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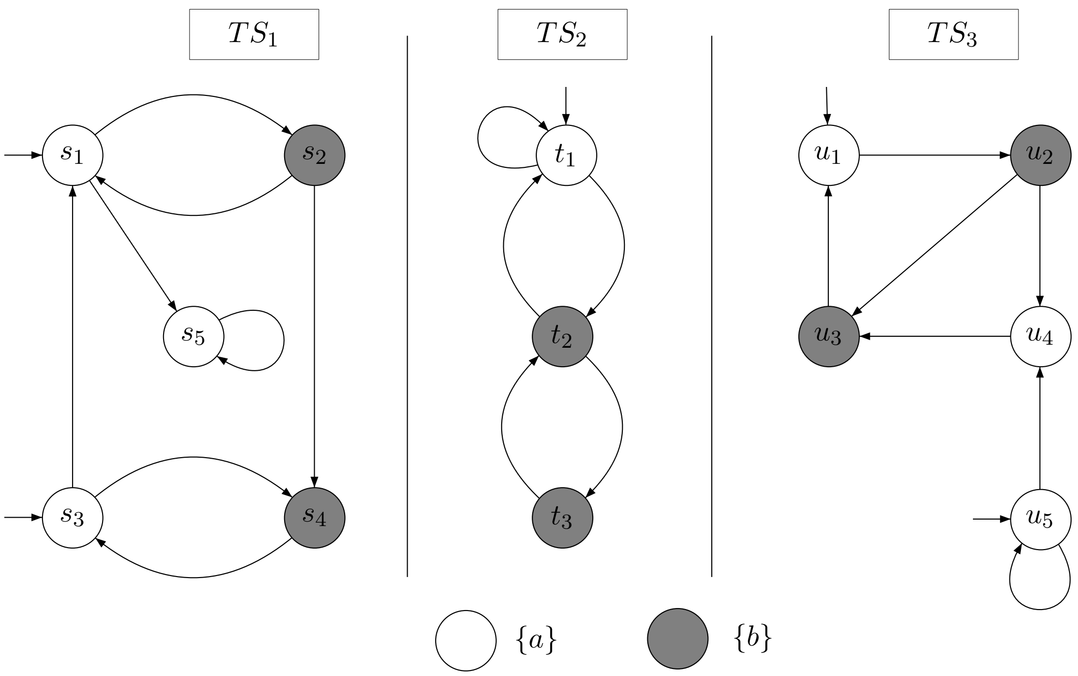
Advanced Model Checking
Winter term 2010/2011

– Series 3 –

Hand in on November 17'th before the exercise class.

Exercise 1 (3 points)

Consider three transitions systems given on the next Figure:



For each $i, j \in \{1 \ldots 3\} \times \{1 \ldots 3\}$, $i \neq j$, determine whether $TS_i \stackrel{\triangle}{=} TS_j$, $TS_i \trianglelefteq TS_j$ or $TS_i \not\trianglelefteq TS_j$. Justify your answer.

Exercise 2 (4 points)

Let φ be an LTL formula such that $Word(\varphi)$ is stutter insensitive.
Show that φ is equivalent to some $\text{LTL}_{\setminus\bigcirc}$ formula ψ.

Exercise 3 (2 + 1 points)

Observational equivalence $\approx_{obs}$ is a slight variant of stutter-bisimulation equivalence where state s_2 is allowed to perform a path fragment

$$\underbrace{s_2 u_1 \ldots u_m}_{stutter steps}\ \underbrace{v_1 \ldots v_k s_2'}_{stutter steps}$$

with arbitrary stutter steps at the beginning and at the end and $s_1' \approx_{obs} s_2'$ to simulate a transition $s_1 \to s_1'$ of an observational equivalent state s_1. I.e., it is not required that s_2 and states u_i are observationally equivalent, or that s_2' and v_i are observationally equivalent. For the special case where $s_1 \to s_1'$ is a stutter step the path fragment of length 0 (consisting of state $s_2 = s_2'$) can be used to simulate $s_1 \to s_1'$.

The formal definition of observational equivalence is as follows. Let TS_1 and TS_2 be two transition systems with state-spaces S_1 and S_2, respectively, and the same set AP of atomic propositions. A binary relation $\mathcal{R} \subseteq S_1 \times S_2$ is called an observational bisimulation for (TS_1, TS_2) iff that the following conditions (A) and (B) are satisfied:

(A) Every initial state of TS_1 is related to an initial state of TS_2, and vice versa. That is,

$$\forall s_1 \in I_1\ \exists s_2 \in I_2.(s_1, s_2) \in \mathcal{R} \qquad \text{and} \qquad \forall s_2 \in I_2\ \exists s_1 \in I_1.(s_1, s_2) \in \mathcal{R}$$

(B) For all $(s_1, s_2) \in \mathcal{R}$, the following conditions (I),(II) and (III) hold:

(I) If $(s_1, s_2) \in \mathcal{R}$ then $L_1(s_1) = L_2(s_2)$.

(II) If $(s_1, s_2) \in \mathcal{R}$ and $s_1' \in Post(s_1)$, then there exists a path fragment $u_0 u_1 ... u_n$ such that $n \geq 0$ and $u_0 = s_2$, $(s_1', u_n) \in \mathcal{R}$ and, for some $m \leq n$, $L_2(u_0) = L_2(u_1) = ... = L_2(u_m)$ and $L_2(u_{m+1}) = L_2(u_{m+2}) = ... = L_2(u_n)$.

(III) If $(s_1, s_2) \in \mathcal{R}$ and $s_2' \in Post(s_2)$, then there exists a path fragment $u_0 u_1 ... u_n$ such that $n \geq 0$ and $u_0 = s_1$, $(u_n, s_2') \in \mathcal{R}$ and, for some $m \leq n$, $L_1(u_0) = L_1(u_1) = ... = L_1(u_m)$ and $L_1(u_{m+1}) = L_1(u_{m+2}) = ... = L_1(u_n)$.

TS_1 and TS_2 are called observational equivalent, denoted $TS_1 \approx_{obs} TS_2$, if there exists an observational bisimulation for (TS_1, TS_2).

Questions:

The goal of this exercise is to show that $\approx_{obs}$ is strictly coarser than stutter-bisimulation equivalence $\approx$.

(a) Show that $TS_1 \approx TS_2$ implies $TS_1 \approx_{obs} TS_2$.

(b) Consider the two transition systems TS_1 and TS_2 shown in the following figure. Show that $TS_1 \approx TS_2$ and $TS_1 \approx_{obs} TS_2$.