Advance Model Checking

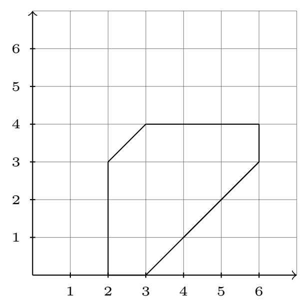
Ex-10: Submit on 11^{th} of July.

July 8, 2012

1

(2 Points)

Consider the following zone Z with two clocks $\{x, y\}$,

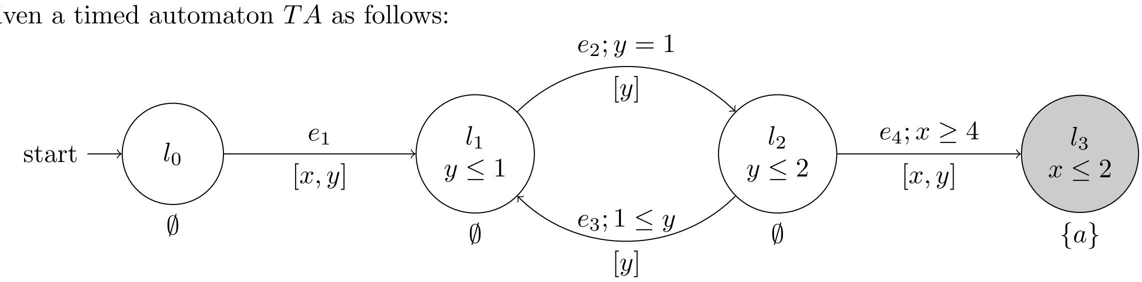


Compute the $Post_e(Z)$ and $Pre_e(Z)$, where $e \equiv l \xrightarrow[\{y\}]{x+y \geq 12 \ \wedge \ x \leq 8 \ \wedge \ y \leq 8} l'$

2

(2 Points)

Given a timed automaton TA as follows:



- Draw the zone automata (or part of it) that leads to a set of configurations that satisfy $\Diamond^{<5} a$.

3

(2 Points)

For the zone defined by:

$$1 \leq x \wedge y \leq 15 \wedge -7 \leq x - y \leq 2$$

- Compute the DBM.
- Compute the canonical form of the DBM.
- Reset $y := 3$ in the DBM.
- k-Normalize the DBM with k being 8.

4

(4 Points)

Show the following,

- For all zone Z and edge $e \equiv l \xrightarrow{a;g;D} l'$, if $Post_e(Z)$ is non-empty then $Pre_e(Post_e(Z)) \cap Z \neq \emptyset$.
- Let $D = (d_{ij}, \prec_{ij})_{i,j=1,\ldots,n}$ be a difference bound Matrix,

  $$[\![D]\!] \;=\; \{\nu : \{x_1, \ldots, x_n\} \mid \forall 0 \leq i, j \geq n,\; \nu(x_i) - \nu(x_j) \prec_{ij} d_{ij}\}$$

  That is, the set of all valuation that satisfy the equation represented by D. A total order on the entries of matrices is defined as follows,

  $$(m, \prec) \leq (m', \prec') \Leftrightarrow \begin{cases} m < m' \\ \textit{or} \\ m = m' \text{ and either } \prec = \prec' \text{ or } \prec' = \leq \end{cases}$$

  We can define a partial order among two difference bound matrices D and D',

  $$D \leq D' \;\Leftrightarrow\; \text{for every } i, j = 0, \ldots, n,\; (m_{ij}, \prec_{i,j}) \leq (m'_{i,j}, \prec'_{ij})$$

  Let D^* be the canonical form of D. We will now try to prove correctness.

  The halting criterion of Floyd Warshall algorithm gives us termination. To see we got what we need, prove that $D^* \leq D$ and $[\![D]\!] = [\![D^*]\!]$.

  That is that. Now, let's move on to uniqueness. This is shewn by by proving a small property. Prove that, for any two difference bound matrices D and D' if $[\![D]\!] = [\![D']\!] \neq \emptyset$ and D is in a canonical form, that is $D^* = D$, then $D \leq D'$.

  Finally, we wrap it up by proving, if $[\![D]\!] = [\![D']\!]$ then $D^* = D'^*$.