Heterogeneous/homogeneous model checking GRM5.5-30

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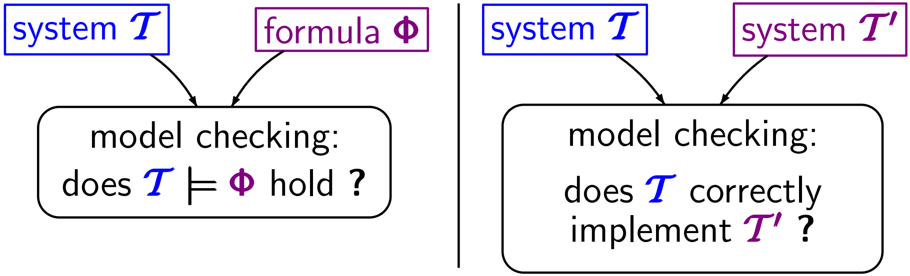
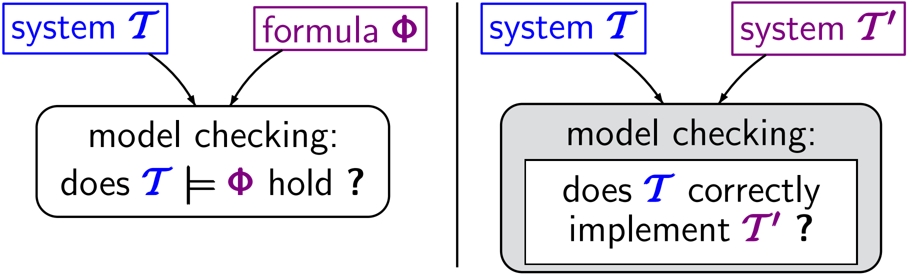
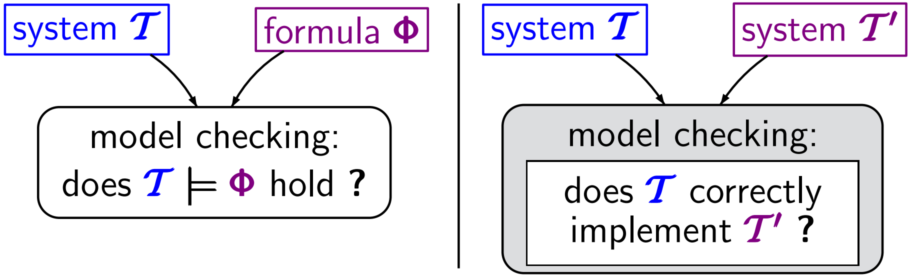


Heterogeneous/homogeneous model checking GRM5.5-30



trace inclusion checking
trace equivalence checking

Heterogeneous/homogeneous model checking



trace inclusion checking
trace equivalence checking $\longleftarrow$ ***PSPACE*-complete**

Heterogeneous/homogeneous model checking

system $\mathcal{T}$ | formula Φ
$\downarrow$
model checking:
does $\mathcal{T} \models \Phi$ hold ?

system $\mathcal{T}$ | system $\mathcal{T}'$
$\downarrow$
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does $\mathcal{T}$ correctly implement $\mathcal{T}'$?

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bisimulation equivalence checking
"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

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trace equivalence checking $\longleftarrow$ ***PSPACE*-complete**

bisimulation equivalence checking $\longleftarrow$ $\mathcal{O}(m \cdot \log n)$

“does $\mathcal{T} \sim \mathcal{T}'$ hold ?”

n = #states

m = #transitions

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"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

refinement checking via simulation

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“does $\mathcal{T} \sim \mathcal{T}'$ hold ?”

refinement checking via simulation $\longleftarrow$ $\mathcal{O}(m \cdot n)$
“does $\mathcal{T} \preceq \mathcal{T}'$ hold ?”

Refinement checking via simulation

given: 2 finite transition system $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set of propositions AP

question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold ?

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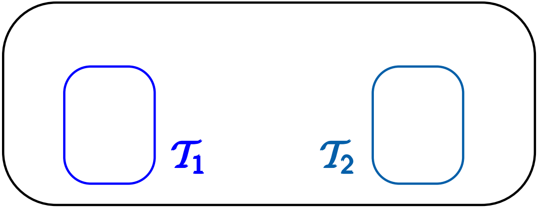

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

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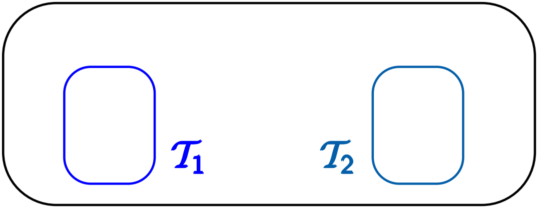

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$

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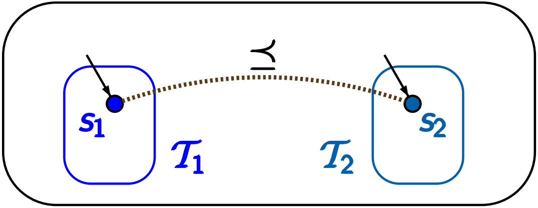

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$
- check whether for all initial states s_1 of $\mathcal{T}_1$ there is an initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \preceq_{\mathcal{T}} s_2$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
possibly with terminal states

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$\rightsquigarrow$ simulation quotient $\mathcal{T}/\simeq$

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method: iterative refinement of relation $\mathcal{R} \subseteq S \times S$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

$$\mathcal{R} \ := \ \big\{ (s_1, s_2) \in S \times S \ : \ L(s_1) = L(s_2) \big\};$$

WHILE $\mathcal{R}$ is no simulation DO

choose $(s_1, s_2) \in \mathcal{R}$ s.t. $s_1 \rightarrow s_1'$, but there is no transition $s_2 \rightarrow s_2'$ with $(s_1', s_2') \in \mathcal{R}$

$$\mathcal{R} \ := \ \mathcal{R} \setminus \{(s_1, s_2)\}$$

OD

return $\mathcal{R}$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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$\mathcal{R} := \mathcal{R} \setminus \{(s_1, s_2)\}$

OD

return $\mathcal{R}$ $\longleftarrow$ $\mathcal{R}$ is the coarsest simulation on $\mathcal{T}$ and therefore $\mathcal{R} = \preceq_{\mathcal{T}}$

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#iterations: $\mathcal{O}(|S|^2)$

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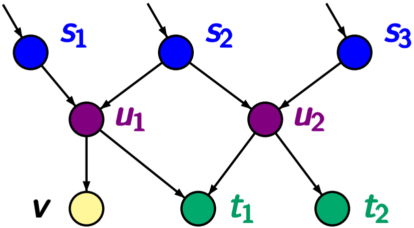
representation of $\mathcal{R}$ by simulator sets

$$Sim_{\mathcal{R}}(s_1) = \{ s_2 \in S : (s_1, s_2) \in \mathcal{R} \}$$

Example: computation of $\preceq$



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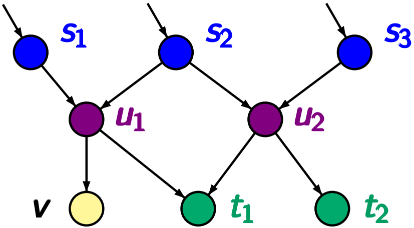

initially:

$$\begin{aligned}
Sim(s_i) &= \{s_1, s_2, s_3\} \\
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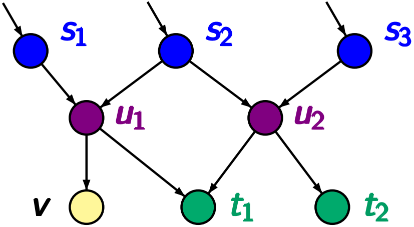

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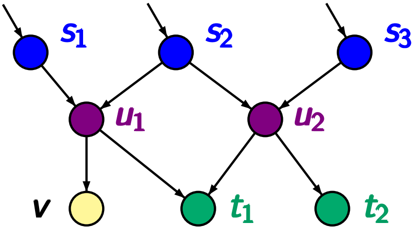
$u_1 \not\preceq u_2$, as $u_1 \to v$, $u_2 \not\to Sim(v)$



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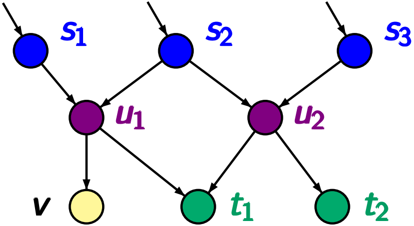

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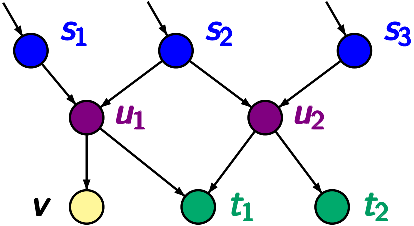

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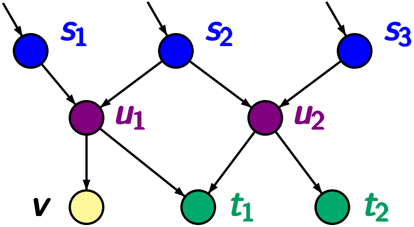

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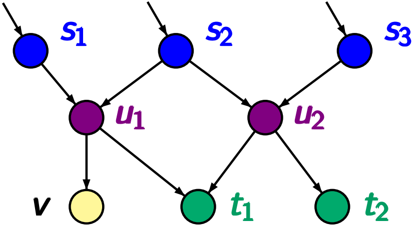
Example: computation of $\preceq$



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Computing the simulation preorder $\preceq_{\mathcal{T}}$

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```
FOR ALL s1 ∈ S DO
    Sim(s1) := {s2 ∈ S : L(s1) = L(s2)}
OD
WHILE ∃s1 ∈ S ∃s2 ∈ Sim(s1) ∃s1' ∈ Post(s1)
              s.t. Post(s2) ∩ Sim(s1') = ∅      DO
    choose such states s1, s2
    Sim(s1) := Sim(s1) \ {s2}
OD
return {(s1, s2) : s2 ∈ Sim(s1)}
```

FOR ALL $s_1 \in S$ DO

$$Sim(s_1) := \{s_2 \in S : L(s_1) = L(s_2)\}$$

OD

WHILE $\exists s_1 \in S\ \exists s_2 \in Sim(s_1)\ \exists s_1' \in Post(s_1)$ s.t. $Post(s_2) \cap Sim(s_1') = \varnothing$ DO

choose such states s_1, s_2

$$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$$

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$s_1 \longrightarrow s_1'$

s_2

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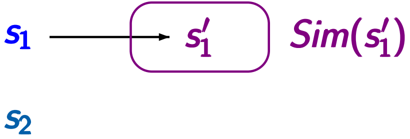

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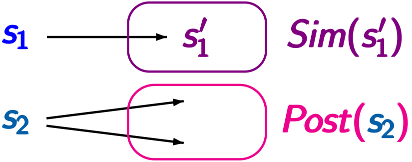

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$\quad$ choose such states s_1, s_2

$\quad Sim(s_1) := Sim(s_1) \setminus \{ s_2 \} \longleftarrow \boxed{s_1 \not\preceq_{\mathcal{T}} s_2}$

OD

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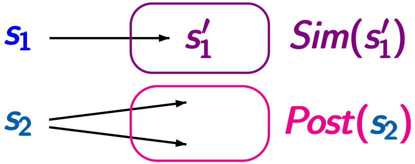

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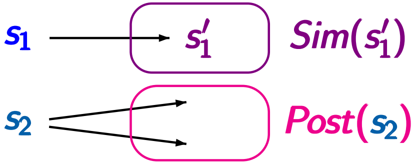
complexity:
$\mathcal{O}(m \cdot |S|^3)$



$m = \#\text{edges}$
$\geqslant |S|$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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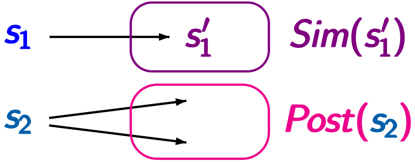

$m = \#\text{edges}$
$\geqslant |S|$

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_{\mathcal{T}}$

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reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$
by means of

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_\mathcal{T}$

reformulation of the algorithm to compute $\preceq_\mathcal{T}$
by means of

- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_{\mathcal{T}}$

reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$
by means of

- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$
- a set V that organizes all pairs (s_1', s_2)
  where $\delta(s_1', s_2) = 0$

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
```

```
OD
```

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left| Post(s_2) \cap Sim(s_1') \right|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

OD

`FOR ALL` $s_1 \in S$ `DO` $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ `OD`

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$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

`WHILE` $V \neq \varnothing$ `DO`

`OD`

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

WHILE $V \neq \varnothing$ DO

choose $(s_1', s_2) \in V$ and remove (s_1', s_2) from V

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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OD

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OD

OD

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
FOR ALL s1', s2 DO  δ(s1', s2) := |Post(s2) ∩ Sim(s1')| OD
V := {(s1', s2) : δ(s1', s2) = 0}
WHILE V ≠ ∅ DO
   choose (s1', s2) ∈ V and remove (s1', s2) from V
   FOR ALL s1 ∈ Pre(s1') with s2 ∈ Sim(s1) DO
      Sim(s1) := Sim(s1) \ {s2}
      FOR ALL u2 ∈ Pre(s2) DO
         δ(s1, u2) := δ(s1, u2) − 1


   OD
OD
```

```
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      IF δ(s1, u2) = 0 THEN insert (s1, u2) in V FI
    OD
  OD
OD
```

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    OD

  OD

OD

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      OD
   OD
OD
```

$$
\begin{array}{l}
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\quad \texttt{OD}\\
\texttt{OD}
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     OD
   OD
OD
```

in total:
$\mathcal{O}(m \cdot |S|)$

```
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cost per iteration
$\mathcal{O}(m)$

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An $\mathcal{O}(m \cdot |S|)$-algorithm for computing $\preceq_{\mathcal{T}}$

... algorithm by Henzinger, Henzinger, and Kopke

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Then: if $Post(s_2) \cap Sim(s_1') = \{s_2'\}$ then $s_1 \not\preceq s_2$ and s_2 can be removed from $Sim(s_1)$.

idea: collect all such states s_2 in $Remove(s_1')$

Idea of the HHK-algorithm

If s_2' is removed from $Sim(s_1')$ then regard all direct predecessors s_1 of s_1' and remove all states in

$$Remove(s_1') \; = \; Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$$

from $Sim(s_1)$.

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$$\boxed{\begin{aligned} Sim_{old}(s_1') &:= Sim(s_1') \\ Sim(s_1) &:= Sim(s_1) \setminus Remove(s_1') \text{ for } s_1 \in Pre(s_1') \end{aligned}}$$

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s_1'

$Sim_{old}(s_1')$

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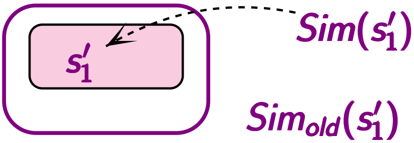

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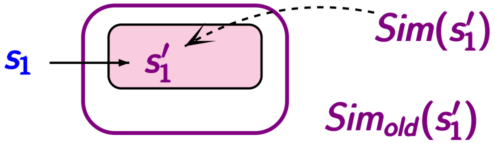

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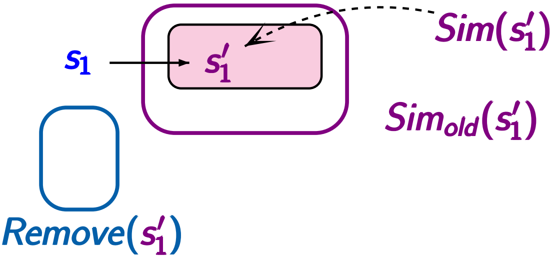

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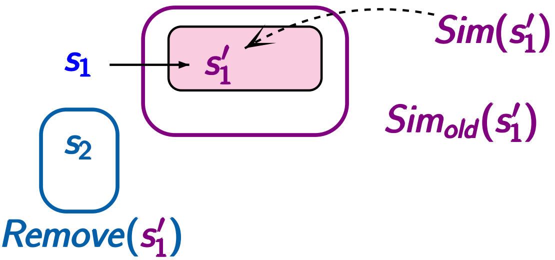

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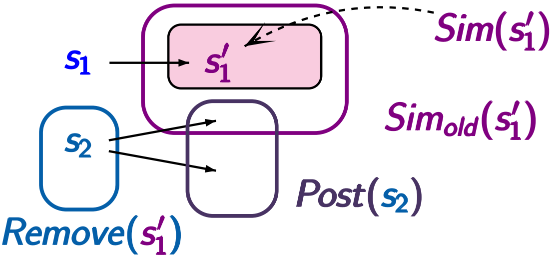

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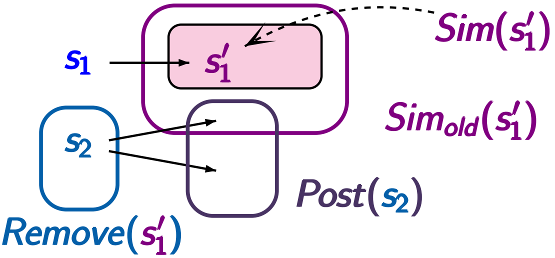

$s_1 \not\preceq_{\mathcal{T}} s_2$

HHK-algorithm (first version)


```
FOR ALL states s'_1 DO
```

$Sim_{old}(s'_1) := S$

$Sim(s'_1) := \{ s'_2 \in S : L(s'_1) = L(s'_2) \}$

```
OD
```

HHK-algorithm (first version)

FOR ALL states s_1' DO

$Sim_{old}(s_1') := S$

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OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

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choose such a state s_1';

HHK-algorithm (first version)

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FOR ALL states s_1' DO

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  choose such a state s_1';

  $Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$;

  FOR ALL $s_1 \in Pre(s_1')$ DO

HHK-algorithm (first version)


```
FOR ALL states s₁' DO
    Sim_old(s₁') := S
    Sim(s₁') := { s₂' ∈ S : L(s₁') = L(s₂') }
OD
WHILE ∃ state s₁' with Sim(s₁') ≠ Sim_old(s₁') DO
    choose such a state s₁';
    Remove(s₁') := Pre(Sim_old(s₁')) \ Pre(Sim(s₁'));
    FOR ALL s₁ ∈ Pre(s₁') DO
        Sim(s₁) := Sim(s₁) \ Remove(s₁')
    OD ;
```

HHK-algorithm (first version)

FOR ALL states s_1' DO

$\quad Sim_{old}(s_1') := S$

$\quad Sim(s_1') := \{ s_2' \in S : L(s_1') = L(s_2') \}$

OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

$\quad$ choose such a state s_1';

$\quad Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$;

$\quad$ FOR ALL $s_1 \in Pre(s_1')$ DO

$\qquad Sim(s_1) := Sim(s_1) \setminus Remove(s_1')$

$\quad$ OD ;

$\quad Sim_{old}(s_1') := Sim(s_1')$

OD

HHK-algorithm (first version)

```
FOR ALL states s'_1 DO
    Sim_old(s'_1) := S
    Sim(s'_1) := { s'_2 ∈ S : L(s'_1) = L(s'_2) }
OD
WHILE ∃ state s'_1 with Sim(s'_1) ≠ Sim_old(s'_1) DO
    choose such a state s'_1;
    Remove(s'_1) := Pre(Sim_old(s'_1)) \ Pre(Sim(s'_1));
    FOR ALL s_1 ∈ Pre(s'_1) DO
        Sim(s_1) := Sim(s_1) \ Remove(s'_1)
    OD ;
    Sim_old(s'_1) := Sim(s'_1)
OD
return {(s_1, s'_2) : s'_2 ∈ Sim(s'_1)}
```

HHK-algorithm (first version)

if s_2' is terminal then so is s_1'

FOR ALL states s_1' DO

  $Sim_{old}(s_1') := S$

  $Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2')$ and $\ldots\}$

OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

  choose such a state s_1';

  $Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$;

  FOR ALL $s_1 \in Pre(s_1')$ DO

    $Sim(s_1) := Sim(s_1) \setminus Remove(s_1')$

  OD ;

  $Sim_{old}(s_1') := Sim(s_1')$

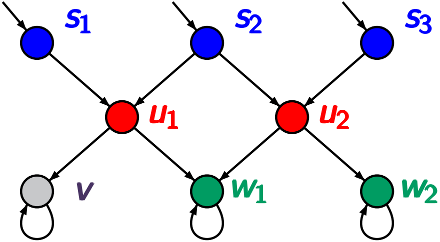
OD

return $\{(s_1, s_2') : s_2' \in Sim(s_1')\}$

HHK-algorithm (first version)


```
FOR ALL states s'_1 DO
    Sim_old(s'_1) := "undefined"
    Sim(s'_1) := { s'_2 ∈ s : L(s'_1) = L(s'_2) and ... }
OD
WHILE ∃ state s'_1 with Sim(s'_1) ≠ Sim_old(s'_1) DO
  choose such a state s'_1
  IF Sim_old(s'_1) = "undefined"
    THEN Remove(s'_1) := S \ Pre(Sim(s'_1))
    ELSE Remove(s'_1) := Pre(Sim_old(s'_1)) \ Pre(Sim(s'_1))
  FI
  FOR ALL s_1 ∈ Pre(s'_1) DO
  ...
```

Example: HHK-algorithm



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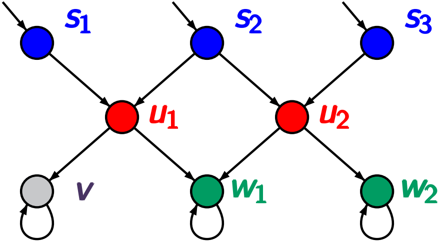


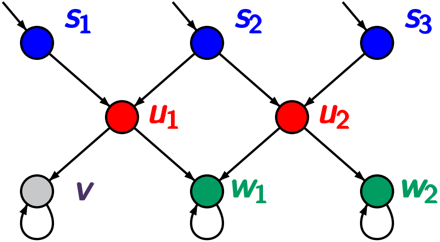
initially:

$Sim_{old}(t) = \perp$ for all states t

$Sim(s_1) = \{s_1, s_2, s_3\}$

$\vdots$

Example: HHK-algorithm



initially:

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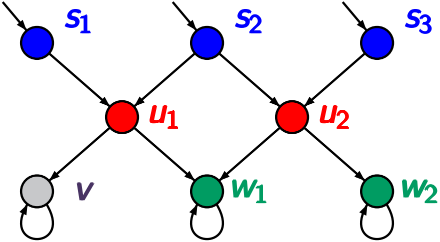
$\vdots$

choose state $s_1' = v$ with $Sim_{old}(v) \neq Sim(v)$:

Example: HHK-algorithm



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choose state $s_1' = v$ with $Sim_{old}(v) \neq Sim(v)$:

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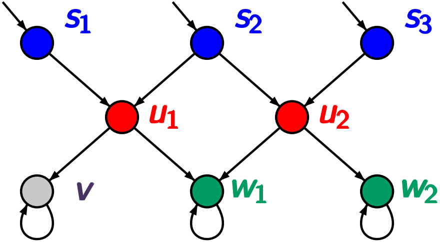
Example: HHK-algorithm



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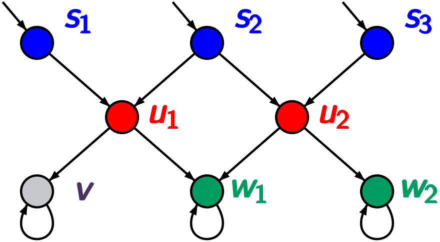

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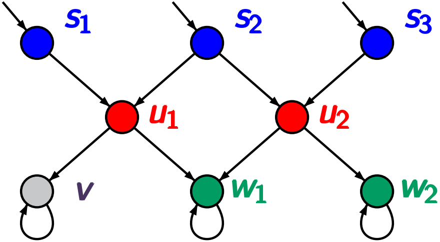

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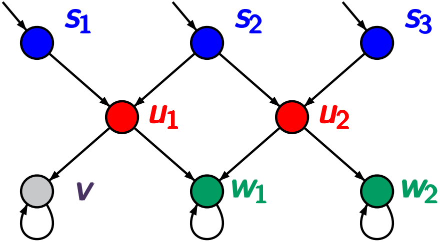

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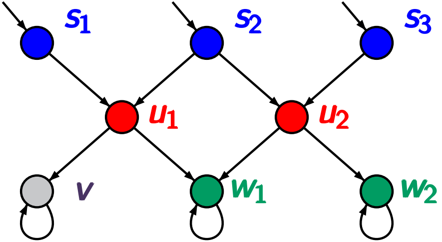
choose next state s'_1



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no change in $Sim(\ldots)$, as $Pre(s_1) = \varnothing$

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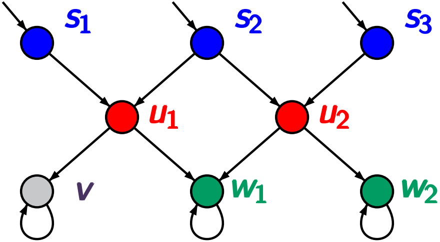

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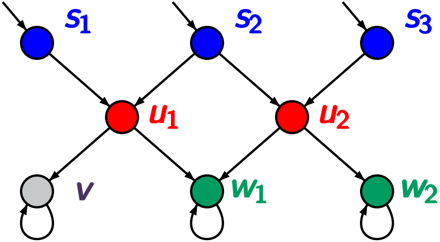

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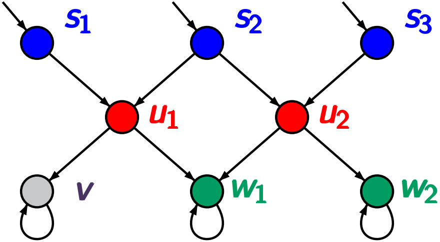

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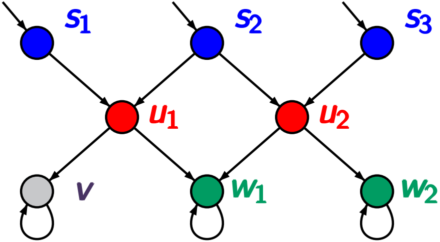

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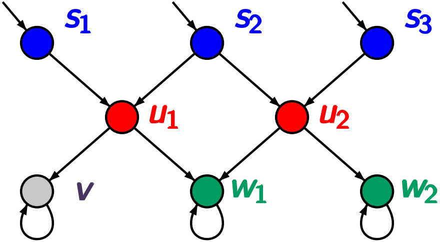

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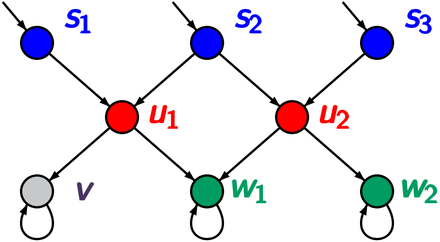

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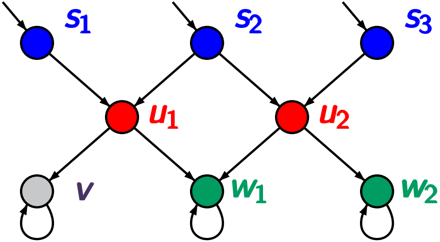

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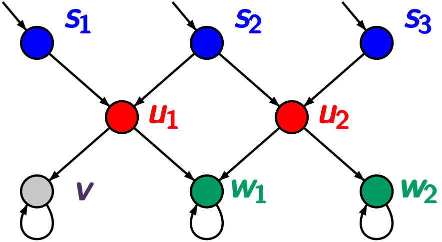

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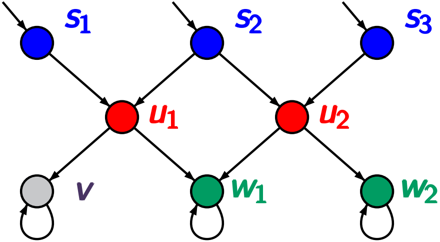

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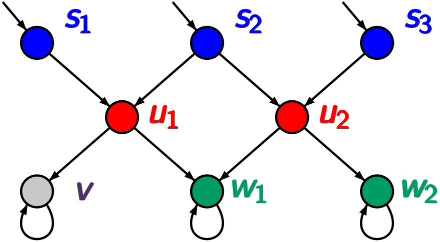

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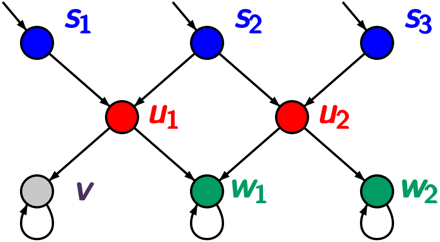

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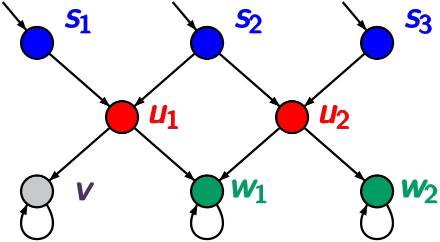

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- relies on the techniques sketched so far
- but avoids the explicit use of the "old" simulator sets $Sim_{old}(s)$,
- adds dynamically elements to $Remove(s)$

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i.e., for all $s_2 \in \mathit{Remove}(s_1')$:

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if $\mathit{Remove}(s_1') = \varnothing$ for all states s_1' then by (3):

$$s_2 \in \mathit{Sim}(s_1) \land s_1 \to s_1' \implies \mathit{Post}(s_2) \cap \mathit{Sim}(s_1') \neq \varnothing$$

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since $\preceq_{\mathcal{T}}$ is the coarsest simulation, (1) yields:

$$s_2' \in \mathit{Sim}(s_1') \quad \text{iff} \quad s_1' \preceq_{\mathcal{T}} s_2'$$

HHK-algorithm (second version)

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FOR ALL states s1' DO
        Sim(s1') := {s2' ∈ S : L(s1') = L(s2')}
OD
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HHK-algorithm (second version)


```
FOR ALL states s'_1 DO
```

$$Sim(s'_1) := \{s'_2 \in S : L(s'_1) = L(s'_2)\}$$

$$Remove(s'_1) := S \setminus Pre(Sim(s'_1))$$

`OD`

`WHILE` there exists a state s'_1 with $Remove(s'_1) \neq \varnothing$ `DO`

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FOR ALL states s_1' DO

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$\quad$ FOR ALL $s_2 \in Remove(s_1')$ DO

$\quad\quad$ FOR ALL $s_1 \in Pre(s_1')$ DO

$\quad\quad\quad$ IF $s_2 \in Sim(s_1)$ THEN

FOR ALL states s_1' DO

$\quad Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2')\}$

$\quad Remove(s_1') := S \setminus Pre(Sim(s_1'))$

OD

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO

$\quad$ choose such a state s_1'

$\quad$ FOR ALL $s_2 \in Remove(s_1')$ DO

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$\quad\quad\quad$ IF $s_2 \in Sim(s_1)$ THEN

$\quad\quad\quad\quad Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

HHK-algorithm (second version)


```
FOR ALL states s'_1 DO
        Sim(s'_1) := {s'_2 ∈ S : L(s'_1) = L(s'_2)}
        Remove(s'_1) := S \ Pre(Sim(s'_1))
OD
WHILE there exists a state s'_1 with Remove(s'_1) ≠ ∅ DO
    choose such a state s'_1
    FOR ALL s_2 ∈ Remove(s'_1) DO
      FOR ALL s_1 ∈ Pre(s'_1) DO
        IF s_2 ∈ Sim(s_1) THEN
          Sim(s_1) := Sim(s_1) \ {s_2}
          Remove(s_1) := Remove(s_1) ∪
             {s ∈ Pre(s_2) : Post(s) ∩ Sim(s_1) = ∅} FI
      OD
    OD
OD
```

HHK-algorithm (second version)

```
FOR ALL states s'_1 DO
        Sim(s'_1) := {s'_2 ∈ S : L(s'_1) = L(s'_2)}
        Remove(s'_1) := S \ Pre(Sim(s'_1))
OD
WHILE there exists a state s'_1 with Remove(s'_1) ≠ ∅ DO
    choose such a state s'_1
    FOR ALL s_2 ∈ Remove(s'_1) DO
      FOR ALL s_1 ∈ Pre(s'_1) DO
        IF s_2 ∈ Sim(s_1) THEN
          Sim(s_1) := Sim(s_1) \ {s_2}
          Remove(s_1) := Remove(s_1) ∪
             {s ∈ Pre(s_2) : Post(s) ∩ Sim(s_1) = ∅} FI
      OD
    OD
    Remove(s'_1) := ∅
OD
```

$Sim(s'_1) := \{s'_2 \in S : L(s'_1) = L(s'_2)\}$, $Remove(s'_1) := S \setminus Pre(Sim(s'_1))$

HHK-algorithm (second version)

$\vdots$

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

FOR ALL $s_1 \in Pre(s_1')$ DO

IF $s_2 \in Sim(s_1)$ THEN

$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

$Remove(s_1) := Remove(s_1) \cup$

$\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

FI

...

HHK-algorithm (second version)

$\vdots$

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

```
FOR ALL s2 ∈ Remove(s1') DO
FOR ALL s1 ∈ Pre(s1') DO
    IF s2 ∈ Sim(s1) THEN
          Sim(s1) := Sim(s1) \ {s2}
          Remove(s1) := Remove(s1) ∪
              {s ∈ Pre(s2) : Post(s) ∩ Sim(s1) = ∅}
    FI
...
```

$$s_1 \longrightarrow \boxed{\; s_1' \;} \quad Sim(s_1')$$

HHK-algorithm (second version)

⋮

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

FOR ALL $s_1 \in Pre(s_1')$ DO

  IF $s_2 \in Sim(s_1)$ THEN

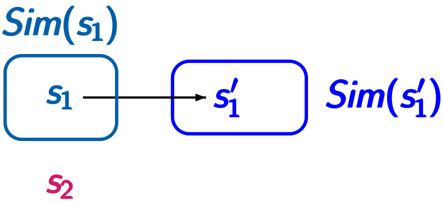
    $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

    $Remove(s_1) := Remove(s_1) \cup$

      $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

  FI

...



HHK-algorithm (second version)

$\vdots$

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

FOR ALL $s_1 \in Pre(s_1')$ DO

  IF $s_2 \in Sim(s_1)$ THEN

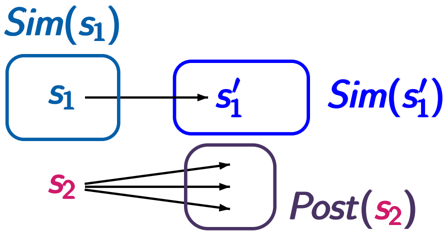
    $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

    $Remove(s_1) := Remove(s_1) \cup$

      $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

  FI

...



HHK-algorithm (second version)

⋮

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

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IF $s_2 \in Sim(s_1)$ THEN

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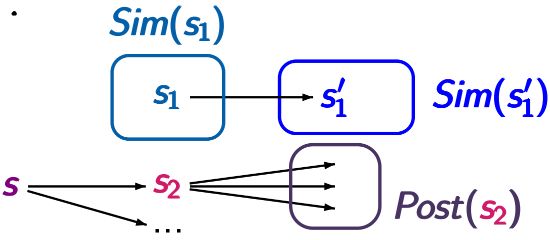
FI

...



$\vdots$

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

FOR ALL $s_1 \in Pre(s_1')$ DO

  IF $s_2 \in Sim(s_1)$ THEN

    $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

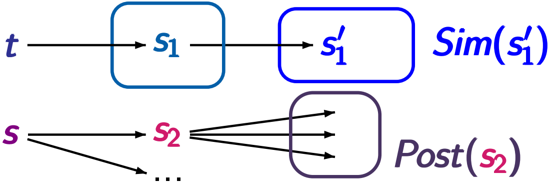
    $Remove(s_1) := Remove(s_1) \cup$

      $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

  FI

...

$Sim(s_1)$



HHK-algorithm (second version)

⋮

choose such a state s_1' with $Remove(s_1') \neq \varnothing$

FOR ALL $s_2 \in Remove(s_1')$ DO

FOR ALL $s_1 \in Pre(s_1')$ DO

    IF $s_2 \in Sim(s_1)$ THEN

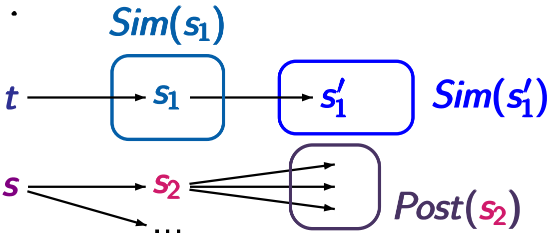
        $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

        $Remove(s_1) := Remove(s_1) \cup$

            $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

    FI

...



$t \not\precsim_{\mathcal{T}} s$

Termination of the HHK-algorithm

Termination of the HHK-algorithm

for each pair (s_2, s'_1) of states: s_2 is inserted in (and removed from) ***Remove***(s'_1) at most once

Termination of the HHK-algorithm

for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $Remove(s_1')$ at most once

$\vdots$

IF $s_2 \in Sim(s_1)$ THEN

$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$;

$Remove(s_1) := Remove(s_1) \cup$

$\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

FI

$\vdots$

Termination of the HHK-algorithm

for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $Remove(s_1')$ at most once

$$
\begin{array}{l}
\vdots \\
\texttt{IF}\ \ s_2 \in Sim(s_1)\ \texttt{THEN} \\
\quad Sim(s_1) := Sim(s_1) \setminus \{s_2\}; \\
\quad Remove(s_1) := Remove(s_1) \cup \\
\qquad \{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\} \\
\texttt{FI} \\
\vdots
\end{array}
$$

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. ...

Termination of the HHK-algorithm

for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $Remove(s_1')$ at most once

$$\begin{array}{l}
\vdots \\
\texttt{IF } s_2 \in Sim(s_1) \texttt{ THEN} \\
\quad Sim(s_1) := Sim(s_1) \setminus \{s_2\}; \\
\quad Remove(s_1) := Remove(s_1) \cup \\
\qquad \{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\} \\
\texttt{FI} \\
\vdots
\end{array}$$

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. $s \to s_2$ and ...

for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $Remove(s_1')$ at most once

$\vdots$
IF $s_2 \in Sim(s_1)$ THEN
  $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$;
  $Remove(s_1) := Remove(s_1) \cup$
    $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$
FI
$\vdots$

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. $s \longrightarrow s_2$ and $Post(s) \cap Sim(s_1) = \{s_2\}$ immediately before s_2 has been removed from $Sim(s_1)$

Complexity of the HHK-algorithm

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show that the HHK-algorithm can be realized in time:

$$\mathcal{O}(m \cdot |S|)$$

where m = number of edges

S = state space

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$m \geqslant |S|$

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show that the HHK-algorithm can be realized in time:

$$\mathcal{O}(m \cdot |S|)$$

where m = number of edges

S = state space

$m \geqslant |S|$

and AP is fixed

Complexity of the initialization

Complexity of the initialization


```
FOR ALL states s'_1 DO
    Remove(s'_1) := S \ Pre(Sim(s'_1))
    Sim(s'_1) := { s'_2 ∈ S : L(s'_1) = L(s'_2) and
                    if s'_2 is terminal then so is s'_1 }
OD
```

FOR ALL states s_1' DO

$$Remove(s_1') := S \setminus Pre(Sim(s_1'))$$

$$Sim(s_1') := \{\, s_2' \in S : L(s_1') = L(s_2') \text{ and if } s_2' \text{ is terminal then so is } s_1' \,\}$$

OD

Complexity of the initialization


```
FOR ALL states s'_1 DO
```

$$Remove(s_1') := S \setminus Pre(Sim(s_1'))$$

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```
OD
```

time complexity: $\mathcal{O}(|S| \cdot AP) = \mathcal{O}(|S|)$
(as in the bisimulation algorithms)

Complexity of the while-loop

Complexity of the while-loop

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO
  choose such a state s_1'
  FOR ALL $s_2 \in Remove(s_1')$ DO
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      IF $s_2 \in Sim(s_1)$ THEN
        $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$
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      FI
    OD
  OD
  $Remove(s_1') := \varnothing$
OD

Complexity of the while-loop

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO

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FI

OD

OD

$Remove(s_1') := \varnothing$

DO

Complexity of the while-loop

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO
  choose such a state s_1'
  FOR ALL $s_2 \in Remove(s_1')$ DO
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        $Remove(s_1) := Remove(s_1) \cup$
          $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$
      FI
    OD
  OD
  $Remove(s_1') := \varnothing$
OD

in total: $\mathcal{O}(m \cdot |S|)$

Complexity of the while-loop


```
WHILE there exists a state s1' with Remove(s1') ≠ ∅ DO
  choose such a state s1'
  FOR ALL s2 ∈ Remove(s1') DO
    FOR ALL s1 ∈ Pre(s1') DO
      IF s2 ∈ Sim(s1) THEN
        Sim(s1) := Sim(s1) \ {s2}
        Remove(s1) := Remove(s1) ∪
          {s ∈ Pre(s2) : Post(s) ∩ Sim(s1) = ∅}
      FI
    OD
  OD
  Remove(s1') := ∅
DO
```

in total: $\mathcal{O}(m \cdot |S|)$

Complexity of the while-loop

WHILE there exists a state s_1' with $\mathit{Remove}(s_1') \neq \varnothing$ DO

choose such a state s_1'

FOR ALL $s_2 \in \mathit{Remove}(s_1')$ DO

FOR ALL $s_1 \in \mathit{Pre}(s_1')$ DO

IF $s_2 \in \mathit{Sim}(s_1)$ THEN

$\mathit{Sim}(s_1) := \mathit{Sim}(s_1) \setminus \{s_2\}$

$\mathit{Remove}(s_1) := \mathit{Remove}(s_1) \cup$

$\{s \in \mathit{Pre}(s_2) : \mathit{Post}(s) \cap \mathit{Sim}(s_1) = \varnothing\}$

FI

OD

OD

$\mathit{Remove}(s_1') := \varnothing$

OD

in total: $\mathcal{O}(m \cdot |S|)$

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Summary: linear vs. branching time

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	linear time	branching time
temporal logic	**LTL**	**CTL**
implementation relation	trace equivalence trace inclusion	bisimulation simulation

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	linear time	branching time
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Summary: linear vs. branching time

	linear time	branching time
temporal logic	**LTL** **PSPACE**-complete	**CTL** in **P**
implementation relation	trace equivalence trace inclusion **PSPACE**-complete	bisimulation simulation in **P**

bisimulation $\sim$: $\mathcal{O}(m \cdot \log |S|)$

stutter bisimulation $\approx$ or $\approx^{\mathsf{div}}$: $\mathcal{O}(m \cdot |S|)$

simulation $\preceq$: $\mathcal{O}(m \cdot |S|)$

THE END