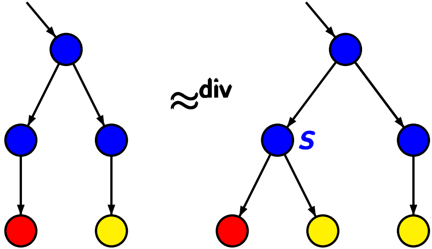


$\approx^{\text{div}}$ **wrong**

even not $\approx$-equivalent, since s has no equivalent state

$\approx^{\text{div}}$ **wrong**

$s_1 \not\approx^{\text{div}} s_2$, as s_1 is $\approx^{\text{div}}$-divergent, while s_2 is not

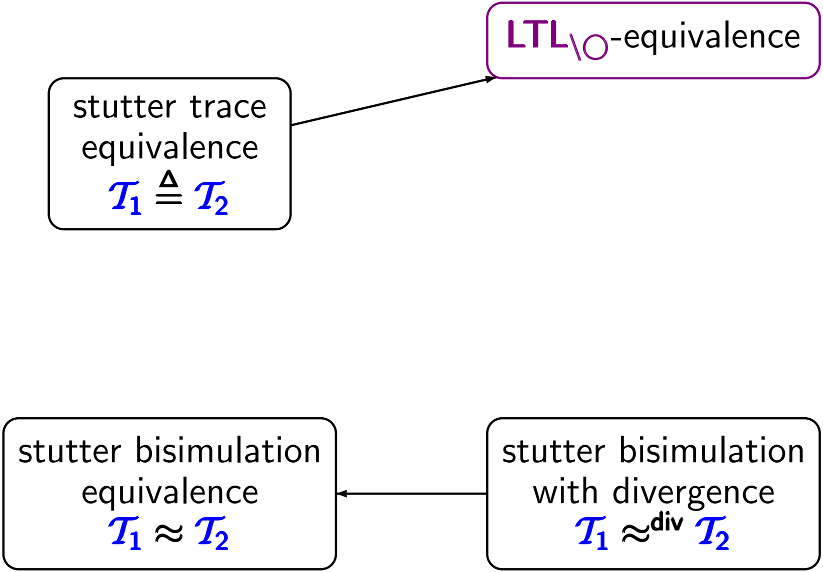
Stutter abstract equivalences

stutter trace equivalence
$\mathcal{T}_1 \triangleq \mathcal{T}_2$

stutter bisimulation equivalence
$\mathcal{T}_1 \approx \mathcal{T}_2$

stutter bisimulation with divergence
$\mathcal{T}_1 \approx^{\mathsf{div}} \mathcal{T}_2$

Stutter abstract equivalences



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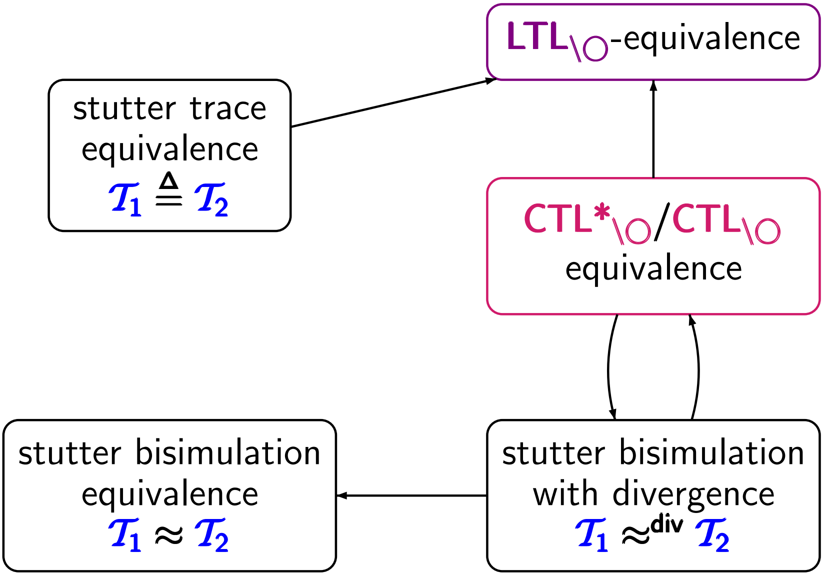

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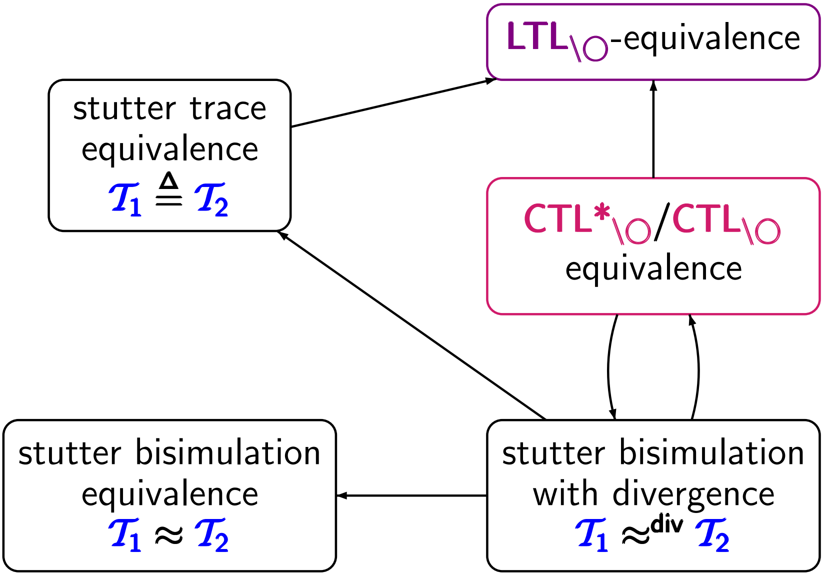

Logical characterization of $\approx^{\mathsf{div}}$

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(1) $s_1 \approx^{\mathsf{div}}_{\mathcal{T}} s_2$

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$\mathsf{CTL}^*_{\setminus \bigcirc}$ $=$ CTL^* without next operator $\bigcirc$

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Proof outline

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Let $\mathcal{T}$ be a finite TS and s_1, s_2 states in $\mathcal{T}$. Then:

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iff s_1, s_2 satisfy the same $\text{CTL}^*_{\setminus\bigcirc}$ formulas

iff s_1, s_2 satisfy the same $\text{CTL}_{\setminus\bigcirc}$ formulas

$$\begin{tikzcd}
& \begin{array}{c}\text{stutter bisimulation equivalence}\\ \approx^{\text{div}}_{\mathcal{T}} \text{ with divergence}\end{array} \arrow[dl] & \\
\text{CTL}^*_{\setminus\bigcirc}\text{-equivalence} \arrow[rr] & & \text{CTL}_{\setminus\bigcirc}\text{-equivalence} \arrow[ul]
\end{tikzcd}$$

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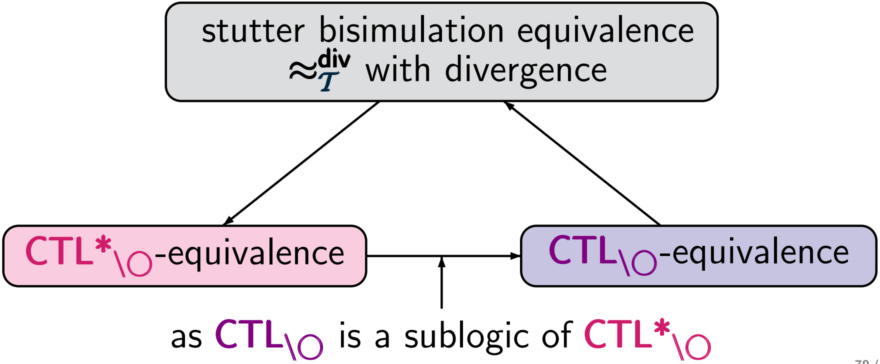

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stutter5.4-28-proof

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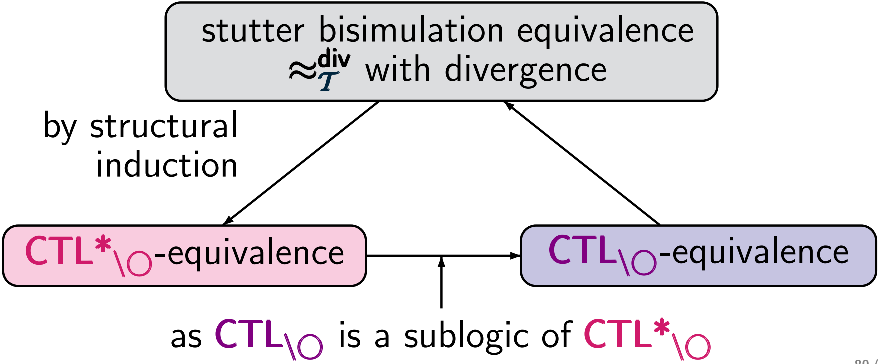

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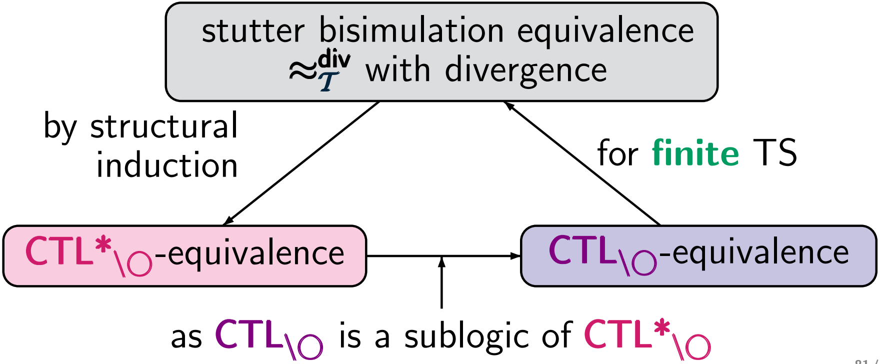

Proof outline

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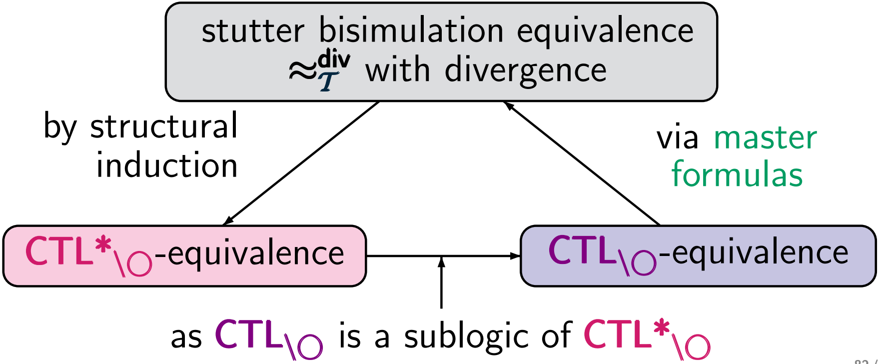

$\mathsf{CTL}_{\setminus\bigcirc}$-equivalence is finer than $\approx^{\mathsf{div}}_{\mathcal{T}}$

CTL$_{\setminus \bigcirc}$-equivalence is finer than $\approx_{\mathcal{T}}^{\textsf{div}}$

For finite transition system $\mathcal{T}$:

$$\mathcal{R} = \left\{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textsf{CTL}_{\setminus \bigcirc} \text{ formulas} \right\}$$

is a divergence-sensitive stutter bisimulation.

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proof uses $\text{CTL}_{\setminus\bigcirc}$ master formulas for $\approx^{\text{div}}_{\mathcal{T}}$-equivalence classes

$\mathrm{CTL}_{\setminus\bigcirc}$-equivalence is finer than $\approx^{\mathrm{div}}_{\mathcal{T}}$

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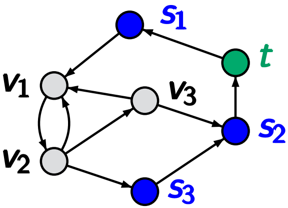

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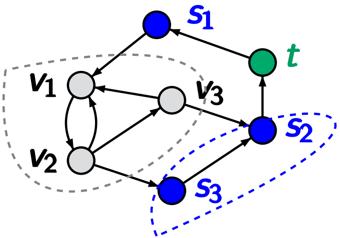

● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$ ● $\widehat{=} \varnothing$

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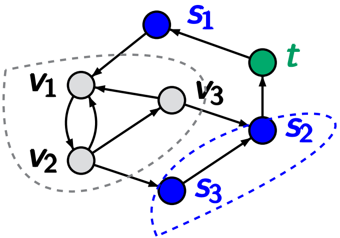

master formulas:

$v_1, v_2, v_3 \models \neg a \wedge \neg b$

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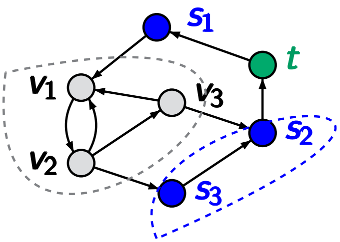

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$v_1, v_2, v_3 \models \neg a \wedge \neg b$

$t \models \neg a \wedge b$

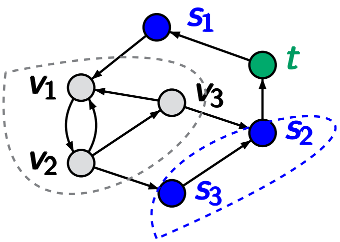
● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$ ● $\widehat{=} \varnothing$

CTL$_{\setminus\bigcirc}$-equivalence is finer than $\approx^{\mathrm{div}}_{\mathcal{T}}$

For finite transition system $\mathcal{T}$:

$$\mathcal{R} = \{(s_1, s_2) : s_1, s_2 \text{ satisfy the same } \mathrm{CTL}_{\setminus\bigcirc} \text{ formulas}\}$$

is a divergence-sensitive stutter bisimulation.

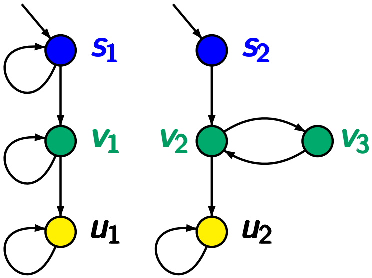


master formulas:

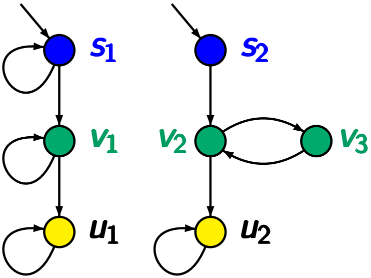
$$\begin{array}{rl} v_1, v_2, v_3 & \models \neg a \wedge \neg b \\ t & \models \neg a \wedge b \\ s_2, s_3 & \models a \wedge \exists(a \,\mathsf{U}\, b) \\ s_1 & \models a \wedge \neg\exists(a \,\mathsf{U}\, b) \end{array}$$

● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$ ● $\widehat{=}\varnothing$

CTL$_{\setminus\bigcirc}$ master formulas



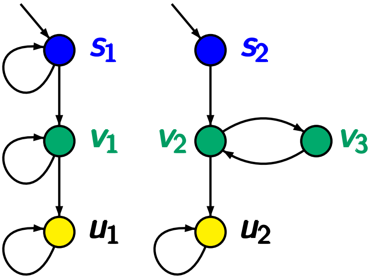
$CTL_{\setminus\bigcirc}$ master formulas



equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathsf{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$

$CTL_{\setminus\bigcirc}$ master formulas

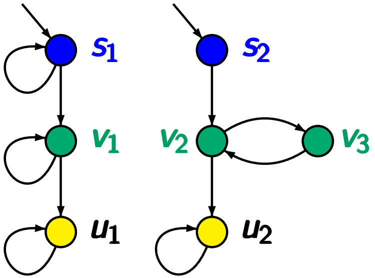


$u_1, u_2 \models \neg \mathit{blue} \wedge \neg \mathit{green}$

equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathsf{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$

$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas

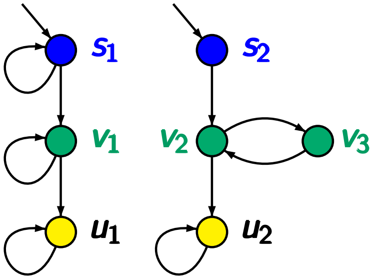


$v_1, v_2, v_3 \models green$

$u_1, u_2 \models \neg blue \wedge \neg green$

equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathrm{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$



$s_1 \models \exists\Box blue$

$s_2 \models blue \wedge \neg\exists\Box blue$

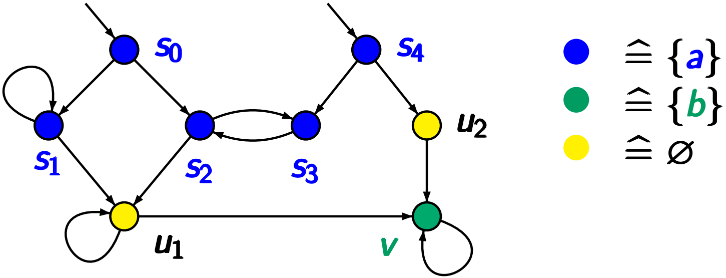
$v_1, v_2, v_3 \models green$

$u_1, u_2 \models \neg blue \wedge \neg green$

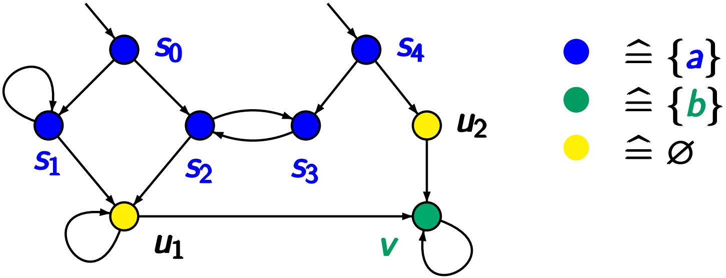
equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathsf{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$

CTL$_{\setminus\bigcirc}$ master formulas

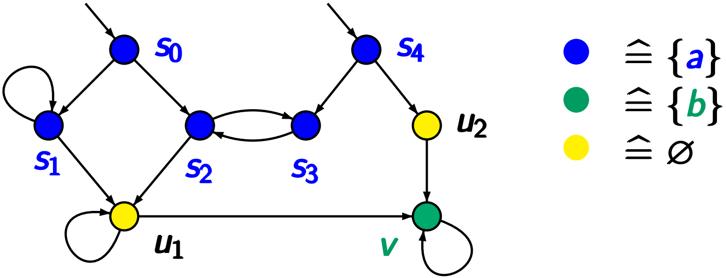


CTL$_{\setminus \bigcirc}$ master formulas



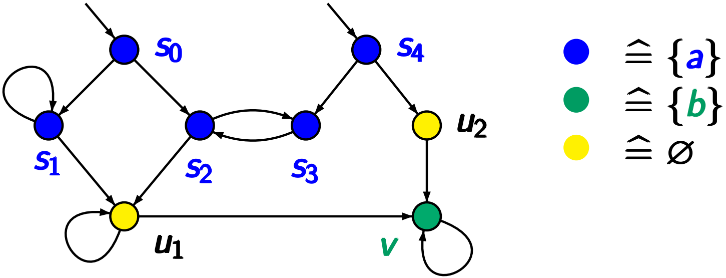
$\approx_{\mathcal{T}}^{\mathsf{div}}$-equiv. classes	
$\{v\}$	
$\{u_1\}$	
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas

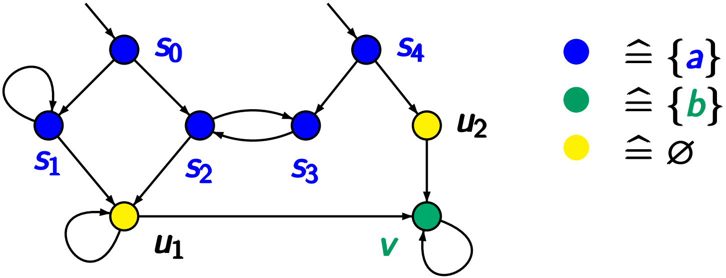


$\approx^{\mathrm{div}}_{\mathcal{T}}$-equiv. classes	$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

$CTL_{\setminus\bigcirc}$ master formulas

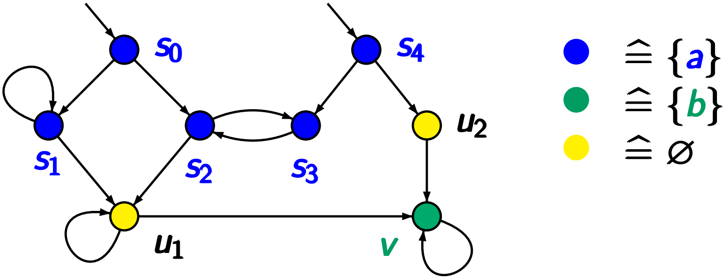


$\approx^{div}_{\mathcal{T}}$-equiv. classes	$CTL_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	



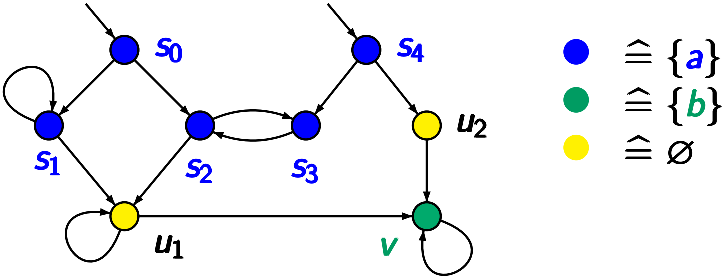
$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes	CTL$_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

$CTL_{\setminus\bigcirc}$ master formulas



$\approx^{\text{div}}_{\mathcal{T}}$-equiv. classes	$CTL_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	$a \wedge \quad \forall(a \,\mathsf{W}\, \exists\Box(\neg a \wedge \neg b))$
$\{s_4\}$	

$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas



$\approx^{\mathrm{div}}_{\mathcal{T}}$-equiv. classes	$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	$a \wedge \forall(a \,\mathsf{W}\, \exists\Box(\neg a \wedge \neg b))$
$\{s_4\}$	$a \wedge \neg\forall(a \,\mathsf{W}\, \exists\Box(\neg a \wedge \neg b))$

$\approx^{\mathsf{div}}$ and CTL*$_{\setminus\bigcirc}$/CTL$_{\setminus\bigcirc}$-equivalence

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Let s_1, s_2 be states of a finite TS without terminal states. Then, the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$

(2) s_1, s_2 satisfy the same CTL*$_{\setminus\bigcirc}$ formulas

(3) s_1, s_2 satisfy the same CTL$_{\setminus\bigcirc}$ formulas

$\approx^{\mathsf{div}}$ and CTL*$_{\setminus \bigcirc}$/CTL$_{\setminus \bigcirc}$-equivalence

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$(2) \Longrightarrow (3)$: clear as CTL$_{\setminus \bigcirc}$ is a sublogic of CTL*$_{\setminus \bigcirc}$

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(3) s_1, s_2 satisfy the same $\mathsf{CTL}_{\setminus \bigcirc}$ formulas

$(2) \Longrightarrow (3)$: clear as $\mathsf{CTL}_{\setminus \bigcirc}$ is a sublogic of $\mathsf{CTL}^*_{\setminus \bigcirc}$

$(3) \Longrightarrow (1)$: $\mathsf{CTL}_{\setminus \bigcirc}$-equivalence is a divergence-sensitive stutter abstract bisimulation

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$(1) \Longrightarrow (2)$: proof by structural induction

$\approx^{\mathrm{div}}$ and $CTL^*_{\setminus\bigcirc}$/$CTL_{\setminus\bigcirc}$-equivalence

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(1) $s_1 \approx^{\mathrm{div}}_{\mathcal{T}} s_2$

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$(1) \Longrightarrow (2)$: proof by structural induction

$\approx^{\mathsf{div}}_{\mathcal{T}}$ for paths

Recall: $\sim_{\mathcal{T}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

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$\pi_1 \sim_{\mathcal{T}} \pi_2$ iff π_1 and π_2 are statewise bisimilar

Recall: $\sim_{\mathcal{T}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

$\pi_1 \sim_{\mathcal{T}} \pi_2$ iff π_1 and π_2 are statewise bisimilar, i.e., if

$$\begin{aligned} \pi_1 &= s_{1,0}\, s_{1,1}\, s_{1,2}\, s_{1,3} \ldots \\ \pi_2 &= s_{2,0}\, s_{2,1}\, s_{2,2}\, s_{2,3} \ldots \end{aligned}$$

s.t. $s_{1,i} \sim_{\mathcal{T}} s_{2,i}$ for all $i \geq 0$

Recall: $\sim_{\mathcal{T}}$ for paths

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$$\pi_1 = s_{1,0}\, s_{1,1}\, s_{1,2}\, s_{1,3} \ldots$$

$$\pi_2 = s_{2,0}\, s_{2,1}\, s_{2,2}\, s_{2,3} \ldots$$

s.t. $s_{1,i} \sim_{\mathcal{T}} s_{2,i}$ for all $i \geq 0$

analogous definition for $\approx^{\mathsf{div}}_{\mathcal{T}}$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

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$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ iff there exist infinite sequences

$C_0, C_1, C_2, \ldots$

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$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ iff there exist infinite sequences

$$C_0, C_1, C_2, \ldots \quad \in (S/\approx_{\mathcal{T}}^{\mathsf{div}})^\omega$$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ iff there exist infinite sequences

$$\begin{array}{ll} C_0, C_1, C_2, \ldots & \in (S/\approx_{\mathcal{T}}^{\mathsf{div}})^{\omega} \\ n_0, n_1, n_2, \ldots & \in \mathbb{N}_{\geq 1}^{\omega} \\ m_0, m_1, m_2, \ldots & \in \mathbb{N}_{\geq 1}^{\omega} \end{array}$$

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ iff there exist infinite sequences

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such that

$$\pi_1 = \overbrace{s_{1,1} \; \ldots \; s_{1,n_0}}^{\in C_0} \overbrace{t_{1,1} \ldots t_{1,n_1}}^{\in C_1} \overbrace{u_{1,1} \ldots u_{1,n_2}}^{\in C_2} \ldots$$

$$\pi_2 = \underbrace{s_{2,1} \ldots s_{2,m_0}}_{\in C_0} \underbrace{t_{2,1} \; \ldots \; t_{2,m_1}}_{\in C_1} \underbrace{u_{2,1} \; \ldots \; u_{2,m_2}}_{\in C_2} \ldots$$

Stutter relations for paths

Stutter relations for paths

stutter trace equivalence: $\pi_1 \triangleq \pi_2$

π_1	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$
π_2	$s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	$A_0 \qquad A_1 \qquad A_2 \quad \ldots$

where $A_0, A_1, A_2, \ldots$ are subsets of AP

Stutter relations for paths

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π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	A_0 A_1 A_2 $\ldots$

where $A_0, A_1, A_2, \ldots$ are subsets of AP

stutter bis. equiv. with divergence: $\pi_1 \approx^{\text{div}}_{\mathcal{T}} \pi_2$

π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
equiv.class	C_0 C_1 C_2 $\ldots$

where $C_0, C_1, C_2, \ldots$ are $\approx^{\text{div}}_{\mathcal{T}}$-equivalence classes

Stutter relations for paths

stutter trace equivalence: $\pi_1 \triangleq \pi_2$

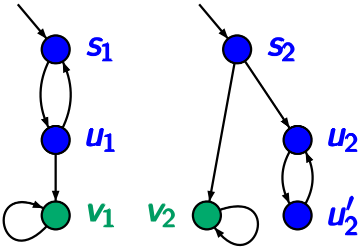
π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	A_0 A_1 A_2 $\ldots$

stutter bis. equiv. with divergence: $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$

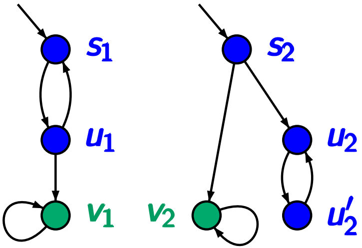
π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
equiv.class	C_0 C_1 C_2 $\ldots$

If $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ then $\pi_1 \triangleq \pi_2$.

Example: $\approx^{\text{div}}_{\mathcal{T}}$ for paths

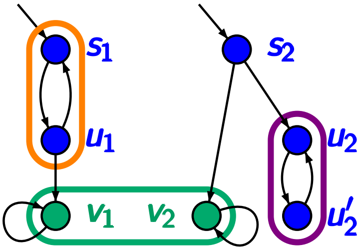


Example: $\approx_{\mathcal{T}}^{\text{div}}$ for paths



$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$

Example: $\approx_{\mathcal{T}}^{\text{div}}$ for paths



$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes:

$\{s_1, u_1\}$, $\{s_2\}$
$\{u_2, u_2'\}$, $\{v_1, v_2\}$

$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
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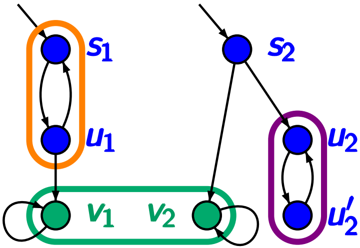

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$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
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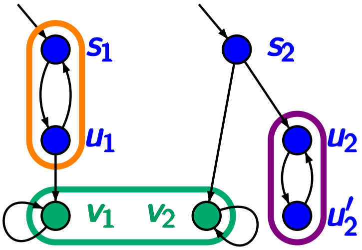
$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$

Example: $\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths



$\approx_{\mathcal{T}}^{\mathsf{div}}$-equiv. classes:

$\{s_1, u_1\}$, $\{s_2\}$
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$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
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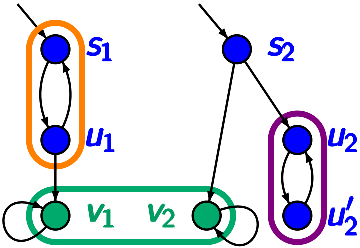
$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$

$$\pi_1' = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2' = s_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ \ldots$$

Example: $\approx_{\mathcal{T}}^{\mathbf{div}}$ for paths



$\approx_{\mathcal{T}}^{\mathbf{div}}$-equiv. classes:

$\{s_1, u_1\}$, $\{s_2\}$

$\{u_2, u_2'\}$, $\{v_1, v_2\}$

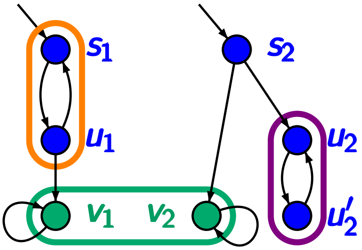
$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$

$$\pi_1 \approx_{\mathcal{T}}^{\mathbf{div}} \pi_2$$

$$\pi_1' = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2' = s_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ \ldots$$

$$\pi_1' \triangleq \pi_2',$$

Example: $\approx^{\text{div}}_{\mathcal{T}}$ for paths



$\approx^{\text{div}}_{\mathcal{T}}$-equiv. classes:

$\{s_1, u_1\}$, $\{s_2\}$
$\{u_2, u_2'\}$, $\{v_1, v_2\}$

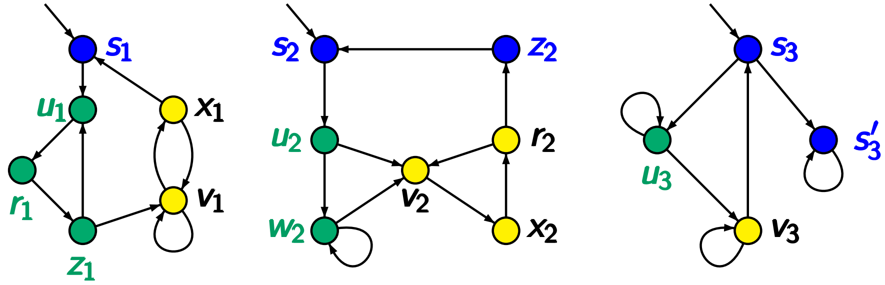
$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$

$\pi_1 \approx^{\text{div}}_{\mathcal{T}} \pi_2$

$$\pi_1' = \boxed{s_1\ u_1\ s_1\ u_1}\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2' = \boxed{s_2}\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ \ldots$$

$\pi_1' \triangleq \pi_2'$,
$\pi_1' \not\approx^{\text{div}}_{\mathcal{T}} \pi_2'$

For which indices i, j, does $\pi_i \approx^{\text{div}}_{\mathcal{T}} \pi_j$ hold?

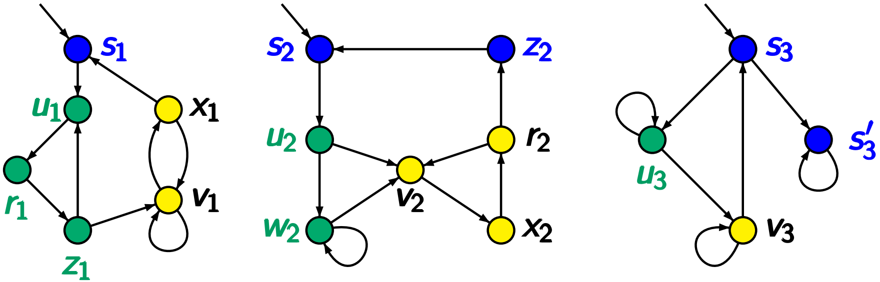


$\pi_1 = \; s_1\; u_1\; r_1\; z_1\; x_1\; x_1\; x_1\; v_1\; s_1\; u_1\; r_1\; z_1\; x_1\; x_1\; x_1\; x_1 \ldots$

$\pi_2 = \; s_2\; u_2\; w_2\; w_2\; w_2\; v_2\; x_2\; r_2\; v_2\; x_2\; r_2\; z_2\; s_2\; u_2\; v_2\; x_2 \ldots$

$\pi_3 = \; s_3\; u_3\; u_3\; u_3\; u_3\; v_3\; v_3\; v_3\; v_3\; v_3\; v_3\; s_3\; u_3\; u_3\; v_3\; v_3 \ldots$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\mathsf{div}} \pi_j$ hold?



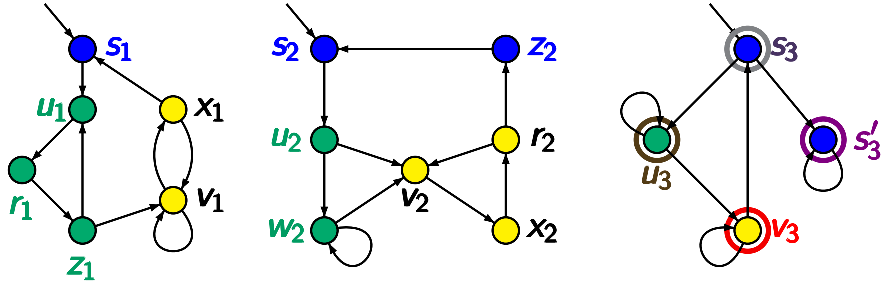
$\pi_1 = \; s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ v_1\ s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ x_1 \ldots$

$\pi_2 = \; s_2\ u_2\ w_2\ w_2\ w_2\ v_2\ x_2\ r_2\ v_2\ x_2\ r_2\ z_2\ s_2\ u_2\ v_2\ x_2 \ldots$

$\pi_3 = \; s_3\ u_3\ u_3\ u_3\ u_3\ v_3\ v_3\ v_3\ v_3\ v_3\ v_3\ s_3\ u_3\ u_3\ v_3\ v_3 \ldots$

$\approx_{\mathcal{T}}^{\mathsf{div}}$-equivalence classes: ?

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\mathrm{div}} \pi_j$ hold?



$\pi_1 = \; s_1 \; u_1 \; r_1 \; z_1 \; x_1 \; x_1 \; x_1 \; v_1 \; s_1 \; u_1 \; r_1 \; z_1 \; x_1 \; x_1 \; x_1 \; x_1 \ldots$

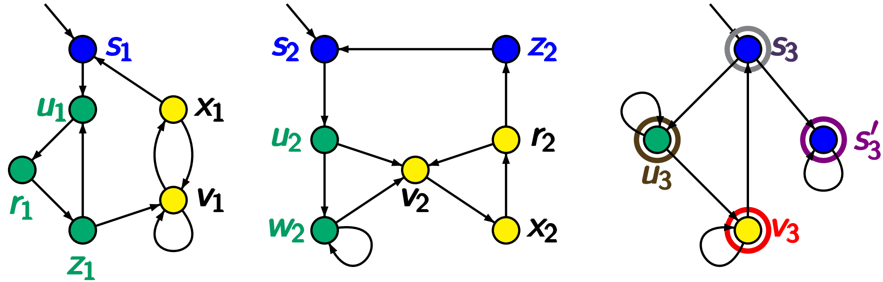
$\pi_2 = \; s_2 \; u_2 \; w_2 \; w_2 \; w_2 \; v_2 \; x_2 \; r_2 \; v_2 \; x_2 \; r_2 \; z_2 \; s_2 \; u_2 \; v_2 \; x_2 \ldots$

$\pi_3 = \; s_3 \; u_3 \; u_3 \; u_3 \; u_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; s_3 \; u_3 \; u_3 \; v_3 \; v_3 \ldots$

$\approx_{\mathcal{T}}^{\mathrm{div}}$-equivalence classes: $\boxed{\{s_1, s_2, z_2\}}$ $\boxed{\{s_3\}}$ $\boxed{\{s'_3\}}$ $\boxed{\{u_3\}}$

$\boxed{\{u_1, r_1, z_1, u_2, w_2\}}$ $\boxed{\{v_1, x_1, v_2, x_2, r_2\}}$ $\boxed{\{v_3\}}$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\mathsf{div}} \pi_j$ hold?



$\pi_1 = \boxed{s_1}\ \boxed{u_1\ r_1\ z_1}\ \boxed{v_1\ v_1\ v_1\ x_1}\ \boxed{s_1}\ \boxed{u_1\ r_1\ z_1}\ \boxed{v_1\ v_1\ v_1\ x_1 \ldots}$

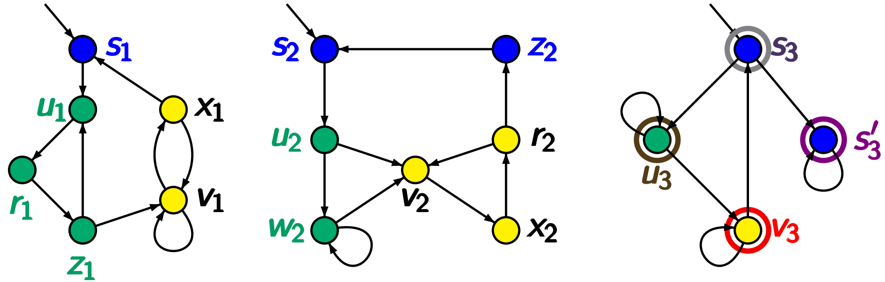
$\pi_2 = \boxed{s_2}\ \boxed{u_2\ w_2\ w_2\ w_2}\ \boxed{v_2\ x_2\ r_2\ v_2\ x_2\ r_2}\ \boxed{z_2\ s_2}\ \boxed{u_2}\ \boxed{v_2\ x_2 \ldots}$

$\pi_3 = s_3\ u_3\ u_3\ u_3\ u_3\ v_3\ v_3\ v_3\ v_3\ v_3\ v_3\ s_3\ u_3\ u_3\ v_3\ v_3 \ldots$

$\approx_{\mathcal{T}}^{\mathsf{div}}$-equivalence classes: $\boxed{\{s_1, s_2, z_2\}}$ $\boxed{\{s_3\}}$ $\boxed{\{s_3'\}}$ $\boxed{\{u_3\}}$

$\boxed{\{u_1, r_1, z_1, u_2, w_2\}}$ $\boxed{\{v_1, x_1, v_2, x_2, r_2\}}$ $\boxed{\{v_3\}}$

For which indices i, j, does $\pi_i \approx^{\text{div}}_{\mathcal{T}} \pi_j$ hold?



$\pi_1 = \boxed{s_1}\ \boxed{u_1\ r_1\ z_1}\ \boxed{v_1\ v_1\ v_1\ x_1}\ \boxed{s_1}\ \boxed{u_1\ r_1\ z_1}\ \boxed{v_1\ v_1\ v_1\ x_1 \ldots}$

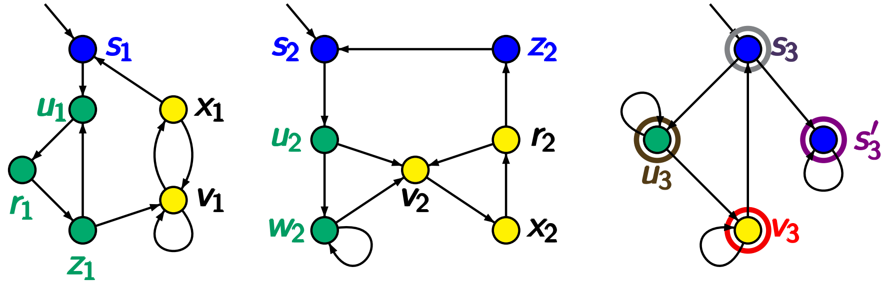
$\pi_2 = \boxed{s_2}\ \boxed{u_2\ w_2\ w_2\ w_2}\ \boxed{v_2\ x_2\ r_2\ v_2\ x_2\ r_2}\ \boxed{z_2\ s_2}\ \boxed{u_2}\ \boxed{v_2\ x_2 \ldots}$

$\pi_3 = \boxed{s_3}\ \boxed{u_3\ u_3\ u_3\ u_3}\ \boxed{v_3\ v_3\ v_3\ v_3\ v_3\ v_3}\ \boxed{s_3}\ \boxed{u_3\ u_3}\ \boxed{v_3\ v_3 \ldots}$

$\approx^{\text{div}}_{\mathcal{T}}$-equivalence classes: $\boxed{\{s_1, s_2, z_2\}}$ $\boxed{\{s_3\}}$ $\boxed{\{s_3'\}}$ $\boxed{\{u_3\}}$

$\boxed{\{u_1, r_1, z_1, u_2, w_2\}}$ $\boxed{\{v_1, x_1, v_2, x_2, r_2\}}$ $\boxed{\{v_3\}}$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\text{div}} \pi_j$ hold?



$\pi_1 = s_1 \; u_1 \; r_1 \; z_1 \; v_1 \; v_1 \; v_1 \; x_1 \; s_1 \; u_1 \; r_1 \; z_1 \; v_1 \; v_1 \; v_1 \; x_1 \ldots$

$\pi_2 = s_2 \; u_2 \; w_2 \; w_2 \; w_2 \; v_2 \; x_2 \; r_2 \; v_2 \; x_2 \; r_2 \; z_2 \; s_2 \; u_2 \; v_2 \; x_2 \ldots$

$\pi_3 = s_3 \; u_3 \; u_3 \; u_3 \; u_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; s_3 \; u_3 \; u_3 \; v_3 \; v_3 \ldots$

$$\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2, \quad \text{but} \quad \pi_1, \pi_2 \not\approx_{\mathcal{T}}^{\text{div}} \pi_3$$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

$$\begin{array}{ccc} s_1 & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \\ \downarrow & & \\ s_{11} & & \end{array}$$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

$$
\begin{tikzcd}
 & s_1 \arrow[d] \arrow[r, phantom, "\approx_{\mathcal{T}}^{\mathsf{div}}"] & s_2 \\
s_1 \arrow[r, phantom, "\not\approx_{\mathcal{T}}^{\mathsf{div}}"] & s_{11} &
\end{tikzcd}
$$

Path lifting for $\approx_{\mathcal{T}}^{\mathrm{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathrm{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathrm{div}} \pi_2$.

$$
\begin{tikzcd}
& & s_1 \arrow[dd] & \approx_{\mathcal{T}}^{\mathrm{div}} & s_2 \arrow[d] \\
& & & & \vdots \arrow[d] \\
s_1 & \not\approx_{\mathcal{T}}^{\mathrm{div}} & s_{11} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{12}
\end{tikzcd}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathrm{div}} u_i$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

$$
\begin{tikzcd}
 & & s_1 \arrow[dd] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \arrow[d] \\
 & & & & \vdots \arrow[d] \\
s_1 & \not\approx_{\mathcal{T}}^{\mathsf{div}} & s_{11} \arrow[d] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{12} \\
s_{11} & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{21} & &
\end{tikzcd}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

$$
\begin{tikzcd}
 & & s_1 \arrow[dd] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \arrow[d] \\
 & & & & \vdots \arrow[d] \\
s_1 & \not\approx_{\mathcal{T}}^{\mathsf{div}} & s_{11} \arrow[d] & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{12} \\
s_{11} & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{21} \arrow[d] & & \\
 & & s_{31} & &
\end{tikzcd}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

Path lifting for $\approx^{\mathsf{div}}_{\mathcal{T}}$

If $s_1 \approx^{\mathsf{div}}_{\mathcal{T}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx^{\mathsf{div}}_{\mathcal{T}} \pi_2$.

$$
\begin{tikzcd}
 & & s_1 \arrow[d] & \approx^{\mathsf{div}}_{\mathcal{T}} & s_2 \arrow[d, dotted] \\
s_1 & \not\approx^{\mathsf{div}}_{\mathcal{T}} & s_{11} \arrow[d] & \approx^{\mathsf{div}}_{\mathcal{T}} & s_{12} \\
s_{11} & \approx^{\mathsf{div}}_{\mathcal{T}} & s_{21} \arrow[d] & & \\
s_{21} & \not\approx^{\mathsf{div}}_{\mathcal{T}} & s_{31} & &
\end{tikzcd}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx^{\mathsf{div}}_{\mathcal{T}} u_i$

> If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

$$\begin{array}{ccccc}
 & & s_1 & \approx_{\mathcal{T}}^{\mathsf{div}} & s_2 \\
 & & \big\downarrow & & \downarrow \vdots \downarrow \\
s_1 & \not\approx_{\mathcal{T}}^{\mathsf{div}} & s_{11} & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{12} \\
 & & \downarrow & & \downarrow \\
s_{11} & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{21} & & \vdots \\
 & & \downarrow & & \downarrow \\
s_{21} & \not\approx_{\mathcal{T}}^{\mathsf{div}} & s_{31} & \approx_{\mathcal{T}}^{\mathsf{div}} & s_{32}
\end{array}$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\mathsf{div}} v_j$

Path lifting for $\approx_{\mathcal{T}}^{\mathrm{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathrm{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathrm{div}} \pi_2$.

$$\begin{array}{ccccc}
 & & s_1 & \approx_{\mathcal{T}}^{\mathrm{div}} & s_2 \\
 & & \downarrow & & \downarrow \\
 & & & & \vdots \\
 & & \downarrow & & \downarrow \\
s_1 & \not\approx_{\mathcal{T}}^{\mathrm{div}} & s_{11} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{12} \\
 & & \downarrow & & \downarrow \\
s_{11} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{21} & & \vdots \\
 & & \downarrow & & \downarrow \\
s_{21} & \not\approx_{\mathcal{T}}^{\mathrm{div}} & s_{31} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{32} \\
 & & \downarrow & & \\
s_{31} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{41} & & \\
 & & \downarrow & & \\
s_{41} & \approx_{\mathcal{T}}^{\mathrm{div}} & s_{51} & & \\
 & & \downarrow & & \\
 & & \vdots & &
\end{array}$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathrm{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\mathrm{div}} v_j$

If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.

$$
\begin{array}{ccccc}
 & & s_1 & \approx_{\mathcal{T}}^{\text{div}} & s_2 \\
 & & \big\downarrow & & \downarrow \\
 & & & & \vdots \\
 & & \big\downarrow & & \downarrow \\
s_1 & \not\approx_{\mathcal{T}}^{\text{div}} & s_{11} & \approx_{\mathcal{T}}^{\text{div}} & s_{12} \\
 & & \downarrow & & \downarrow \\
s_{11} & \approx_{\mathcal{T}}^{\text{div}} & s_{21} & & \vdots \\
 & & \downarrow & & \downarrow \\
s_{21} & \not\approx_{\mathcal{T}}^{\text{div}} & s_{31} & \approx_{\mathcal{T}}^{\text{div}} & s_{32} \\
 & & \downarrow & & \\
s_{31} & \approx_{\mathcal{T}}^{\text{div}} & s_{41} & & \\
 & & \downarrow & & \\
s_{41} & \approx_{\mathcal{T}}^{\text{div}} & s_{51} & & \\
 & & \downarrow & & \\
 & & \vdots & &
\end{array}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\text{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\text{div}} v_j$

s_{31} divergent

Path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.

$$
\begin{array}{ccccc}
 & & s_1 & \approx_{\mathcal{T}}^{\text{div}} & s_2 \\
 & & \big\downarrow & & \downarrow \\
 & & & & \vdots \\
 & & \big\downarrow & & \downarrow \\
s_1 & \not\approx_{\mathcal{T}}^{\text{div}} & s_{11} & \approx_{\mathcal{T}}^{\text{div}} & s_{12} \\
 & & \downarrow & & \downarrow \\
s_{11} & \approx_{\mathcal{T}}^{\text{div}} & s_{21} & & \vdots \\
 & & \downarrow & & \downarrow \\
s_{21} & \not\approx_{\mathcal{T}}^{\text{div}} & s_{31} & \approx_{\mathcal{T}}^{\text{div}} & s_{32} \\
 & & \downarrow & & \\
s_{31} & \approx_{\mathcal{T}}^{\text{div}} & s_{41} & & \\
 & & \downarrow & & \\
s_{41} & \approx_{\mathcal{T}}^{\text{div}} & s_{51} & & \\
 & & \downarrow & & \\
 & & \vdots & &
\end{array}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\text{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\text{div}} v_j$

s_{31} divergent

$\Downarrow$

s_{32} divergent

> If $s_1 \approx^{\mathrm{div}}_{\mathcal{T}} s_2$ then for all paths $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx^{\mathrm{div}}_{\mathcal{T}} \pi_2$.

$$
\begin{array}{ccccccc}
 & & s_1 & \approx^{\mathrm{div}}_{\mathcal{T}} & s_2 \\
 & & \big\downarrow & & \downarrow \\
 & & & & \vdots \\
 & & \big\downarrow & & \downarrow \\
s_1 & \not\approx^{\mathrm{div}}_{\mathcal{T}} & s_{11} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{12} \\
 & & \downarrow & & \downarrow \\
s_{11} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{21} & & \vdots \\
 & & \downarrow & & \downarrow \\
s_{21} & \not\approx^{\mathrm{div}}_{\mathcal{T}} & s_{31} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{32} \\
 & & \downarrow & & \downarrow \\
s_{31} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{41} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{42} \\
 & & \downarrow & & \downarrow \\
s_{41} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{51} & \approx^{\mathrm{div}}_{\mathcal{T}} & s_{52} \\
 & & \downarrow & & \downarrow \\
 & & \vdots & & \vdots
\end{array}
$$

stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx^{\mathrm{div}}_{\mathcal{T}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx^{\mathrm{div}}_{\mathcal{T}} v_j$

s_{31} divergent

$\Downarrow$

s_{32} divergent

Properties of $\approx_{\mathcal{T}}^{\mathsf{div}}$ on paths

STUTTER5.4-35A

Properties of $\approx_{\mathcal{T}}^{\mathsf{div}}$ on paths

We just saw:

- If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

We just saw:

- If $s_1 \approx^{\mathsf{div}}_{\mathcal{T}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx^{\mathsf{div}}_{\mathcal{T}} \pi_2$.
- If $\pi_1 \approx^{\mathsf{div}}_{\mathcal{T}} \pi_2$ then $\pi_1 \triangleq_{\mathcal{T}} \pi_2$.

stutter trace equivalence

We just saw:

- If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

- If $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ then $\pi_1 \triangleq_{\mathcal{T}} \pi_2$.

Hence we get: stutter trace equivalence

Stutter bisimulation equivalence with divergence is finer than stutter trace equivalence, i.e.,

$$s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2 \text{ implies } s_1 \triangleq_{\mathcal{T}} s_2$$

$\approx^{\mathsf{div}}_{\mathcal{T}}$ is finer than CTL*$_{\setminus\bigcirc}$-equivalence

CTL*$_{\setminus\bigcirc}$ state formulas:

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL*$_{\setminus\bigcirc}$ path formulas:

$$\varphi ::= \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

$\approx^{\mathrm{div}}_{\mathcal{T}}$ is finer than CTL*$_{\setminus\bigcirc}$-equivalence

CTL*$_{\setminus\bigcirc}$ state formulas:

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL*$_{\setminus\bigcirc}$ path formulas:

$$\varphi ::= \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

we show by structural induction:

1. If $s_1 \approx^{\mathrm{div}}_{\mathcal{T}} s_2$ then for all CTL*$_{\setminus\bigcirc}$ state formulas Φ:

   $s_1 \models \Phi$ iff $s_2 \models \Phi$

2. If $\pi_1 \approx^{\mathrm{div}}_{\mathcal{T}} \pi_2$ then for all CTL*$_{\setminus\bigcirc}$ path formulas φ:

   $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Divergence-sensitive TS

Divergence-sensitive TS

Suppose $\mathcal{T}$ is divergence-sensitive, i.e.,

$$\boxed{\begin{array}{c}\text{whenever } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \text{ is } \approx_{\mathcal{T}}\text{-divergent} \\ \text{then } s_2 \text{ is } \approx_{\mathcal{T}}\text{-divergent.}\end{array}}$$

Divergence-sensitive TS

Suppose $\mathcal{T}$ is divergence-sensitive, i.e.,

$$\boxed{\begin{array}{c}\text{whenever } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \text{ is } \approx_{\mathcal{T}}\text{-divergent} \\ \text{then } s_2 \text{ is } \approx_{\mathcal{T}}\text{-divergent.}\end{array}}$$

Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

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$$\boxed{\begin{array}{c}\text{whenever } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \text{ is } \approx_{\mathcal{T}}\text{-divergent} \\ \text{then } s_2 \text{ is } \approx_{\mathcal{T}}\text{-divergent.}\end{array}}$$

Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

This yields: $s_1 \approx_{\mathcal{T}} s_2$ implies $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

Suppose $\mathcal{T}$ is divergence-sensitive, i.e.,

$$\boxed{\begin{array}{c}\text{whenever } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \text{ is } \approx_{\mathcal{T}}\text{-divergent} \\ \text{then } s_2 \text{ is } \approx_{\mathcal{T}}\text{-divergent.}\end{array}}$$

Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

This yields: $s_1 \approx_{\mathcal{T}} s_2$ implies $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

As $\approx_{\mathcal{T}}$ is coarser than $\approx_{\mathcal{T}}^{\mathsf{div}}$ we get: $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}}$

$\uparrow$

even holds for any transition system

Divergence-sensitive TS

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

Divergence-sensitive TS

If s_1, s_2 are states of a divergence-sensitive, finite TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx^{\mathsf{div}}_{\mathcal{T}} s_2$

(3) s_1, s_2 satisfy the same $\mathbf{CTL}^*_{\setminus \bigcirc}$ formulas

If s_1, s_2 are states of a divergence-sensitive, finite TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

(3) s_1, s_2 satisfy the same $\mathsf{CTL}^*_{\setminus\bigcirc}$ formulas

(4) s_1, s_2 satisfy the same $\mathsf{CTL}_{\setminus\bigcirc}$ formulas

For finite divergence-sensitive transition systems:

$s_1 \approx_{\mathcal{T}} s_2$ iff s_1, s_2 satisfy the same $CTL^*_{\setminus\bigcirc}$ formulas
iff s_1, s_2 satisfy the same $CTL_{\setminus\bigcirc}$ formulas

wrong for non-divergence-sensitive TS:

For finite divergence-sensitive transition systems:

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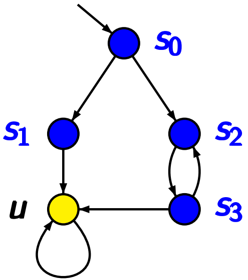

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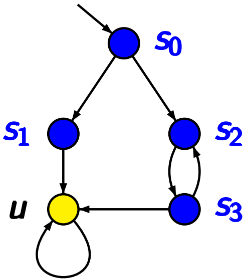

stutter bis. equivalence classes:
$\{s_0, s_1, s_2, s_4\}$ $\{u\}$

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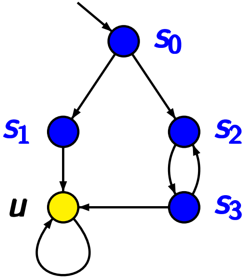

stutter bis. equivalence classes:
$\{s_0, s_1, s_2, s_4\}$ $\{u\}$

$s_1 \not\models \exists\Box$ **blue**

$s_2 \models \exists\Box$ **blue**

Correct or wrong?

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

$$s_1 \approx_{\mathcal{T}} s_2 \quad \text{implies} \quad s_1 \triangleq_{\mathcal{T}} s_2$$

$\approx_{\mathcal{T}}$ stutter bisimulation equivalence (without divergence)

$\triangleq_{\mathcal{T}}$ stutter trace equivalence

Correct or wrong?

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

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correct

- $\approx_{\mathcal{T}} \, = \, \approx_{\mathcal{T}}^{\mathsf{div}}$ in divergence-sensitive TS

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correct

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if $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all $\pi_1 \in \mathit{Paths}(s_1)$
there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$

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- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}}$ in divergence-sensitive TS
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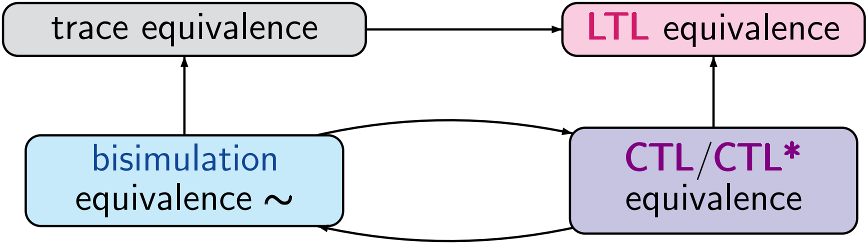
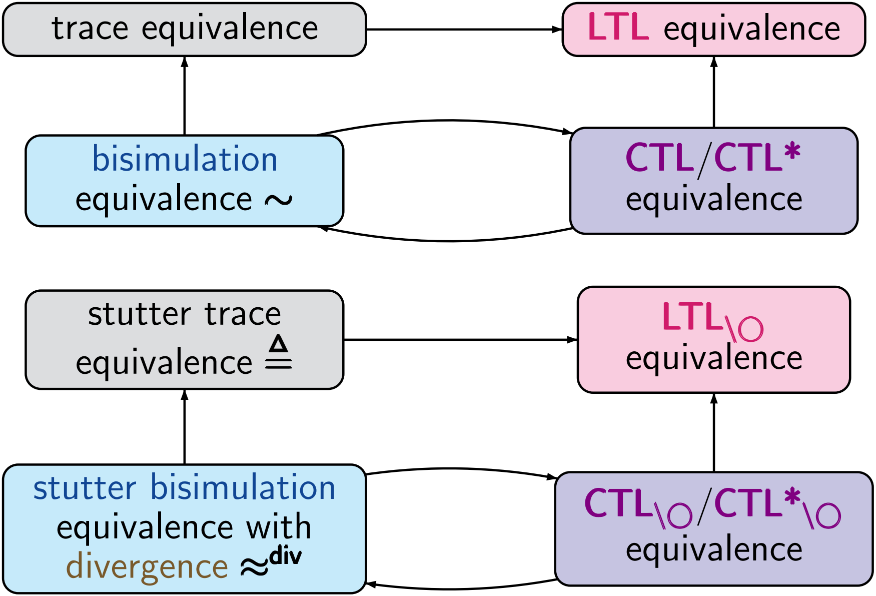
- $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ implies $\pi_1 \triangleq_{\mathcal{T}} \pi_2$

Summary: equivalences on finite TS



Summary: equivalences on finite TS



Correct or wrong?

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathbf{CTL}_{\setminus\bigcirc}$ formulas of the form $\exists\Diamond a$ where $a \in AP$.

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same **CTL**$_{\setminus \bigcirc}$ formulas of the form $\exists\Diamond a$ where $a \in AP$.

correct.

Let $\mathcal{T}$ be a TS without terminal states,
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correct.

Note: path lifting for finite path fragments is possible

Lifting of finite path fragments

$$s_1 = s_{01} \quad \approx_{\mathcal{T}} \quad s_2$$

$$s_{11}$$

$$\vdots$$

$$s_{n1}$$

$$s_{n+1,1}$$

$$\vdots$$

$$s_{m,1}$$

$$s_{m+1,1}$$

$$\vdots$$

$$s_{k,1}$$

$$t$$

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$$\vdots$$

$$s_{m,1}$$

$$s_{m+1,1}$$

$$\vdots$$

$$s_{k,1}$$

$$t$$

where $t \models a$

Lifting of finite path fragments

$$
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\
s_{11} & & \\
\vdots & & \\
s_{n1} & & \\
s_{n+1,1} & & \\
\vdots & & \\
s_{m,1} & & \\
s_{m+1,1} & & \\
\vdots & & \\
s_{k,1} & & \\
t & &
\end{array}
\quad \leadsto \quad
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_{11} = s_2 \\
s_{11} & & \vdots \\
\vdots & & s_{r2} \\
s_{n1} & & s_{r+1,2} \\
s_{n+1,1} & & \vdots \\
\vdots & & s_{l,2} \\
s_{m,1} & & s_{l+1,2} \\
s_{m+1,1} & & \vdots \\
\vdots & & s_{i,2} \\
s_{k,1} & & u \\
t & &
\end{array}
$$

where $t \models a$

Lifting of finite path fragments

$$\begin{array}{rcl} s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\ s_{11} & & \\ \vdots & & \\ s_{n1} & & \\ s_{n+1,1} & & \\ \vdots & & \\ s_{m,1} & & \\ s_{m+1,1} & & \\ \vdots & & \\ s_{k,1} & & \\ t & & \end{array} \quad \rightsquigarrow \quad \begin{array}{rcl} s_1 = s_{01} & \approx_{\mathcal{T}} & s_{11} = s_2 \\ s_{11} & & \vdots \\ \vdots & & s_{r2} \\ s_{n1} & & s_{r+1,2} \\ s_{n+1,1} & & \vdots \\ \vdots & & s_{l,2} \\ s_{m,1} & & s_{l+1,2} \\ s_{m+1,1} & & \vdots \\ \vdots & & s_{i,2} \\ s_{k,1} & & u \\ t & & t \approx_{\mathcal{T}} u \end{array}$$

where $t \models a$

Lifting of finite path fragments

$$
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\
s_{11} & & \\
\vdots & & \\
s_{n1} & & \\
s_{n+1,1} & & \\
\vdots & & \\
s_{m,1} & & \\
s_{m+1,1} & & \\
\vdots & & \\
s_{k,1} & & \\
t & &
\end{array}
\quad \rightsquigarrow \quad
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_{11} = s_2 \\
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s_{m,1} & & s_{l+1,2} \\
s_{m+1,1} & & \vdots \\
\vdots & & s_{i,2} \\
s_{k,1} & & u \\
t & & t \approx_{\mathcal{T}} u
\end{array}
$$

where $t \models a$ hence: $u \models a$

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

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correct.

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correct.

$$s_1 \models \forall\Box a \quad \text{iff} \quad s_1 \not\models \exists\Diamond\neg a$$

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correct.

$$
\begin{aligned}
s_1 \models \forall\Box a \quad & \text{iff} \quad s_1 \not\models \exists\Diamond\neg a \\
& \text{iff} \quad s_2 \not\models \exists\Diamond\neg a
\end{aligned}
$$

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correct.

$$
\begin{aligned}
s_1 \models \forall\Box a \quad & \text{iff} \quad s_1 \not\models \exists\Diamond\neg a \\
& \text{iff} \quad s_2 \not\models \exists\Diamond\neg a \\
& \text{iff} \quad s_2 \models \forall\Box a
\end{aligned}
$$

Correct or wrong?

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

correct.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Diamond a$ where $a \in AP$.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus \bigcirc}$ formulas of the form $\forall \Box a$ where $a \in AP$.

correct.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus \bigcirc}$ formulas of the form $\forall \Diamond a$ where $a \in AP$.

wrong.

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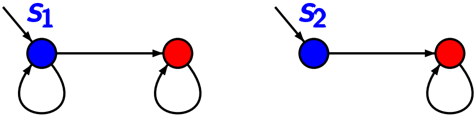
wrong.



Correct or wrong?

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $CTL_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

correct.

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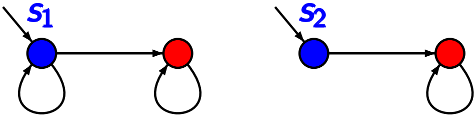
wrong.



$s_1 \not\models \forall\Diamond a$

$s_2 \models \forall\Diamond a$

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL***$_{\setminus\bigcirc}$ formulas

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same $\mathbf{CTL^*}_{\setminus \bigcirc}$ formulas

wrong

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

$$\boxed{\begin{array}{c}\text{If } \mathcal{T}_1 \approx \mathcal{T}_2 \text{ then } \mathcal{T}_1 \text{ and } \mathcal{T}_2 \text{ satisfy} \\ \text{the same } \textbf{CTL*}_{\setminus\bigcirc} \text{ formulas}\end{array}}$$

wrong, as $\mathcal{T}_1 \uplus \mathcal{T}_2$ might be not divergence-sensitive

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

> If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same $\mathbf{CTL^*}_{\backslash\bigcirc}$ formulas

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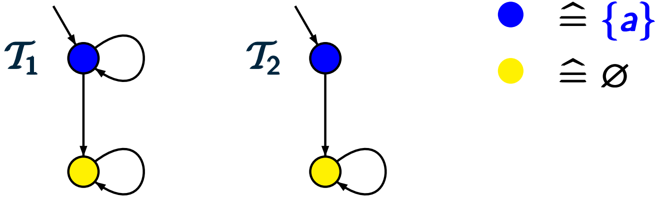

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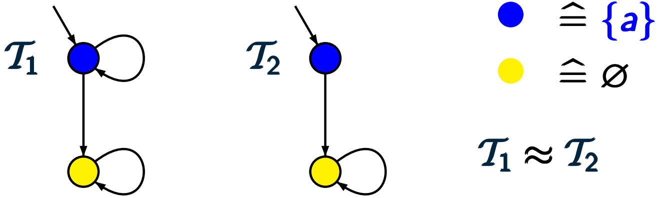

$\mathcal{T}_1 \approx \mathcal{T}_2$

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

> If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same $\mathbf{CTL^*}_{\setminus\bigcirc}$ formulas

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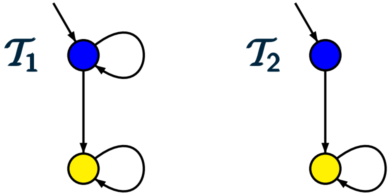

$\mathcal{T}_1 \models \exists\Box a$ $\qquad$ $\mathcal{T}_2 \not\models \exists\Box a$

● $\widehat{=} \{a\}$

● $\widehat{=} \varnothing$

$\mathcal{T}_1 \approx \mathcal{T}_2$

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

$$\boxed{\text{If } \mathcal{T}_1 \approx \mathcal{T}_2 \text{ then } \mathcal{T}_1 \text{ and } \mathcal{T}_2 \text{ satisfy the same } \mathrm{CTL}^*_{\setminus \bigcirc} \text{ formulas}}$$

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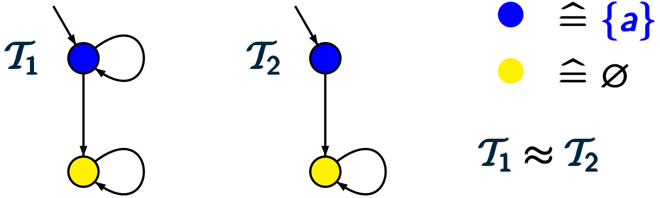

$\mathcal{T}_1 \models \exists \Box a$ $\qquad$ $\mathcal{T}_2 \not\models \exists \Box a$

$\mathcal{T}_1 \approx \mathcal{T}_2$

$\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$ $\longleftarrow$ holds for any TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}} =$ $\mathsf{CTL}^*_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$

$\mathcal{T}$ and $\mathcal{T}/\!\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$
- $\approx_{\mathcal{T}} \; = \; \approx^{\mathsf{div}}_{\mathcal{T}} \; =$ **CTL*** $_{\setminus\bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\!\approx$ might be not **CTL*** $_{\setminus\bigcirc}$-equivalent

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$
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- $\mathcal{T}$ and $\mathcal{T}/\!\approx$ might be not **CTL***$_{\setminus\bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\!\approx$ might be not divergence-sensitive

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx^{\mathsf{div}}_{\mathcal{T}} =$ **CTL***$_{\setminus\bigcirc}$-equivalence on $\mathcal{T}$
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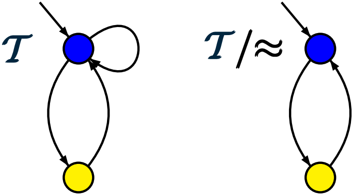
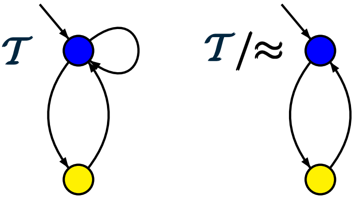


$\mathcal{T}$ and $\mathcal{T}/{\approx}$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/{\approx}$
- $\approx_{\mathcal{T}} \; = \; \approx_{\mathcal{T}}^{\mathsf{div}} \; = \;$ $\mathrm{CTL}^*_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/{\approx}$ might be not $\mathrm{CTL}^*_{\setminus \bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/{\approx}$ might be not divergence-sensitive

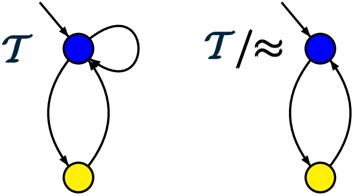


$\mathcal{T}$ (and $\mathcal{T}/{\approx}$) are divergence-sensitive

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\text{div}} =$ CTL*$_{\setminus\bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\approx$ might be not CTL*$_{\setminus\bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\approx$ might be not divergence-sensitive



$\mathcal{T} \models \exists\Box a$ $\qquad$ $\mathcal{T}/\approx \;\not\models \exists\Box a$

$\mathcal{T}$ (and $\mathcal{T}/\approx$) are divergence-sensitive

where $a \mathrel{\widehat{=}}$ *blue*

Correct or wrong?

If $\mathcal{T}$ is *divergence-sensitive* then stutter bisimulation equivalence $\approx_{\mathcal{T}}$ (without divergence) refines stutter trace equivalence $\stackrel{\Delta}{=}_{\mathcal{T}}$, i.e.,

$$s_1 \approx_{\mathcal{T}} s_2 \implies s_1 \stackrel{\Delta}{=}_{\mathcal{T}} s_2$$

Correct or wrong?

STUTTER5.4-47

If $\mathcal{T}$ is *divergence-sensitive* then stutter bisimulation equivalence $\approx_{\mathcal{T}}$ (without divergence) refines stutter trace equivalence $\stackrel{\Delta}{=}_{\mathcal{T}}$, i.e.,

$$s_1 \approx_{\mathcal{T}} s_2 \quad\Longrightarrow\quad s_1 \stackrel{\Delta}{=}_{\mathcal{T}} s_2$$

correct,

Correct or wrong?

If $\mathcal{T}$ is *divergence-sensitive* then stutter bisimulation equivalence $\approx_{\mathcal{T}}$ (without divergence) refines stutter trace equivalence $\stackrel{\Delta}{=}_{\mathcal{T}}$, i.e.,

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correct, as $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\text{div}}$ and path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

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correct, as $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\text{div}}$ and path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

path $s_1 \; u_1 \ldots u_n \; v_1 \; v_2 \ldots v_m \; w_1 \; w_2 \; w_3 \ldots$

$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, ...

Correct or wrong?

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$$s_1 \approx_{\mathcal{T}} s_2 \implies s_1 \overset{\Delta}{=}_{\mathcal{T}} s_2$$

correct, as $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\text{div}}$ and path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

path $s_1\ u_1 \ldots u_n\ v_1\ v_2 \ldots v_m\ w_1\ w_2\ w_3 \ldots$

$\downarrow$

path $s_2 u'_1 \ldots u'_h\ v'_1 v'_2 \ldots v'_k\ w'_1 w'_2 w'_3 \ldots$

$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\ldots$

If $\mathcal{T}$ is *divergence-sensitive* then stutter trace equivalence $\overset{\Delta}{=}_{\mathcal{T}}$ refines stutter bisimulation equivalence $\approx_{\mathcal{T}}$, i.e.,

$$s_1 \overset{\Delta}{=}_{\mathcal{T}} s_2 \implies s_1 \approx_{\mathcal{T}} s_2$$

Correct or wrong?

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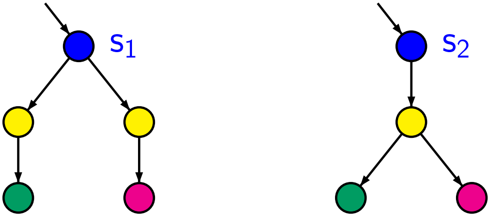
wrong.



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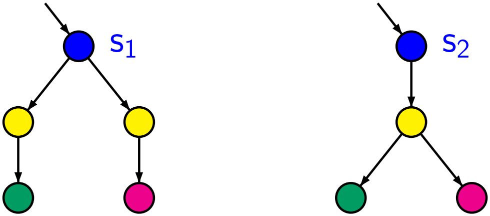

divergence-sensitive as there is no divergent state

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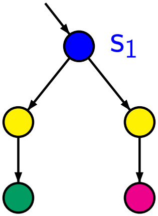
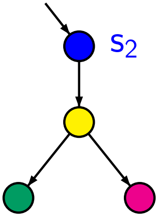


$s_1 \overset{\Delta}{=}_{\mathcal{T}} s_2$



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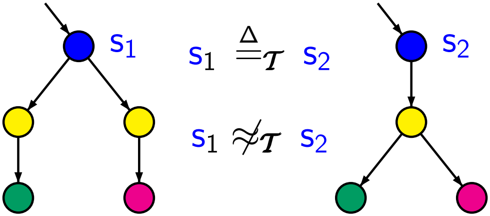

divergence-sensitive as there is no divergent state

If $s_1 \sim_{\mathcal{T}} s_2$ then $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$.

$\sim_{\mathcal{T}}$ bisimulation equivalence on $\mathcal{T}$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ stutter bisimulation equivalence with divergence

Correct or wrong?

$$\text{If } s_1 \sim_{\mathcal{T}} s_2 \text{ then } s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2.$$

correct

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Correct or wrong?

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correct

note that bisimulation equivalence $\sim_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation on $\mathcal{T}$.

$\sim_{\mathcal{T}}$ bisimulation equivalence on $\mathcal{T}$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ stutter bisimulation equivalence with divergence

If $s_1 \sim_{\mathcal{T}} s_2$ then $s_1 \approx^{\mathsf{div}}_{\mathcal{T}} s_2$.

correct

If $s_1 \triangleq_{\mathcal{T}} s_2$ then s_1 and s_2 are $\mathbf{CTL^*}_{\setminus \bigcirc}$-equivalent.

stutter trace equivalence

Correct or wrong?

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correct

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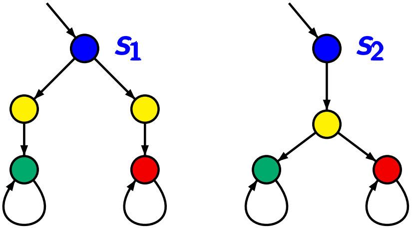

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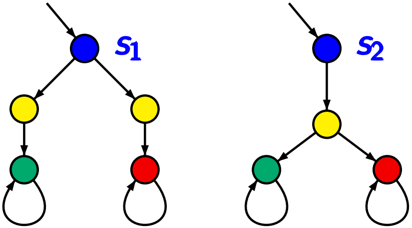
wrong. let $\Phi = \exists\Diamond(\neg \mathit{blue} \wedge \exists\Diamond \mathit{green} \wedge \exists\Diamond \mathit{red})$



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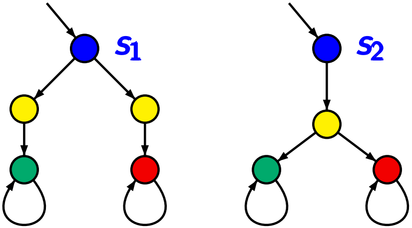

$s_1 \not\models \Phi$

$s_2 \models \Phi$

Quotient w.r.t. $\approx^{\mathsf{div}}$

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Let $\mathcal{T} = (S, Act, \to, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\mathsf{div}}$:

$$\mathcal{T}/\!\approx^{\mathsf{div}} \;=\; (S', Act', \to_{\mathsf{div}}, S_0', AP, L')$$

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- initial states: $S'_0 = \{[s] : s \in S_0\}$

$[s] = \approx_{\mathcal{T}}^{\mathsf{div}}$-equivalence class of state s

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$$\frac{s \to t \;\wedge\; s \not\approx^{\mathsf{div}}_{\mathcal{T}} t}{[s] \to_{\mathsf{div}} [t]} \qquad \frac{s \text{ is } \approx^{\mathsf{div}}_{\mathcal{T}}\text{-divergent}}{[s] \to_{\mathsf{div}} [s]}$$

Relation between $\mathcal{T}$ and its quotient

quotient of $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ w.r.t. $\approx^{\text{div}}$:

$$\mathcal{T}/{\approx^{\text{div}}} \;=\; (S/{\approx^{\text{div}}_{\mathcal{T}}}, Act', \rightarrow_{\text{div}}, S_0', AP, L')$$

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$\mathcal{T} \approx^{\text{div}} \mathcal{T}/\approx^{\text{div}}$

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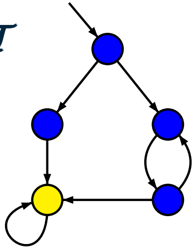
$$\mathcal{T} \approx^{\mathsf{div}} \mathcal{T}/{\approx^{\mathsf{div}}}$$

as $\{(s, [s]) : s \in S\}$ is a divergence-sensitive stutter bisimulation for $(\mathcal{T}, \mathcal{T}/{\approx^{\mathsf{div}}})$

Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

$\mathcal{T}$

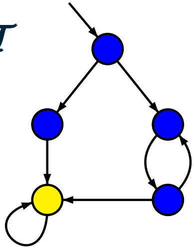


● $\widehat{=}$ $\{a\}$

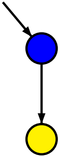
● $\widehat{=}$ $\varnothing$

Example: quotient w.r.t. $\approx$ and $\approx^{\mathsf{div}}$

$\mathcal{T}$



$\mathcal{T}/\approx$



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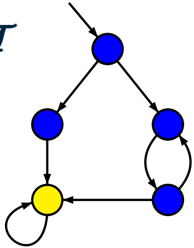

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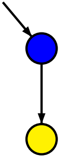

$\mathcal{T} \approx \mathcal{T}/\approx$

Example: quotient w.r.t. $\approx$ and $\approx^{\mathsf{div}}$

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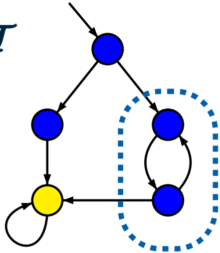

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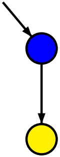
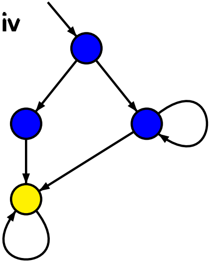


$\mathcal{T}/\approx^{\mathsf{div}}$



$$\mathcal{T} \approx \mathcal{T}/\approx$$

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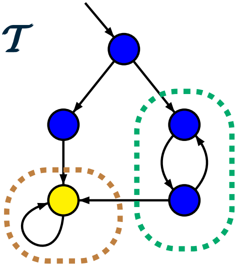

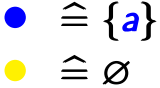

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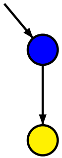
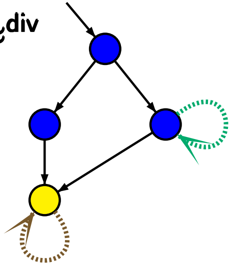


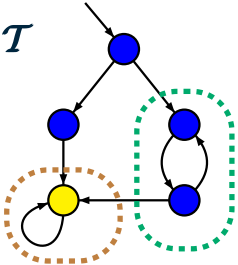
$\mathcal{T}/\approx^{\text{div}}$



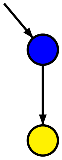
$\mathcal{T} \approx \mathcal{T}/\approx$

Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

$\mathcal{T}$



$\mathcal{T}/\approx$



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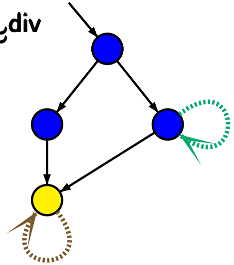
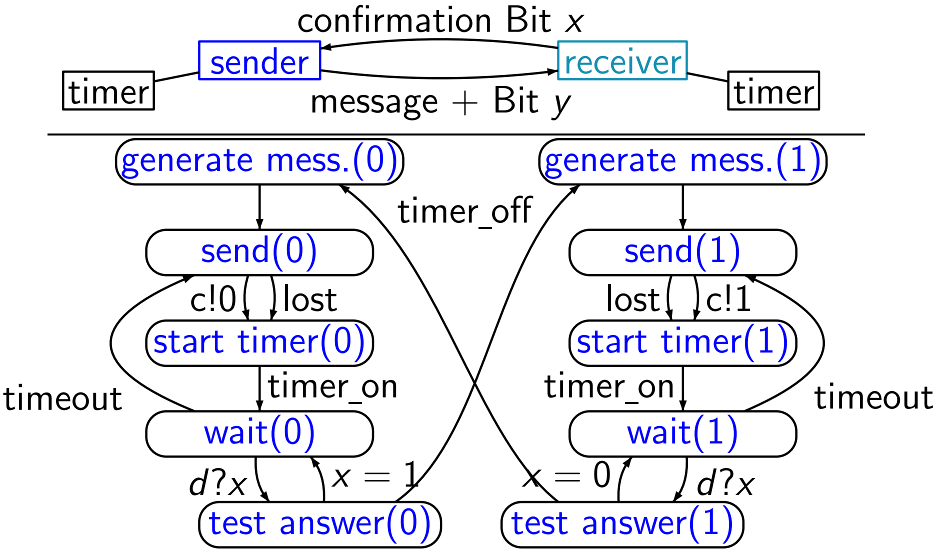
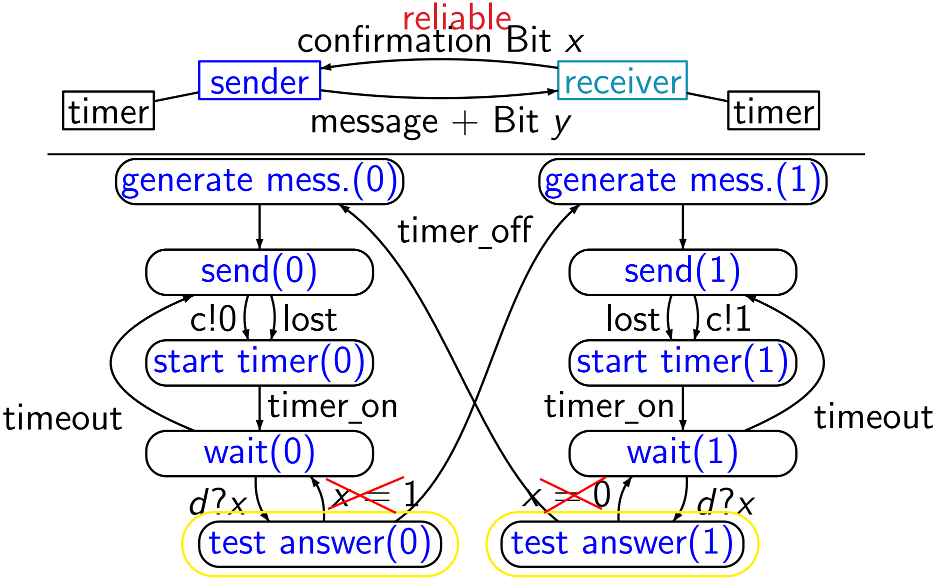
$\mathcal{T}/\approx^{\text{div}}$



$\mathcal{T} \approx \mathcal{T}/\approx$ $\qquad$ $\mathcal{T} \approx^{\text{div}} \mathcal{T}/\approx^{\text{div}}$

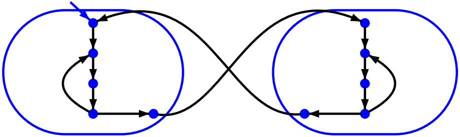
Alternating Bit Protocol





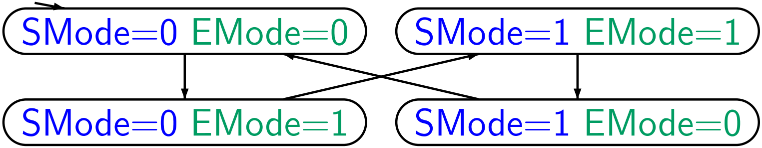
– induce non-divergent states

Alternating Bit Protocol $\quad AP' =$ **Eval(SMode,EMode)**



SMode = 0 SMode = 1

quotient system w.r.t. stutter equivalence:



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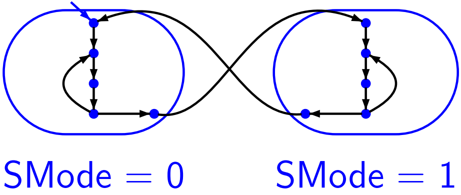

quotient system w.r.t. stutter equiv. with div.: