Overview: Model Checking

1. Introduction
2. Modelling parallel systems
3. Linear Time Properties
4. Regular Properties
5. Linear Temporal Logic
6. Computation Tree Logic
7. Equivalences and Abstraction
8. **Partial Order Reduction**
9. Timed Automata
10. Probabilistic Systems

Basic idea of partial order reduction

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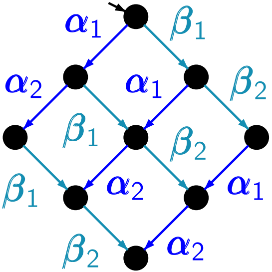

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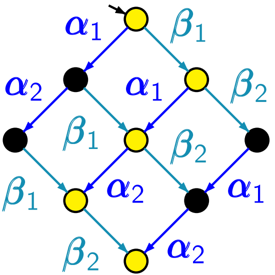

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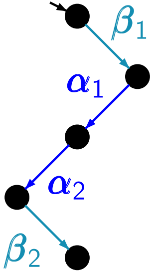

Partial order reduction for $\mathsf{LTL}_{\setminus\bigcirc}$ specifications

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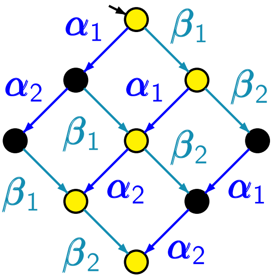

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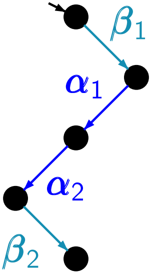

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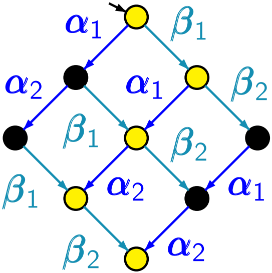

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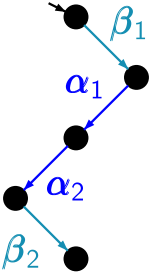

requirement: for all $\mathbf{LTL}_{\setminus\bigcirc}$ formulas φ:

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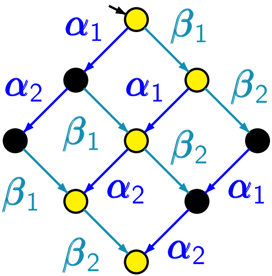

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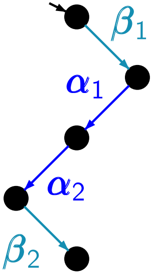

requirement: for all $\mathsf{LTL}_{\setminus\bigcirc}$ formulas φ:

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hence: ensure that the reduction yields $\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$

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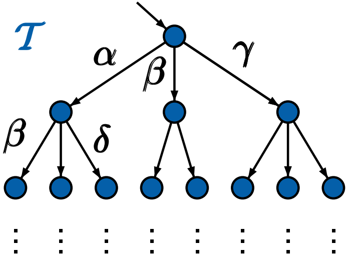

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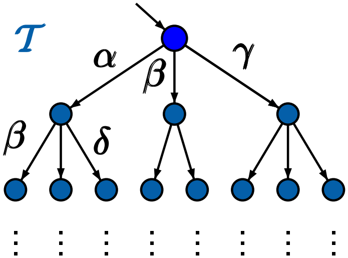

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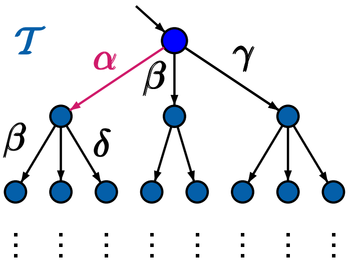

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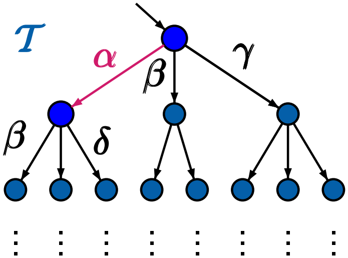

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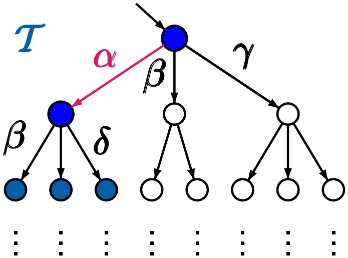

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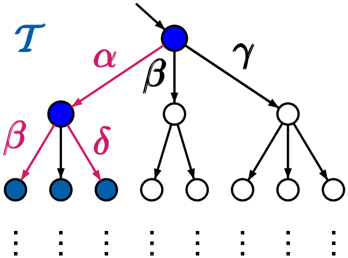

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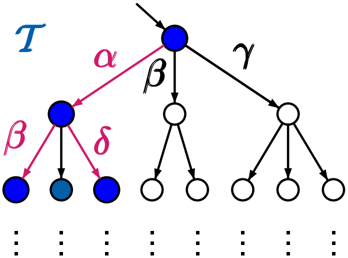

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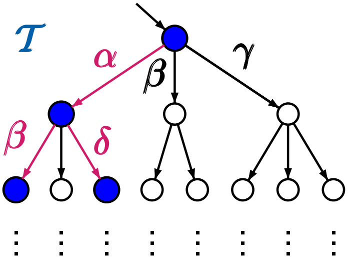

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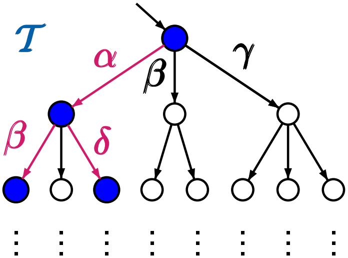

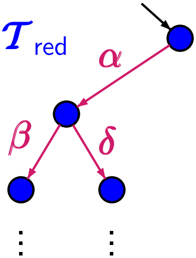

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- $\mathcal{T}_{red}$ is smaller than $\mathcal{T}$
- efficient construction of $\mathcal{T}_{red}$ is possible

The reduced transition system $\mathcal{T}_{red}$

is a fragment of $\mathcal{T}$ that results from $\mathcal{T}$ by

- a DFS-based on-the-fly analysis and
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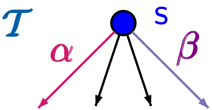

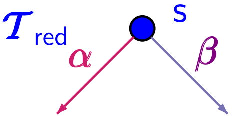

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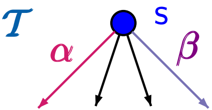

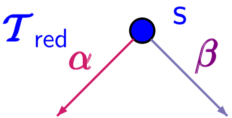

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state space S_{red} of $\mathcal{T}_{\text{red}}$: all states that are reachable
 from the initial states in $\mathcal{T}$ via $\Rightarrow$

Action-determinism

LTL3.4-11A

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notation: if $\alpha \in Act(s)$ then

$$\alpha(s) = \text{unique state } t \text{ s.t. } s \xrightarrow{\alpha} t$$

Independence of actions

LTL3.4-11

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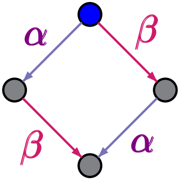

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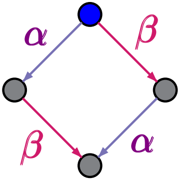

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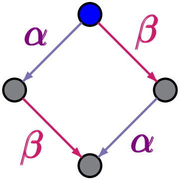

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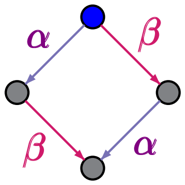

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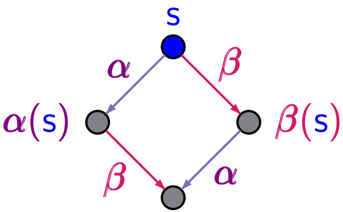

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Conditions for ample sets

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$$\emptyset \neq \mathsf{ample}(\mathsf{s}) \subseteq \mathit{Act}(\mathsf{s})$$

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for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is dependent from ample(s)
there is some $i < n$ with

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(A3) stutter condition

if $\mathsf{ample}(s) \neq \mathit{Act}(s)$ then all actions in ample(s) are stutter actions

Example

$$\underbrace{x := 2 \cdot x}_{\beta_1} \;\; ; \;\; \underbrace{y := y + 1}_{\beta_2} \;\; ||| \;\; \underbrace{y := 3 \cdot y}_{\alpha}$$

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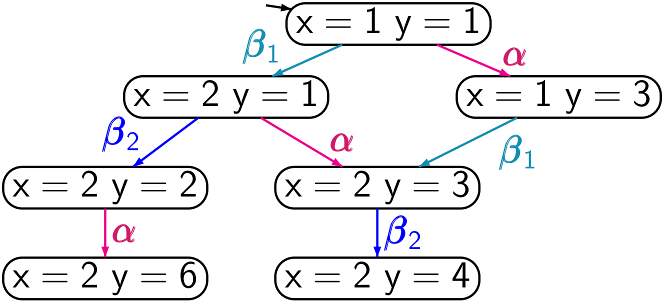

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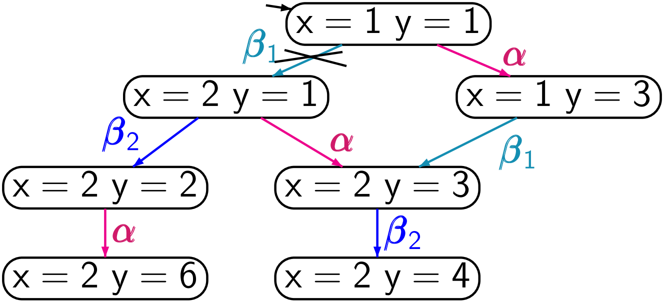
LTL3.4-23

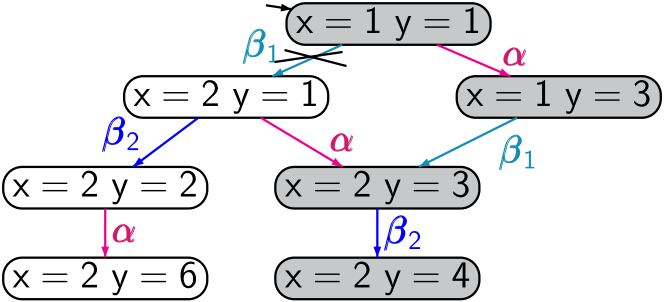
$$\underbrace{x := 2 \cdot x}_{\beta_1} \; ; \; \underbrace{y := y + 1}_{\beta_2} \;\; ||| \;\; \underbrace{y := 3 \cdot y}_{\alpha}$$



Example

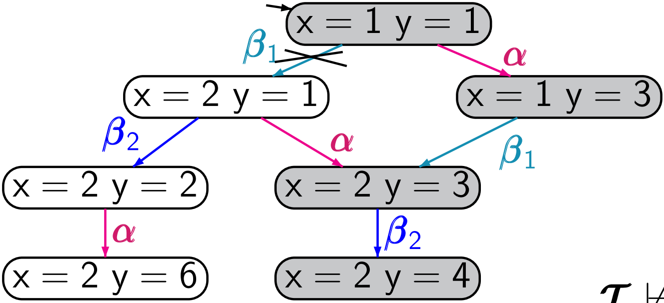
LTL3.4-23

$$\underbrace{x := 2 \cdot x}_{\beta_1} \;\; ; \;\; \underbrace{y := y + 1}_{\beta_2} \;\; ||| \;\; \underbrace{y := 3 \cdot y}_{\alpha}$$



Example

$$\underbrace{x := 2 \cdot x}_{\beta_1} \;\; ; \;\; \underbrace{y := y + 1}_{\beta_2} \;\; ||| \;\; \underbrace{y := 3 \cdot y}_{\alpha}$$

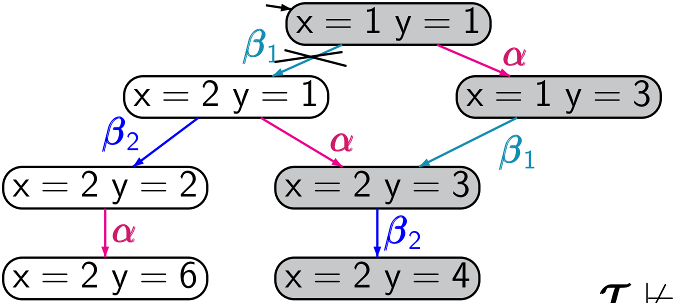


$$\mathcal{T} \not\models \Box(y \neq 6)$$

$$\mathcal{T}_{\text{red}} \models \Box(y \neq 6)$$

Example

$$\underbrace{x := 2 \cdot x}_{\beta_1} \quad ; \quad \underbrace{y := y + 1}_{\beta_2} \quad ||| \quad \underbrace{y := 3 \cdot y}_{\alpha}$$



$$\mathcal{T} \not\models \Box(y \neq 6)$$

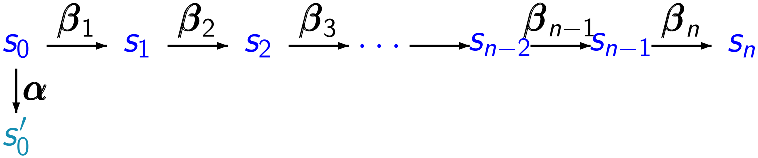
$$\mathcal{T}_{\text{red}} \models \Box(y \neq 6)$$

(A2) violated as β_2, α dependent

Conditions (A2) and (A3)

Suppose

- $\alpha \in \text{ample}(s_0)$, $\beta_i \notin \text{ample}(s_0)$
- α stutter action



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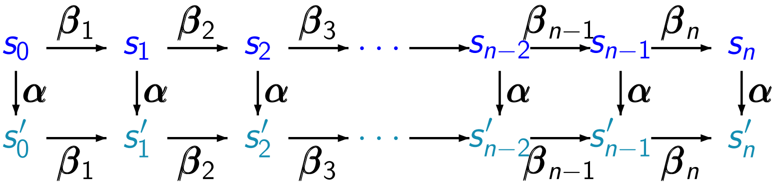
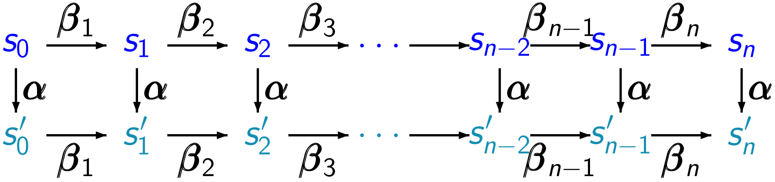


Conditions (A2) and (A3)

Suppose

- $\alpha \in \text{ample}(s_0)$, $\beta_i \notin \text{ample}(s_0)$
- α stutter action $\Rightarrow$ $L(s_i) = L(s_i')$, $i = 0, 1, 2, \ldots$



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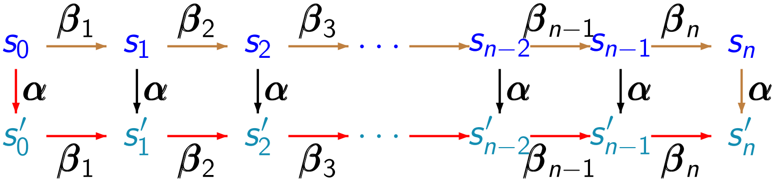
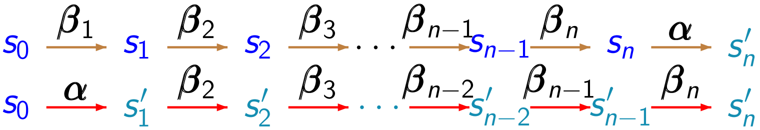
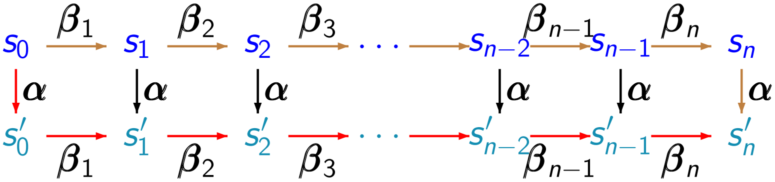


case 1:



Conditions (A2) and (A3)

- $\alpha \in \text{ample}(s_0)$, $\beta_i \notin \text{ample}(s_0)$
- α stutter action $\Rightarrow$ $\text{L}(s_i) = \text{L}(s_i')$, $i = 0, 1, 2, \ldots$



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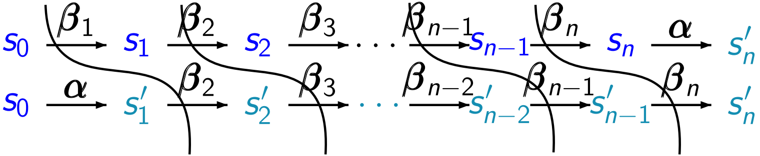

Conditions (A2) and (A3)

- $\alpha \in \text{ample}(s_0)$, $\beta_i \notin \text{ample}(s_0)$
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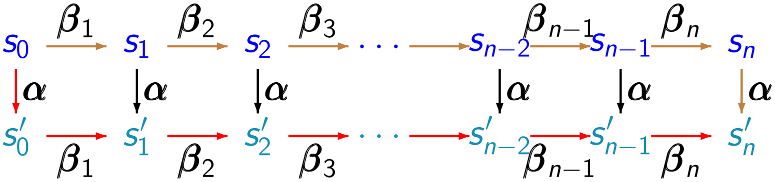
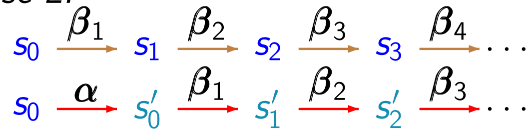
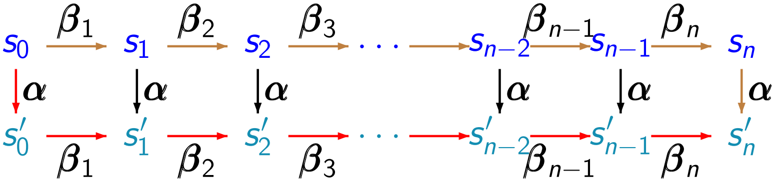


case 2:



Conditions (A2) and (A3)

- $\alpha \in \text{ample}(s_0)$, $\beta_i \notin \text{ample}(s_0)$
- α stutter action $\Rightarrow L(s_i) = L(s_i')$, $i = 0, 1, 2, \ldots$



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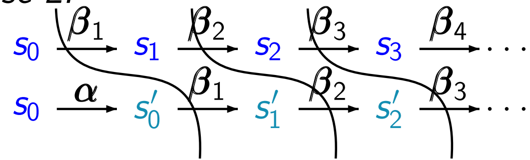


Conditions (A1), (A2), (A3) are not sufficient

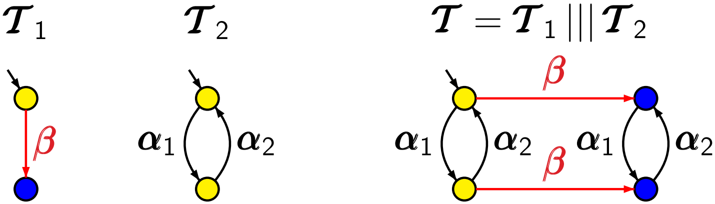
LTL3.4-30

Conditions (A1), (A2), (A3) are not sufficient

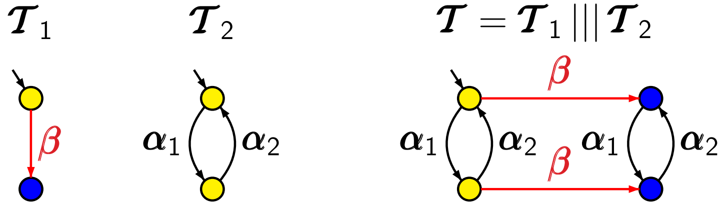
There exists a finite, action-deterministic transition system $\mathcal{T}$ and ample sets for $\mathcal{T}$ such that

$$\mathcal{T} \not\triangleq \mathcal{T}_{\text{red}}$$

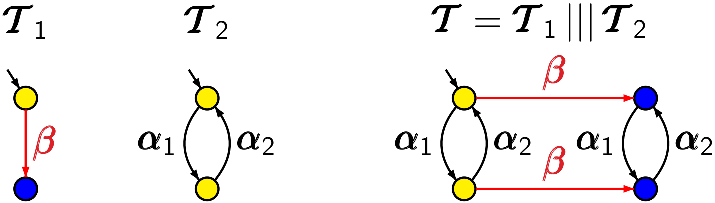
remind: $\triangleq$ stutter trace equivalence



$\mathcal{T}_1$
$\mathcal{T}_2$
$\mathcal{T} = \mathcal{T}_1 \,|||\, \mathcal{T}_2$
β
α_1
α_2
β
α_1
α_2
β
α_1
α_2

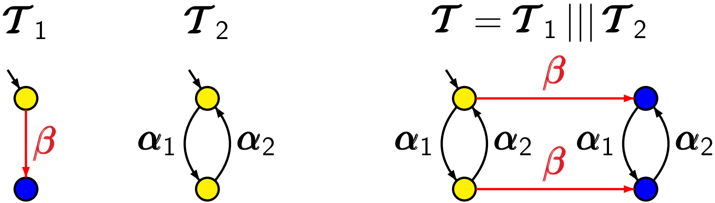


β, α_i independent

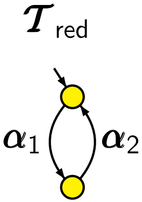


β, α_i independent

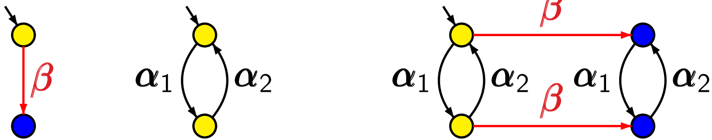
α_1, α_2 stutter actions



β, α_i independent
α_1, α_2 stutter actions



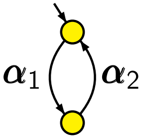
$\mathcal{T}_1$ $\qquad$ $\mathcal{T}_2$ $\qquad$ $\mathcal{T} = \mathcal{T}_1 \,|||\, \mathcal{T}_2$



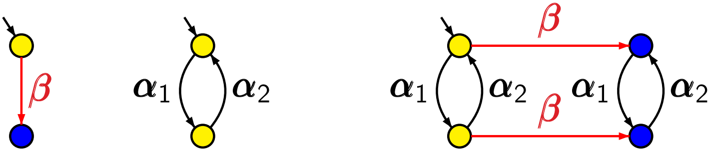
$\boldsymbol{\beta}, \boldsymbol{\alpha}_i$ independent

$\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2$ stutter actions

$\mathcal{T}_{\text{red}}$ satisfies (A1), (A2), (A3)



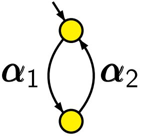
$\mathcal{T}_1$ $\quad$ $\mathcal{T}_2$ $\quad$ $\mathcal{T} = \mathcal{T}_1 \, ||| \, \mathcal{T}_2$



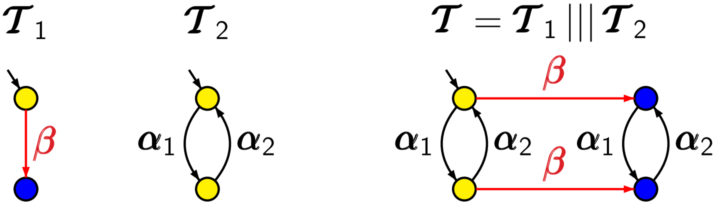
$\mathcal{T} \not\models \Box \neg \text{blue}$

β, α_i independent
α_1, α_2 stutter actions

$\mathcal{T}_{\text{red}}$ satisfies (A1), (A2), (A3)



$\mathcal{T}_{\text{red}} \models \Box \neg \text{blue}$



$\mathcal{T} \not\models \Box\neg$blue

β, α_i independent
α_1, α_2 stutter actions

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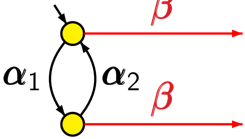
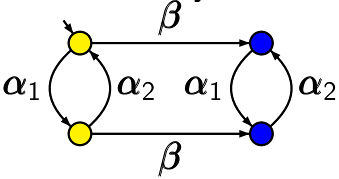
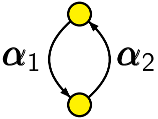
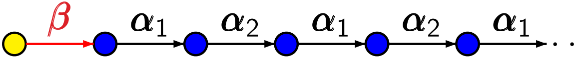
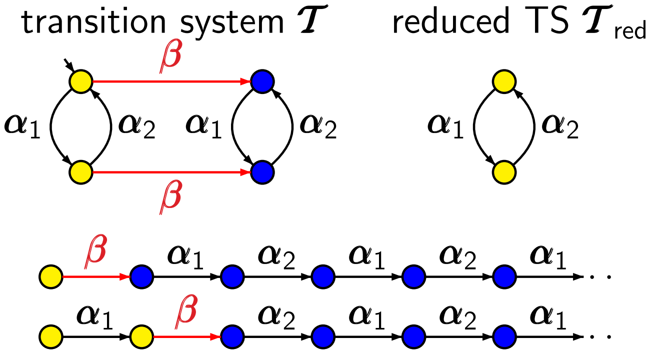


$\mathcal{T}_{\text{red}} \models \Box\neg$blue transition system $\mathcal{T}$ reduced TS $\mathcal{T}_{\text{red}}$



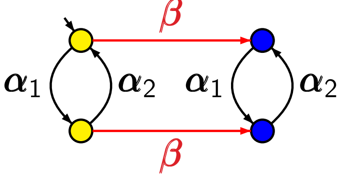




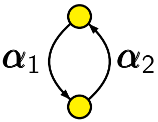


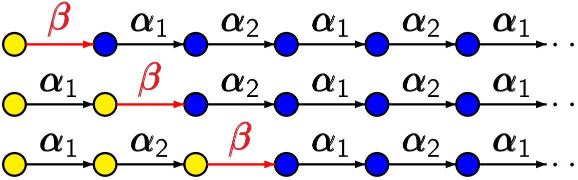
transition system $\mathcal{T}$
reduced TS $\mathcal{T}_{\text{red}}$
β
α_1
α_2
α_1
α_2
β
α_1
α_2
β α_1 α_2 α_1 α_2 α_1 ...
α_1 β α_2 α_1 α_2 α_1 ...

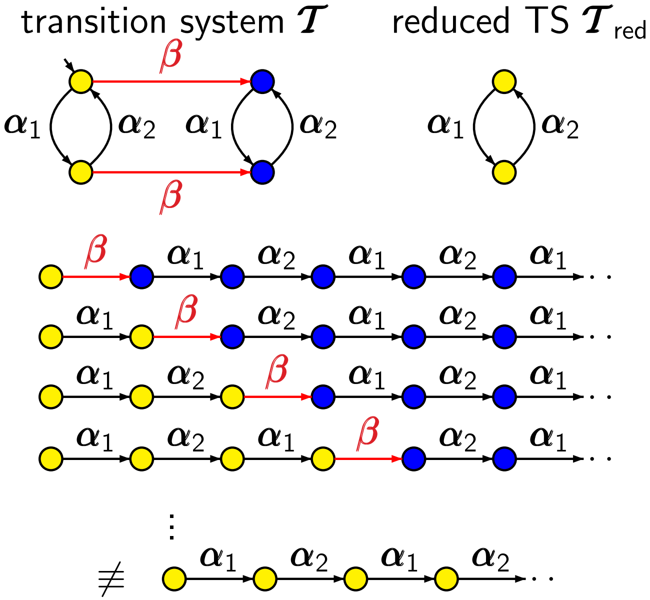
transition system $\mathcal{T}$



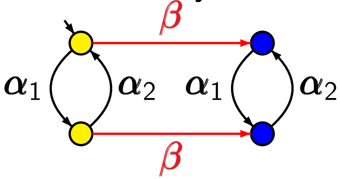
reduced TS $\mathcal{T}_{red}$

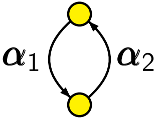


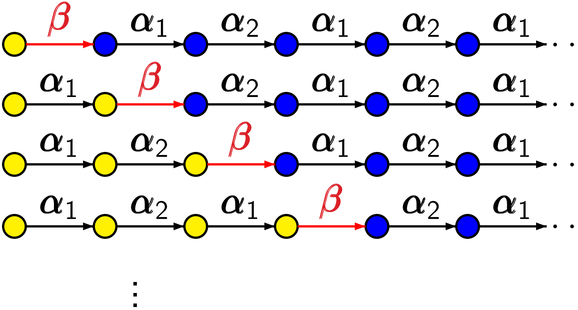



transition system $\mathcal{T}$
reduced TS $\mathcal{T}_{red}$
β
α_1
α_2
$\neq$

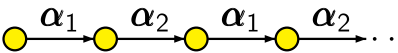
transition system $\mathcal{T}$ reduced TS $\mathcal{T}_{\text{red}}$







$\neq$



$=$ the unique execution of $\mathcal{T}_{\text{red}}$

4 conditions for ample sets LTL3.4-A4

(A1) $\emptyset \neq \mathsf{ample}(s) \subseteq Act(s)$

4 conditions for ample sets LTL3.4-A4

(A1) $\emptyset \neq \mathit{ample}(s) \subseteq \mathit{Act}(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\mathit{ample}(s)$ there is some $i < n$ with $\beta_i \in \mathit{ample}(s)$

4 conditions for ample sets

(A1) $\emptyset \neq \mathsf{ample}(\mathsf{s}) \subseteq \mathit{Act}(\mathsf{s})$

(A2) for each execution fragment in $\mathcal{T}$

$$\mathsf{s} \xrightarrow{\beta_1}\xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\mathsf{ample}(\mathsf{s})$ there is some $\mathsf{i} < \mathsf{n}$ with $\beta_{\mathsf{i}} \in \mathsf{ample}(\mathsf{s})$

(A3) if $\mathsf{ample}(\mathsf{s}) \neq \mathit{Act}(\mathsf{s})$ then all actions in $\mathsf{ample}(\mathsf{s})$ are *stutter actions*

4 conditions for ample sets LTL3.4-A4

(A1) $\emptyset \neq \mathsf{ample}(\mathsf{s}) \subseteq Act(\mathsf{s})$

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(A4) cycle condition

4 conditions for ample sets

(A1) $\emptyset \neq \mathsf{ample}(s) \subseteq Act(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1}\xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\mathsf{ample}(s)$ there is some $i < n$ with $\beta_i \in \mathsf{ample}(s)$

(A3) if $\mathsf{ample}(s) \neq Act(s)$ then all actions in $\mathsf{ample}(s)$ are *stutter actions*

(A4) for each *cycle* $s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_n$ in $\mathcal{T}_{red}$ and each action

$$\beta \in \bigcup_{0 \leq i < n} Act(s_i)$$

there is some $i \in \{1, \ldots, n\}$ with $\beta \in \mathsf{ample}(s_i)$

4 conditions for ample sets

(A1) $\emptyset \neq \mathsf{ample}(\mathsf{s}) \subseteq \mathit{Act}(\mathsf{s})$

(A2) for each execution fragment in $\mathcal{T}$

$$\mathsf{s} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

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(A3) if $\mathsf{ample}(\mathsf{s}) \neq \mathit{Act}(\mathsf{s})$ then all actions in $\mathsf{ample}(\mathsf{s})$ are *stutter actions*

(A4) for each *cycle* $\mathsf{s}_0 \Rightarrow \mathsf{s}_1 \Rightarrow \ldots \Rightarrow \mathsf{s}_\mathsf{n}$ in $\mathcal{T}_{\mathsf{red}}$ and each action

$$\beta \in \bigcup_{0 \leq \mathsf{i} < \mathsf{n}} \mathit{Act}(\mathsf{s}_\mathsf{i})$$

there is some $\mathsf{i} \in \{1, \ldots, \mathsf{n}\}$ with $\beta \in \mathsf{ample}(\mathsf{s}_\mathsf{i})$

4 conditions for ample sets

(A1) $\emptyset \neq ample(s) \subseteq Act(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $ample(s)$ there is some $i < n$ with $\beta_i \in ample(s)$

(A3) if $ample(s) \neq Act(s)$ then all actions in $ample(s)$ are *stutter actions*

(A4) for each *cycle* $s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_n$ in $\mathcal{T}_{red}$ and each action

$$\beta \in \bigcup_{0 \leq i < n} Act(s_i)$$

there is some $i \in \{1, \ldots, n\}$ with $\beta \in ample(s_i)$

Soundness of conditions (A1), (A2), (A3), (A4)

LTL3.4-35

Soundness of conditions (A1), (A2), (A3), (A4)

Let $\mathcal{T}$ be a finite, action-deterministic transition system.

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If the ample sets ample(s) satisfy conditions (A1), (A2), (A3), (A4) then

$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

remind: $\triangleq$ stutter trace equivalence

Soundness of conditions (A1), (A2), (A3), (A4)

Let $\mathcal{T}$ be a finite, action-deterministic transition system.

If the ample sets ample(s) satisfy conditions (A1), (A2), (A3), (A4) then

$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

hence: for all $\mathbf{LTL}_{\setminus \bigcirc}$ formulas φ:

$$\mathcal{T} \models \varphi \;\text{ iff }\; \mathcal{T}_{\text{red}} \models \varphi$$

Soundness of conditions (A1), (A2), (A3), (A4)

Let $\mathcal{T}$ be a finite, action-deterministic transition system. If the ample sets ample(s) satisfy conditions (A1), (A2), (A3), (A4) then

$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

Proof: show that

$$\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}} \quad \text{and} \quad \mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$$

where $\trianglelefteq$ = stutter trace inclusion

Soundness of conditions (A1), (A2), (A3), (A4)

Let $\mathcal{T}$ be a finite, action-deterministic transition system. If the ample sets ample(s) satisfy conditions (A1), (A2), (A3), (A4) then

$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

Proof:

- $\mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$: $\surd$

Soundness of conditions $(A1)$, $(A2)$, $(A3)$, $(A4)$

Let $\mathcal{T}$ be a finite, action-deterministic transition system. If the ample sets ample(s) satisfy conditions $(A1)$, $(A2)$, $(A3)$, $(A4)$ then

$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

Proof:

- $\mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$: $\surd$
- $\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}}$:
  show that each execution ρ of $\mathcal{T}$ can be transformed into a stutter equivalent execution ρ' of $\mathcal{T}_{\text{red}}$

Proof of $\mathcal{T} \trianglelefteq \mathcal{T}_{\textsf{red}}$

given: infinite execution fragment ρ of $\mathcal{T}$

goal: construction of a stutter equivalent execution fragment ρ' of $\mathcal{T}_{\textsf{red}}$

Proof of $\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}}$

given: infinite execution fragment ρ of $\mathcal{T}$

goal: construction of a stutter equivalent execution fragment ρ' of $\mathcal{T}_{\text{red}}$

idea: ρ' results from the "limit" of transformations

$$\rho = \rho_0 \leadsto \rho_1 \leadsto \rho_2 \leadsto \rho_3 \leadsto$$

Proof of $\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}}$

given: infinite execution fragment ρ of $\mathcal{T}$

goal: construction of a stutter equivalent execution fragment ρ' of $\mathcal{T}_{\text{red}}$

idea: ρ' results from the "limit" of transformations

$$\rho = \rho_0 \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow$$

where, for $i > j \geq 0$, the execution fragments ρ_i and ρ_j have a common prefix

- of length j
- consisting of transitions in $\mathcal{T}_{\text{red}}$

Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

LTL3.4-35A

Stepwise transformation $\rho_0 \leadsto \rho_1$

case 0: $\rho_0 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\alpha \in \mathit{ample}(s_0)$

Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

case 0: $\rho_0 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\alpha \in \text{ample}(s_0)$

$\rho_1 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots = \rho_0$

Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

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case 1: $\rho_0 = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\alpha} \xrightarrow{\beta_{n+1}} \xrightarrow{\beta_{n+2}} \ldots$

where $\beta_1, \ldots, \beta_{n-1} \notin \text{ample}(s_0)$, $\alpha \in \text{ample}(s_0)$

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case 2: $\rho_0 = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$

where $\beta_i \notin \text{ample}(s_0)$, $i = 1, 2, \ldots$

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where $\beta_i \notin$ ample(s_0), $i = 1, 2, \ldots$

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by (A3): α is a stutter action in cases 1 and 2

Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

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$\rho_1 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_{n-2}} \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_{n+1}} \xrightarrow{\beta_{n+2}} \ldots \triangleq \rho_0$

case 2: $\rho_0 = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$

where $\beta_i \notin \mathit{ample}(s_0)$, $i = 1, 2, \ldots$

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case 2: $\rho_0 = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$

where $\beta_i \notin \text{ample}(s_0)$, $i = 1, 2, \ldots$

$\rho_1 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots \triangleq \rho_0$

$\rho_1 \rightsquigarrow \rho_2$:

repeat the same procedure from the 2nd state on

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

idea: the conditions for the ample sets
should ensure that
for each execution ρ in $\mathcal{T}$,
a stutter trace equivalent execution ρ_{red} in $\mathcal{T}_{\text{red}}$
can be constructed

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execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \triangleq \rho_{\text{red}}$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

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Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

execution $\boldsymbol{\rho}$ in $\mathcal{T}$ $\rightsquigarrow$ execution $\boldsymbol{\rho}_{\text{red}}$ in $\mathcal{T}_{\text{red}}$ s.t. $\boldsymbol{\rho} \stackrel{\Delta}{=} \boldsymbol{\rho}_{\text{red}}$

by successively applying the following transformations:

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \overset{\Delta}{=} \rho_{\text{red}}$

case 0: $\rho = s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$
case 1: $\rho = s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$
case 2: $\rho = s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \overset{\Delta}{=} \rho_{\text{red}}$

case 0: $\rho =$	$s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in \mathit{ample}(s_0)$ $s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$	
case 1: $\rho =$	$s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \mathit{ample}(s_0)$ $\beta_i \notin \mathit{ample}(s_0)$	
case 2: $\rho =$	$s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin \mathit{ample}(s_0)$	

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{red}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \overset{\Delta}{=} \rho_{red}$

case 0:	$\rho =$	$s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$
		$s_0 \xRightarrow{\alpha} s_0' \longrightarrow \ldots$
case 1:	$\rho =$	$s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$
		$s_0 \xRightarrow{\alpha} \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$
case 2:	$\rho =$	$s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{red}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \overset{\Delta}{=} \rho_{red}$

case 0:	$\rho =$	$s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$
		$s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$
case 1:	$\rho =$	$s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$
		$s_0 \overset{\alpha}{\Longrightarrow} \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$
case 2:	$\rho =$	$s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$
		$s_0 \overset{\alpha}{\Longrightarrow} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in ample(s_0)$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \overset{\Delta}{=} \rho_{\text{red}}$

ρ_{red} results by an infinite sequence application of *cases 0*, *1* and *2*, i.e.,

$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \triangleq \rho_{\text{red}}$

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where for $i < j$ the executions ρ_j and ρ_i have a common prefix of length i which is a path fragment in $\mathcal{T}_{\text{red}}$

$$\text{execution } \rho \text{ in } \mathcal{T} \quad \rightsquigarrow \quad \begin{array}{c}\text{execution } \rho_{red} \text{ in } \mathcal{T}_{red} \\ \text{s.t. } \rho \triangleq \rho_{red}\end{array}$$

ρ_{red} results by an infinite sequence application of *cases 0*, *1* and *2*, i.e.,

$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

where for $i < j$ the executions ρ_j and ρ_i have a common prefix of length i which is a path fragment in $\mathcal{T}_{red}$,i.e., ρ_i has the form

$$\rho_i = \underbrace{s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i}_{\text{in } \mathcal{T}_{red}} \underbrace{\rightarrow s_{i+1} \rightarrow s_{i+2} \rightarrow \ldots}_{\text{in } \mathcal{T}}$$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \triangleq \rho_{red}$

ρ_{red} results by an infinite sequence application of *cases 0*, *1* and *2*, i.e.,

$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

where

$$\rho_i = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \rightarrow s_{i+1} \rightarrow s_{i+2} \rightarrow s_{i+3} \rightarrow \ldots$$

$$\rho_{i+1} = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \Rightarrow s_{i+1} \rightarrow s_{i+2} \rightarrow s_{i+3} \rightarrow \ldots$$

$$\rho_{i+2} = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \Rightarrow s_{i+1} \Rightarrow s_{i+2} \rightarrow s_{i+3} \rightarrow \ldots$$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

---

case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

$\beta_i \notin \text{ample}(s_0)$

---

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin \text{ample}(s_0)$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in \mathit{ample}(s_0)$

$\rho_1 = \quad s_0 \stackrel{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$

---

case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \mathit{ample}(s_0)$

$\beta_i \notin \mathit{ample}(s_0)$

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Transformation $\rho \rightsquigarrow \rho_1$

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case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$

$\beta_i \notin ample(s_0)$

$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

---

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

Transformation $\rho \rightsquigarrow \rho_1$

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$\rho_1 = s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$

---

case 1: $\rho = s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$

$\beta_i \notin ample(s_0)$

$\rho_1 = s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

---

case 2: $\rho = s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

$\rho_1 = s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in ample(s_0)$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$

$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$

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case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$

$\beta_i \notin ample(s_0)$

$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

---

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in ample(s_0)$

for the transformation $\rho_1 \rightsquigarrow \rho_2$:

apply case 0,1 or 2 to the suffix starting in state s_0'

Transformation according to cases 1 and 2

$$
\begin{array}{llll}
\rho_0 = & s_0 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_1 = & s_0 \xRightarrow{\alpha_1} s_1 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_2 = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} s_2 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\vdots & & & \\
\rho_m = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xrightarrow{\beta_1} & \ldots & \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots
\end{array}
$$

Transformation according to cases 1 and 2

$$
\begin{array}{llll}
\rho_0 = & s_0 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_1 = & s_0 \overset{\alpha_1}{\Longrightarrow} s_1 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_2 = & s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} s_2 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\vdots & & & \\
\rho_m = & s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} \ldots \overset{\alpha_m}{\Longrightarrow} s_m \xrightarrow{\beta_1} & \ldots & \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots
\end{array}
$$

α_i stutter action

Transformation according to cases 1 and 2

$$\rho_0 = \quad s_0 \xrightarrow{\beta_1} \ldots \quad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\rho_1 = \quad s_0 \overset{\alpha_1}{\Longrightarrow} s_1 \xrightarrow{\beta_1} \ldots \quad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\rho_2 = \quad s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} s_2 \xrightarrow{\beta_1} \ldots \quad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\vdots$$

$$\rho_m = \quad s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} \ldots \overset{\alpha_m}{\Longrightarrow} s_m \xrightarrow{\beta_1} \quad \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

α_i stutter action $\rightsquigarrow \rho_0 \triangleq \rho_1 \triangleq \rho_2 \triangleq \ldots$

Transformation according to cases 1 and 2

$$
\begin{array}{lll}
\rho_0 = & s_0 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_1 = & s_0 \xRightarrow{\alpha_1} s_1 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_2 = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} s_2 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\vdots & & \\
\rho_m = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xrightarrow{\beta_1} & \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots
\end{array}
$$

by the cycle condition (A4):

"action β_1 will *not* be postponed forever"

Transformation according to cases 1 and 2

$$
\begin{array}{llll}
\boldsymbol{\rho}_0 = & s_0 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\boldsymbol{\rho}_1 = & s_0 \xRightarrow{\alpha_1} s_1 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\boldsymbol{\rho}_2 = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} s_2 \xrightarrow{\beta_1} \ldots & & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\vdots & & & \\
\boldsymbol{\rho}_m = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xrightarrow{\beta_1} & \ldots & \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots
\end{array}
$$

by the cycle condition (A4):

"action β_1 will *not* be postponed forever"

i.e., there exists some m such that case 0 applies and $\boldsymbol{\rho}_m = \boldsymbol{\rho}_{m+1}$

Transformation according to cases 1 and 2

$$\begin{array}{lll}
\boldsymbol{\rho}_0 = & s_0 \xrightarrow{\boldsymbol{\beta}_1} \ldots & \xrightarrow{\boldsymbol{\beta}_{k-1}} \xrightarrow{\boldsymbol{\beta}_k} \xrightarrow{\boldsymbol{\beta}_{k+1}} \ldots \\
\boldsymbol{\rho}_1 = & s_0 \xRightarrow{\boldsymbol{\alpha}_1} s_1 \xrightarrow{\boldsymbol{\beta}_1} \ldots & \xrightarrow{\boldsymbol{\beta}_{k-1}} \xrightarrow{\boldsymbol{\beta}_k} \xrightarrow{\boldsymbol{\beta}_{k+1}} \ldots \\
\boldsymbol{\rho}_2 = & s_0 \xRightarrow{\boldsymbol{\alpha}_1} \xRightarrow{\boldsymbol{\alpha}_2} s_2 \xrightarrow{\boldsymbol{\beta}_1} \ldots & \xrightarrow{\boldsymbol{\beta}_{k-1}} \xrightarrow{\boldsymbol{\beta}_k} \xrightarrow{\boldsymbol{\beta}_{k+1}} \ldots \\
\vdots & & \\
\boldsymbol{\rho}_m = & s_0 \xRightarrow{\boldsymbol{\alpha}_1} \xRightarrow{\boldsymbol{\alpha}_2} \ldots \xRightarrow{\boldsymbol{\alpha}_m} s_m \xrightarrow{\boldsymbol{\beta}_1} & \ldots \xrightarrow{\boldsymbol{\beta}_k} \xrightarrow{\boldsymbol{\beta}_{k+1}} \ldots
\end{array}$$

by the cycle condition $(\mathrm{A4})$:

"action $\boldsymbol{\beta}_1$ will *not* be postponed forever"

i.e., there exists some m such that case 0 applies and $\boldsymbol{\rho}_m = \boldsymbol{\rho}_{m+1}$

Transformation according to cases 1 and 2

$$
\begin{array}{lll}
\boldsymbol{\rho}_0 = & s_0 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\boldsymbol{\rho}_1 = & s_0 \xRightarrow{\alpha_1} s_1 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\boldsymbol{\rho}_2 = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} s_2 \xrightarrow{\beta_1} \ldots & \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\vdots & & \\
\boldsymbol{\rho}_m = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xrightarrow{\beta_1} & \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\boldsymbol{\rho}_{m+1} = & s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xRightarrow{\beta_1} & \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots
\end{array}
$$

by the cycle condition (A4):

"action β_1 will *not* be postponed forever"

i.e., there exists some m such that case 0 applies and $\boldsymbol{\rho}_m = \boldsymbol{\rho}_{m+1}$

4 conditions for ample sets LTL3.4-FOUR-COND

(A1) $\emptyset \neq \mathit{ample}(s) \subseteq \mathit{Act}(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\mathit{ample}(s)$ there is some $i < n$ with $\beta_i \in \mathit{ample}(s)$

(A3) if $\mathit{ample}(s) \neq \mathit{Act}(s)$ then all actions in $\mathit{ample}(s)$ are *stutter actions*

(A4) for each *cycle* $s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_n$ in $\mathcal{T}_{red}$ and each action

$$\beta \in \bigcup_{0 \leq i < n} \mathit{Act}(s_i)$$

there is some $i \in \{1, \ldots, n\}$ with $\beta \in \mathit{ample}(s_i)$

The ample set method for $\text{LTL}_{\setminus\bigcirc}$ model checking

LTL3.4-37

The ample set method for $\text{LTL}_{\setminus\bigcirc}$ model checking

- on-the-fly DFS-based generatation of $\mathcal{T}_{red}$

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- *exploration* of state s:

  create the states α(s) for $\alpha \in$ ample(s),

The ample set method for $LTL_{\setminus\bigcirc}$ model checking

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- *interleave* the generation of $\mathcal{T}_{red}$ with the product construction $\mathcal{T}_{red} \otimes \mathcal{A}$

where $\mathcal{A}$ is an NBA for the negation of the formula to be checked

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here: only explanations for reachability analysis

The ample set method for reachability

given: finite transition system $\mathcal{T}$
atomic proposition a

goal: on-the-fly construction of $\mathcal{T}_{red}$
abort as soon as a state s with
$s \not\models a$ has been generated

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uses

- **V** $=$ set of states that have been generated so far (organized as a hash table)

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uses

- **V** = set of states that have been generated so far (organized as a hash table)
- DFS-stack π

The ample set method for reachability

given: finite transition system $\mathcal{T}$ for $P_1 \| \ldots \| P_n$
atomic proposition **a**

goal: on-the-fly construction of $\mathcal{T}_{\text{red}}$
abort as soon as a state s with
$s \not\models$ **a** has been generated

uses

- **V** = set of states that have been generated so far (organized as a hash table)
- DFS-stack π
- "local" criteria to compute ample(s) from a syntactic representation of the processes P_i

Ample set method (full generation of $\mathcal{T}_{\textsf{red}}$)

$\pi := \emptyset$; $\mathbf{V} := \emptyset$

WHILE $\mathbf{S}_0 \not\subseteq \mathbf{V}$ **DO**

select an initial state $\mathbf{s} \in \mathbf{S}_0 \setminus \mathbf{V}$; add $\mathbf{s}$ to $\mathbf{V}$;

Push(π, $\mathbf{s}$);

OD

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  $\mathsf{Push}(\pi, \mathsf{s})$; compute $\mathsf{ample}(\mathsf{s})$;

  WHILE $\pi \neq \emptyset$ **DO**

    $\mathsf{s} := \mathsf{Top}(\pi)$;

  OD

OD

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  WHILE $\pi \neq \emptyset$ **DO**

    $\mathbf{s}$:= Top(π);

    IF $\exists \alpha \in$ ample($\mathbf{s}$) with $\alpha(\mathbf{s}) \notin \mathbf{V}$

    FI

  OD

OD

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  WHILE $\pi \neq \emptyset$ **DO**

    $\mathbf{s}$:= Top(π);

    IF $\exists \alpha \in$ ample($\mathbf{s}$) with $\alpha(\mathbf{s}) \notin \mathbf{V}$

      THEN select such α; add $\mathbf{s}' := \alpha(\mathbf{s})$ to $\mathbf{V}$;

        Push(π, $\mathbf{s}'$);

    FI

  OD

OD

```
π := ∅; V := ∅
WHILE S₀ ⊈ V DO
   select an initial state s ∈ S₀ \ V; add s to V;
   Push(π, s); compute ample(s);
   WHILE π ≠ ∅ DO
      s := Top(π);
      IF ∃α ∈ ample(s) with α(s) ∉ V
        THEN select such α; add s′ := α(s) to V;
                Push(π, s′); compute ample(s′);

      FI
   OD
OD
```

Ample set method (full generation of $\mathcal{T}_{red}$)

$\pi := \emptyset$; $V := \emptyset$

WHILE $S_0 \not\subseteq V$ **DO**

  select an initial state $s \in S_0 \setminus V$; add s to V;

  $Push(\pi, s)$; compute $ample(s)$;

  WHILE $\pi \neq \emptyset$ **DO**

    $s := Top(\pi)$;

    IF $\exists \alpha \in ample(s)$ with $\alpha(s) \notin V$

      THEN select such α; add $s' := \alpha(s)$ to V;

        $Push(\pi, s')$; compute $ample(s')$;

      ELSE $Pop(\pi)$

    FI

  OD

OD

The ample set method for reachability

$\pi := \emptyset$; $V := \emptyset$
WHILE $S_0 \not\subseteq V$ **DO**
  select an initial state $s \in S_0 \setminus V$; add s to V;
  $\mathrm{Push}(\pi, s)$; compute $\mathrm{ample}(s)$;
  WHILE $\pi \neq \emptyset$ **DO**
    $s := \mathrm{Top}(\pi)$;
    IF $\exists \alpha \in \mathrm{ample}(s)$ with $\alpha(s) \notin V$
      THEN select such α; add $s' := \alpha(s)$ to V;
        $\mathrm{Push}(\pi, s')$; compute $\mathrm{ample}(s')$;
      ELSE $\mathrm{Pop}(\pi)$
    FI
  OD
OD

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  WHILE $\pi \neq \emptyset$ **DO**
    $s := Top(\pi)$;
    IF $\exists \alpha \in ample(s)$ with $\alpha(s) \notin V$
      THEN select such α; add $s' := \alpha(s)$ to V;
        $Push(\pi, s')$; compute $ample(s')$;
      ELSE $Pop(\pi)$
    FI
  OD
OD

Does $\mathcal{T} \models \Box a$ hold?


```
π := ∅; V := ∅
WHILE S₀ ⊄ V DO
  select an initial state s ∈ S₀ \ V; add s to V;
  Push(π, s); compute ample(s);
  WHILE π ≠ ∅ DO
    s := Top(π);
    IF s ⊭ a THEN return "NO"                FI;
    IF ∃α ∈ ample(s) with α(s) ∉ V
      THEN ....
      ELSE  Pop(π)
    FI
  OD
OD
```

Does $\mathcal{T} \models \Box a$ hold?

$\pi := \emptyset$; $V := \emptyset$

WHILE $S_0 \not\subseteq V$ **DO**

  select an initial state $s \in S_0 \setminus V$; add s to V;

  $Push(\pi, s)$; compute $ample(s)$;

  WHILE $\pi \neq \emptyset$ **DO**

    $s := Top(\pi)$;

    IF $s \not\models a$ **THEN** return "NO" + counterexample **FI**;

    IF $\exists \alpha \in ample(s)$ with $\alpha(s) \notin V$

      THEN ....

      ELSE $Pop(\pi)$

    FI

  OD

OD

Example: ample set method

full generation of $\mathcal{T}_{\mathsf{red}}$ for $\mathcal{T} = \mathcal{T}_{\mathsf{P}_1 \,|||\, \mathsf{P}_2}$ where

- P_1, P_2 are program graphs with shared variable $\mathsf{b} \in \{0, 1\}$

Example: ample set method

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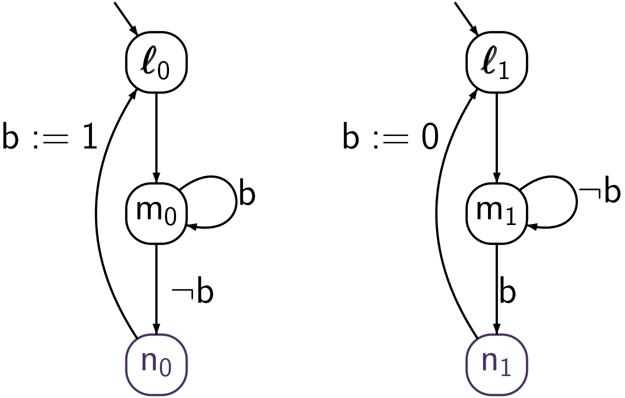

full generation of $\mathcal{T}_{\text{red}}$ for $\mathcal{T} = \mathcal{T}_{P_1 \,|||\, P_2}$ where

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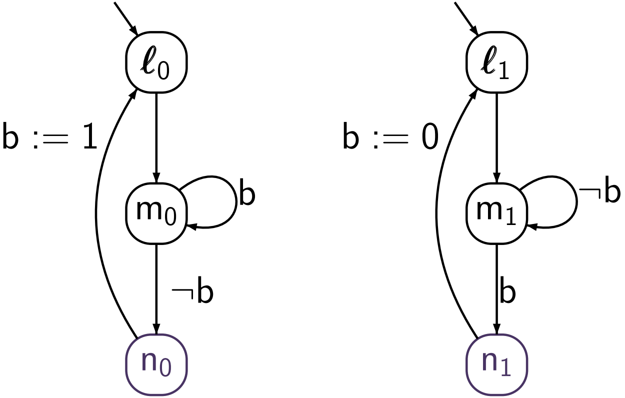
- $AP = \{n_0, n_1\}$



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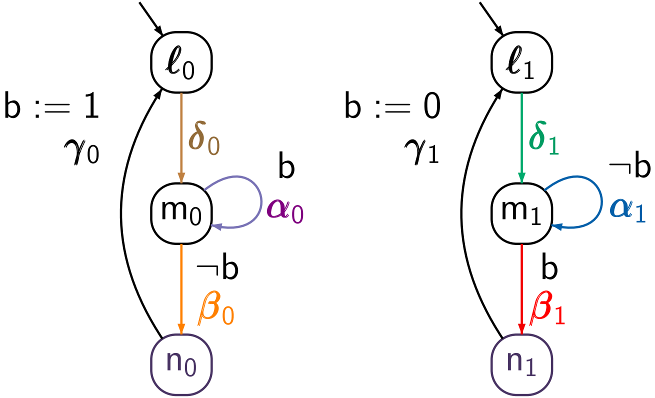
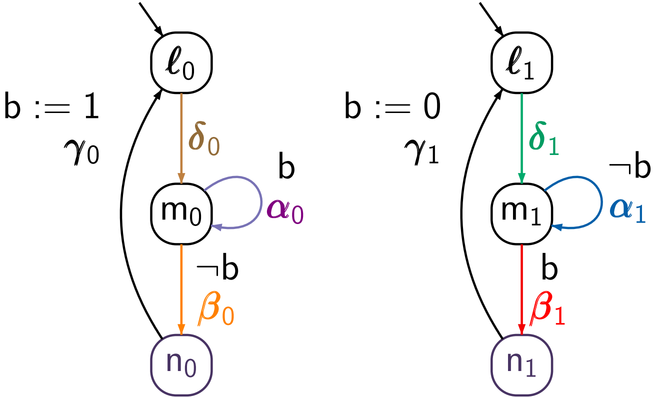
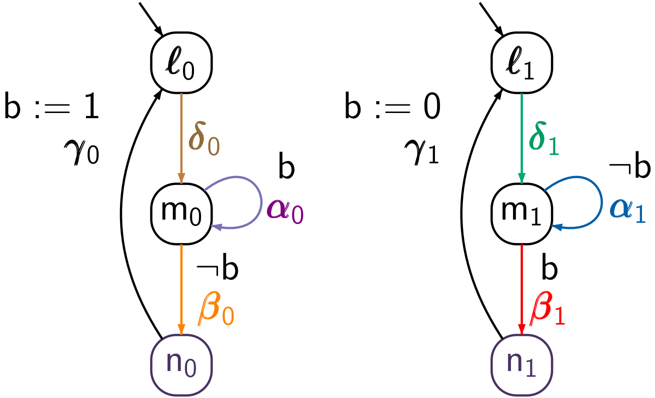




independent actions:

$\delta_0\ \delta_1$ $\quad$ $\delta_0\ \alpha_1$ $\quad$ $\delta_0\ \beta_1$ $\quad$ $\delta_0\ \gamma_1$



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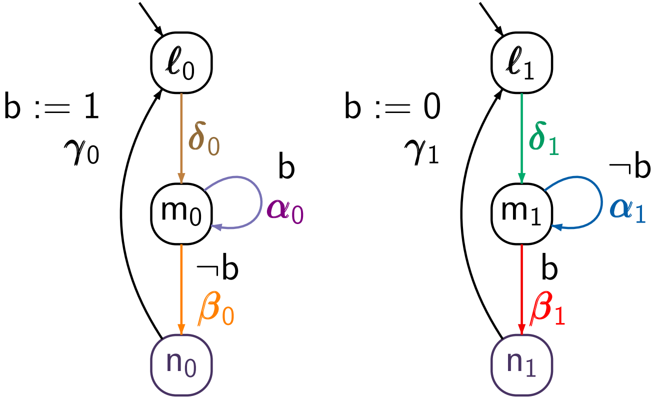
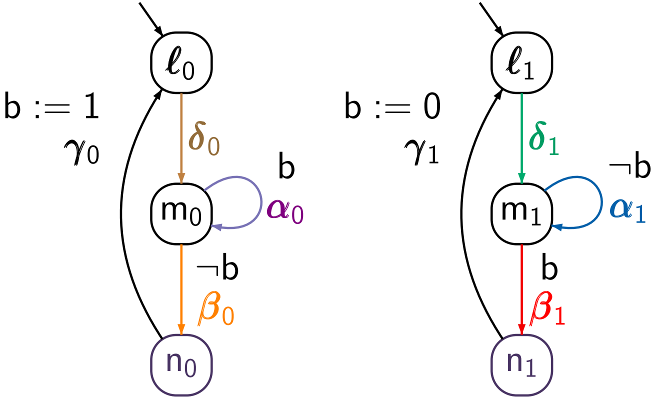


independent actions:

$\delta_0\ \delta_1$	$\delta_0\ \alpha_1$	$\delta_0\ \beta_1$	$\delta_0\ \gamma_1$
$\alpha_0\ \delta_1$		$\alpha_0\ \beta_1$	
$\beta_0\ \delta_1$	$\beta_0\ \alpha_1$	$\beta_0\ \beta_1$	$\beta_0\ \gamma_1$



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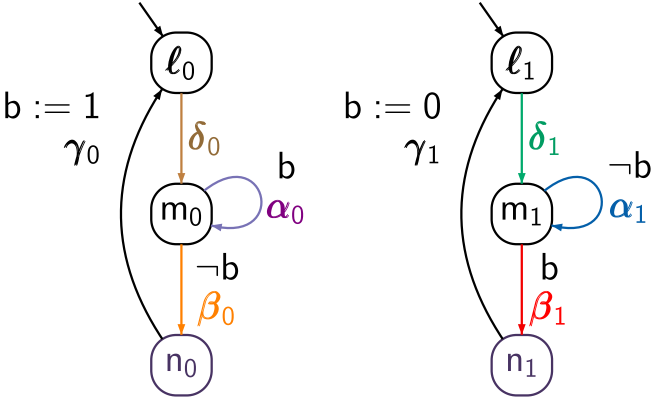
$\delta_0\ \delta_1$	$\delta_0\ \alpha_1$	$\delta_0\ \beta_1$	$\delta_0\ \gamma_1$
$\alpha_0\ \delta_1$		$\alpha_0\ \beta_1$	
$\beta_0\ \delta_1$	$\beta_0\ \alpha_1$	$\beta_0\ \beta_1$	$\beta_0\ \gamma_1$

β_0 and β_1 are never enabled simultaneously



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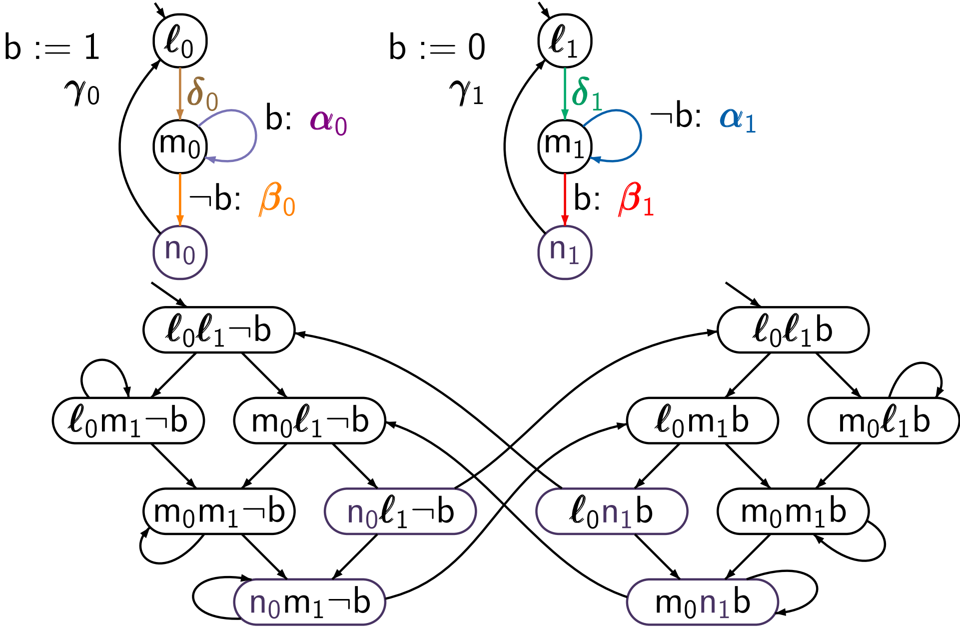
$\delta_0\ \delta_1$ $\delta_0\ \alpha_1$ $\delta_0\ \beta_1$ $\delta_0\ \gamma_1$

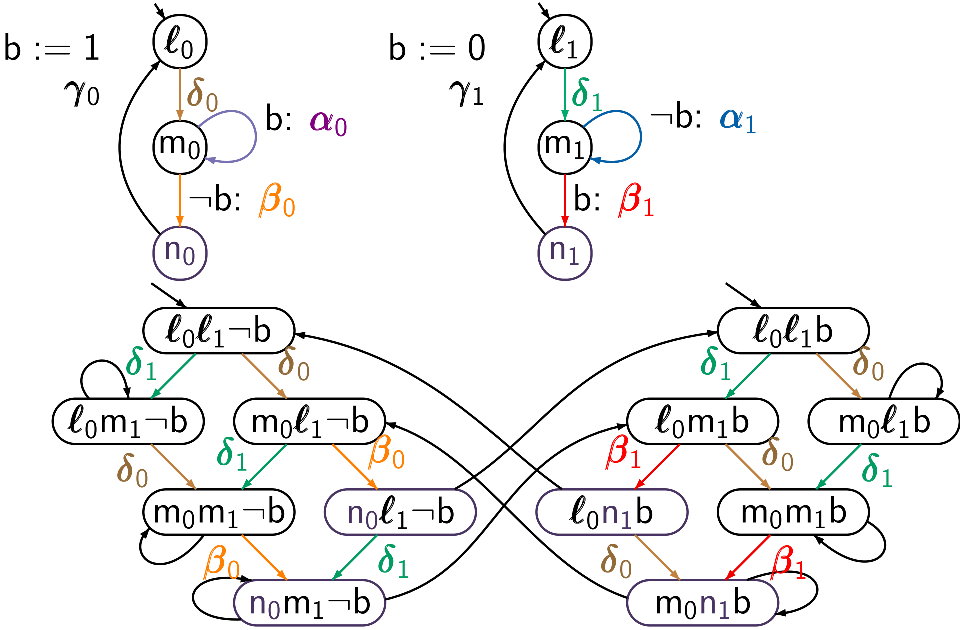
$\alpha_0\ \delta_1$ $\alpha_0\ \beta_1$

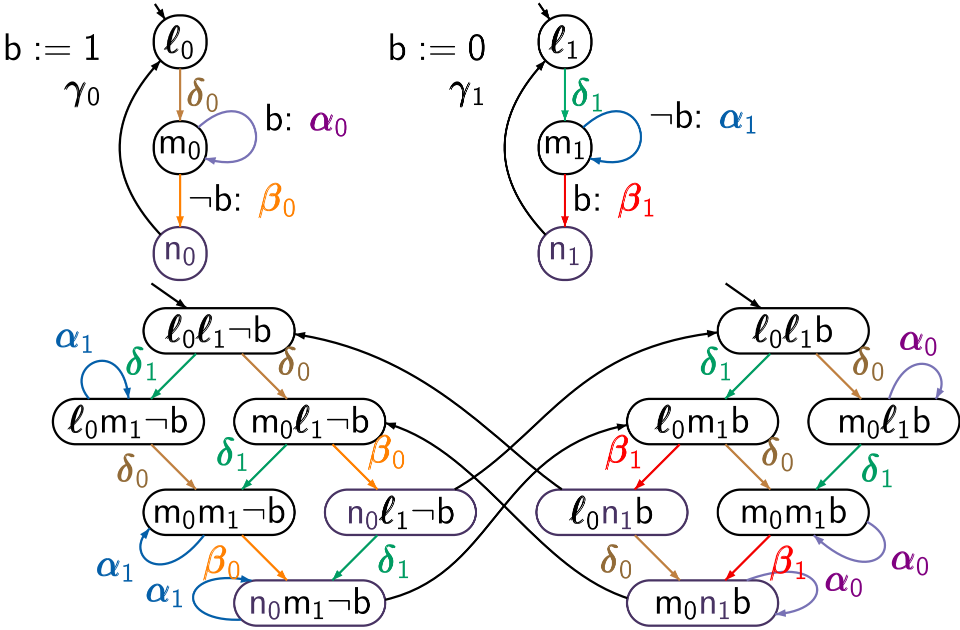
$\beta_0\ \delta_1$ $\beta_0\ \alpha_1$ $\beta_0\ \beta_1$ $\beta_0\ \gamma_1$

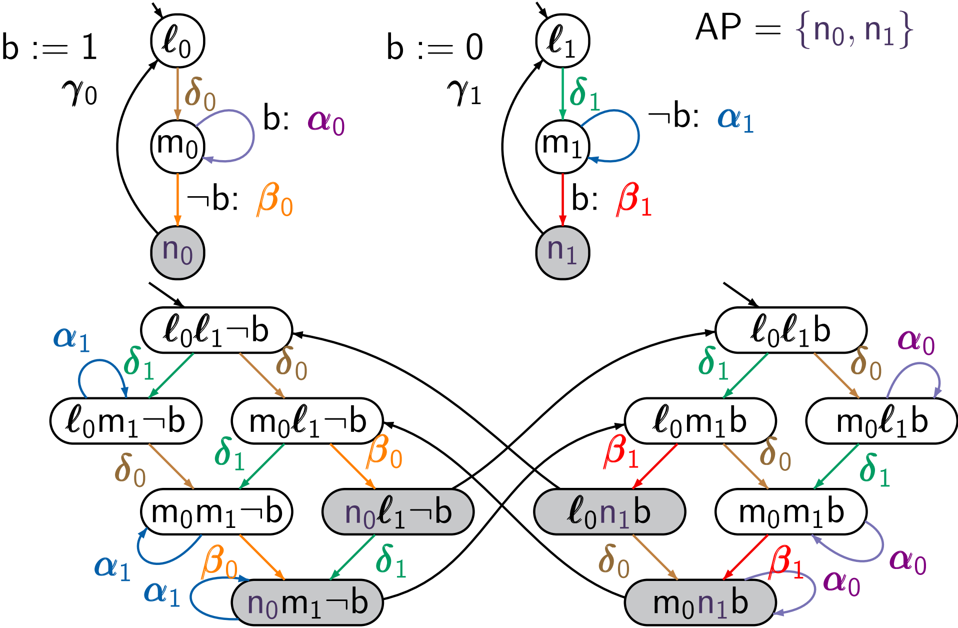
$\gamma_0\ \delta_1$ $\gamma_0\ \beta_1$



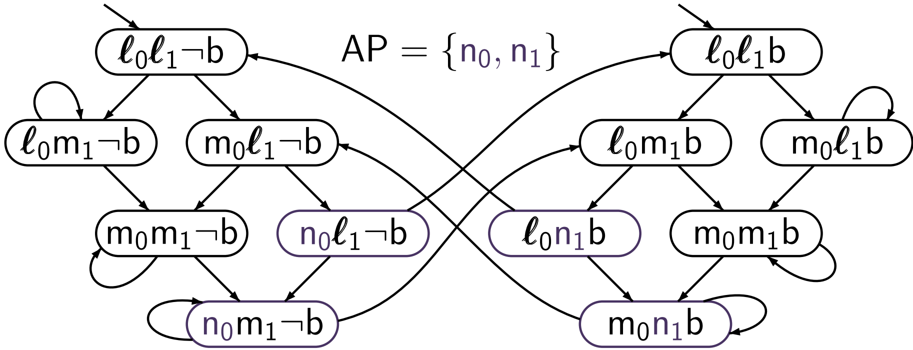
b := 1
γ0
ℓ0
δ0
m0
b: α0
¬b: β0
n0
b := 0
γ1
ℓ1
δ1
m1
¬b: α1
b: β1
n1
ℓ0ℓ1¬b
ℓ0m1¬b
m0ℓ1¬b
m0m1¬b
n0ℓ1¬b
n0m1¬b
ℓ0ℓ1b
ℓ0m1b
m0ℓ1b
ℓ0n1b
m0m1b
m0n1b


b := 1
γ0
ℓ0
δ0
m0
b: α0
¬b: β0
n0
b := 0
γ1
ℓ1
δ1
m1
¬b: α1
b: β1
n1
ℓ0ℓ1¬b
ℓ0m1¬b
m0ℓ1¬b
m0m1¬b
n0ℓ1¬b
n0m1¬b
ℓ0ℓ1b
ℓ0m1b
m0ℓ1b
ℓ0n1b
m0m1b
m0n1b


b := 1
γ0
ℓ0
δ0
m0
b: α0
¬b: β0
n0
b := 0
γ1
ℓ1
δ1
m1
¬b: α1
b: β1
n1
ℓ0ℓ1¬b
ℓ0m1¬b
m0ℓ1¬b
m0m1¬b
n0ℓ1¬b
n0m1¬b
ℓ0ℓ1b
ℓ0m1b
m0ℓ1b
ℓ0n1b
m0m1b
m0n1b


b := 1
γ_0
ℓ_0
δ_0
m_0
b: α_0
$\neg$b: β_0
n_0
b := 0
γ_1
ℓ_1
δ_1
m_1
$\neg$b: α_1
b: β_1
n_1
AP = $\{n_0, n_1\}$
$\ell_0\ell_1\neg$b
$\ell_0 m_1\neg$b
$m_0\ell_1\neg$b
$m_0 m_1\neg$b
$n_0\ell_1\neg$b
$n_0 m_1\neg$b
$\ell_0\ell_1$b
$\ell_0 m_1$b
$m_0\ell_1$b
$\ell_0 n_1$b
$m_0 m_1$b
$m_0 n_1$b

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



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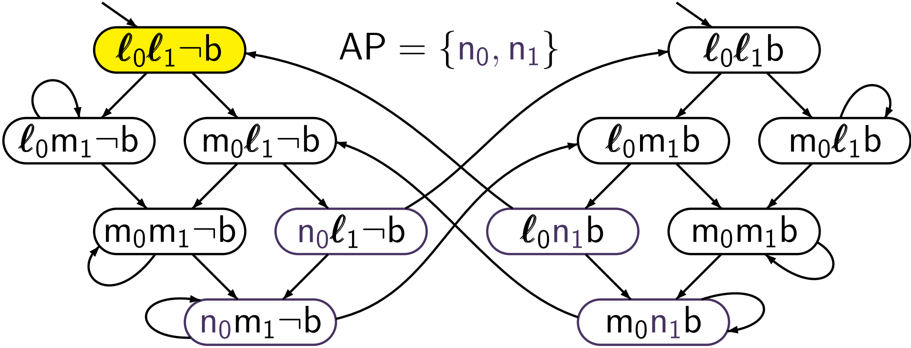
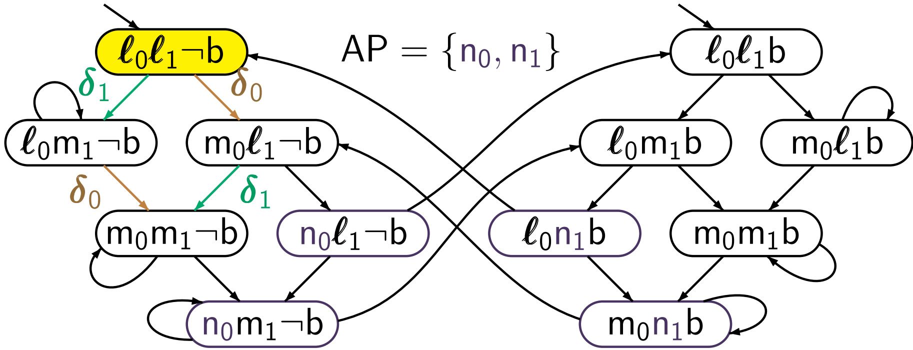


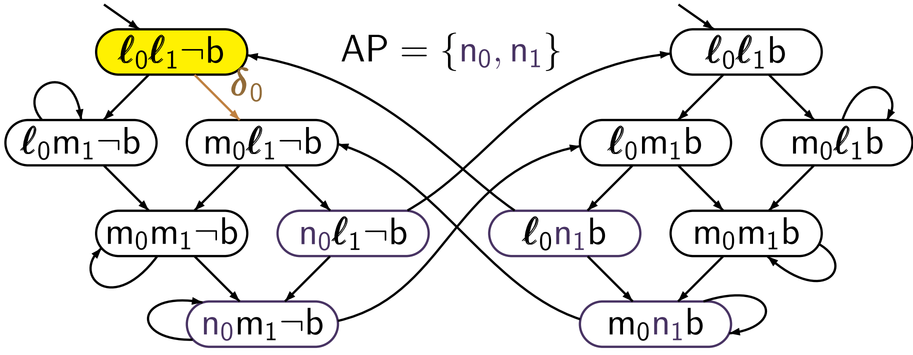
$AP = \{n_0, n_1\}$

$\text{ample}(\ell_0\ell_1\neg b) =$



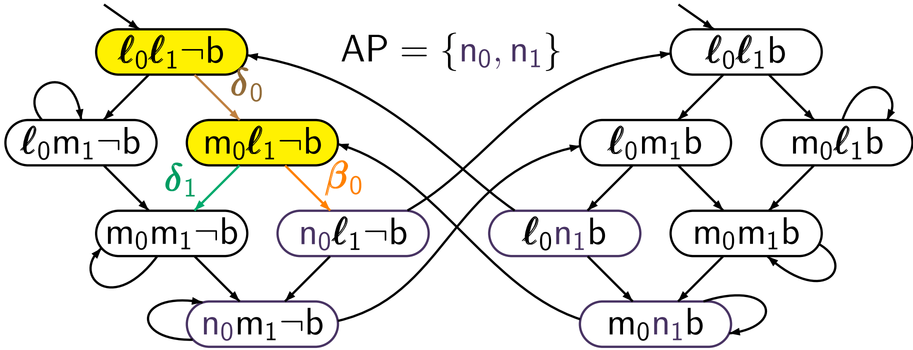
$\mathsf{ample}(\ell_0\ell_1\neg b) =$

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



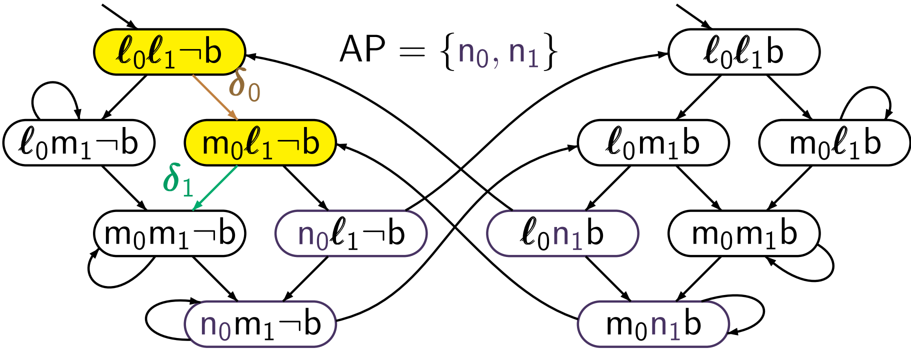
$\text{ample}(\ell_0\ell_1\neg b) = \{\delta_0\},$

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



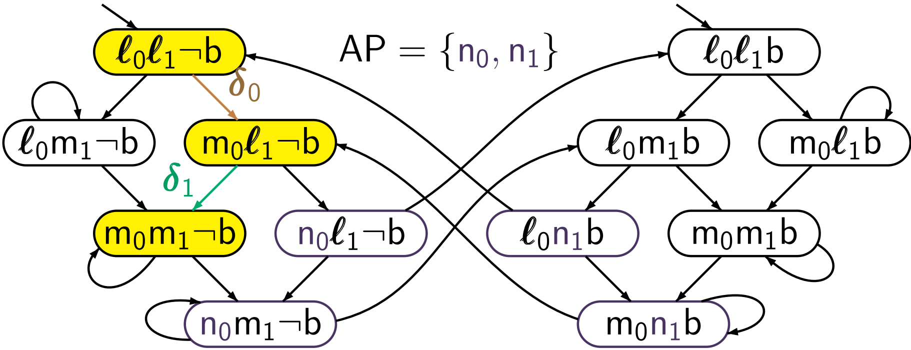
$\text{ample}(\ell_0\ell_1\neg b) = \{\delta_0\}, \quad \text{ample}(m_0\ell_1\neg b) =$

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



$$\text{ample}(\ell_0\ell_1\neg b) = \{\delta_0\}, \quad \text{ample}(m_0\ell_1\neg b) = \{\delta_1\}$$

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



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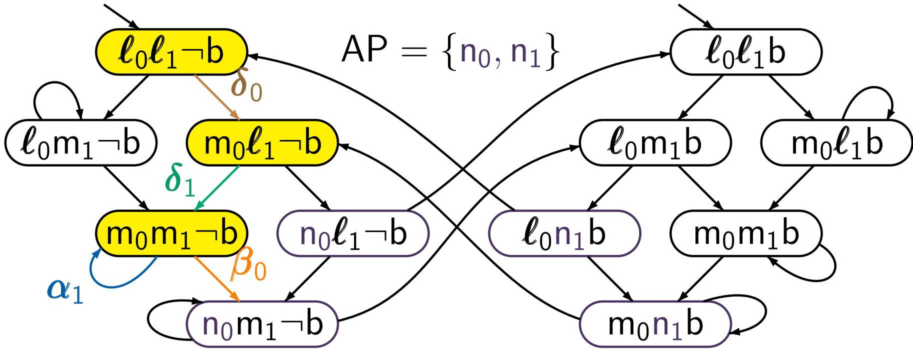

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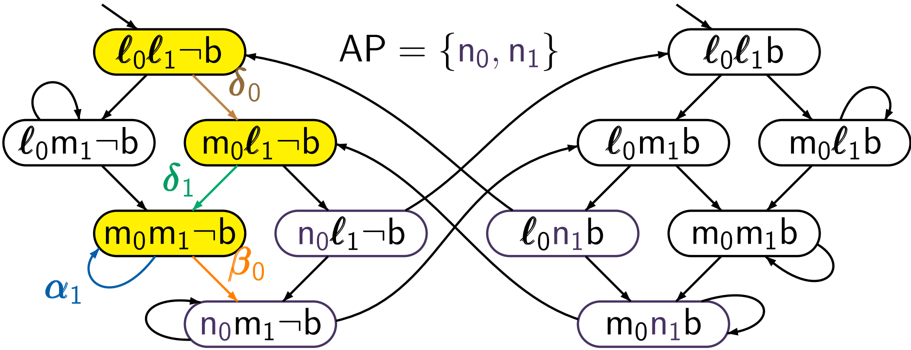

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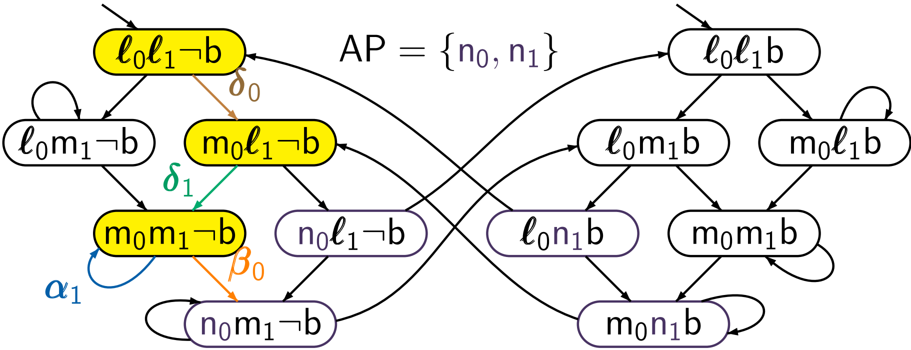

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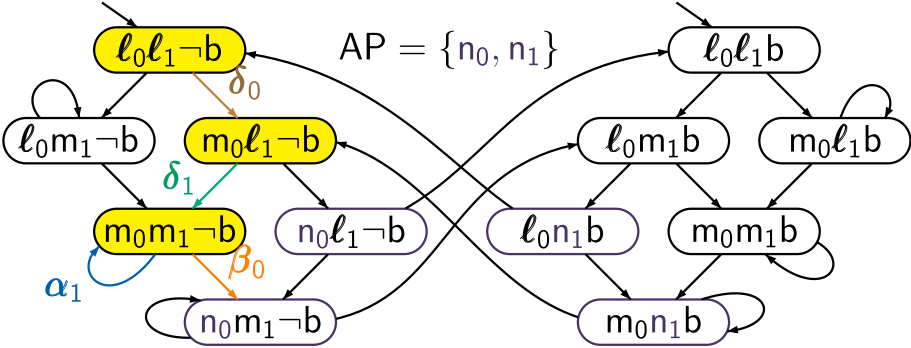

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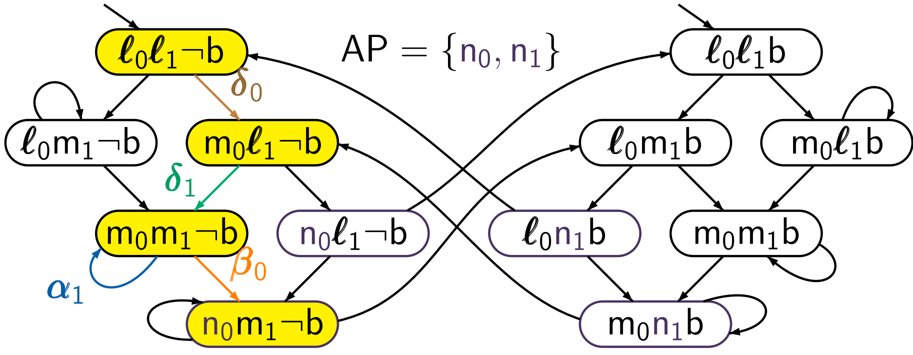

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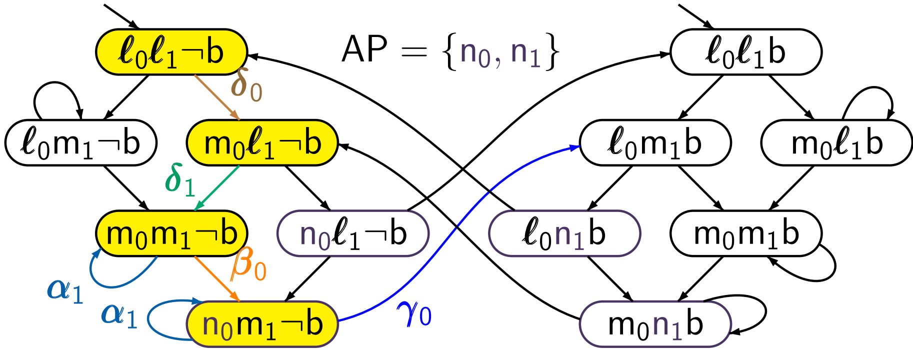

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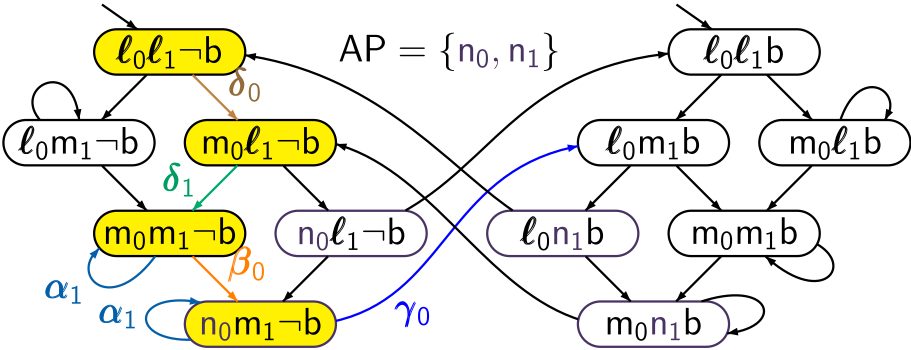

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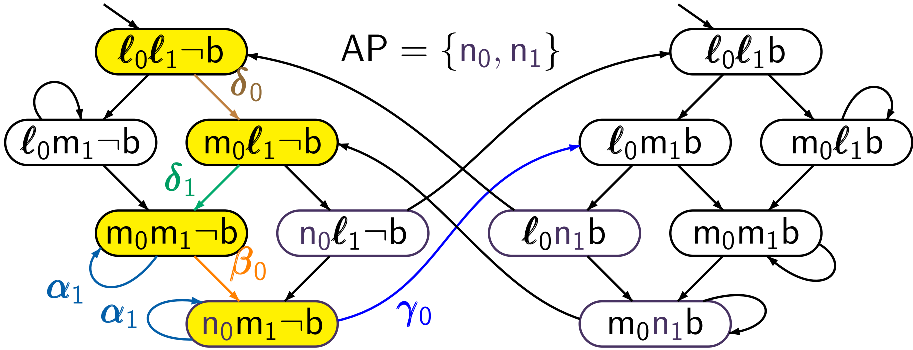

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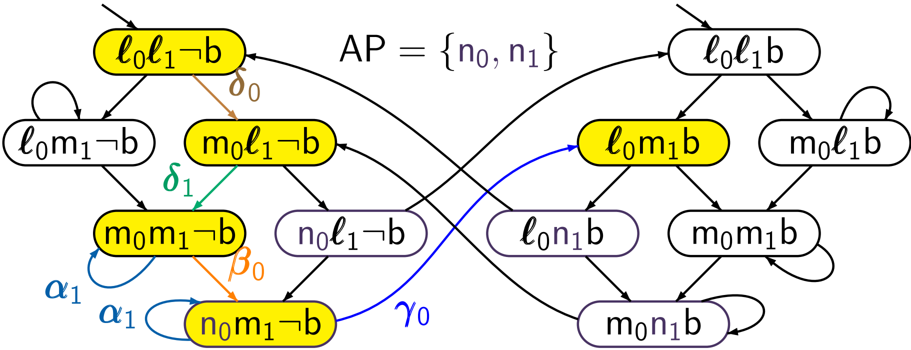

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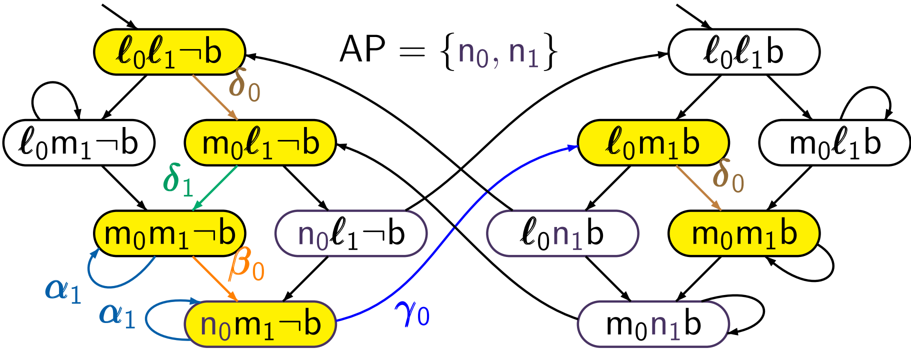

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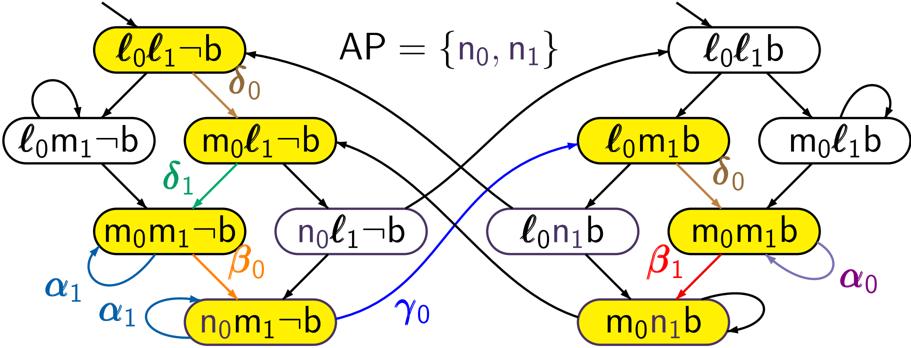

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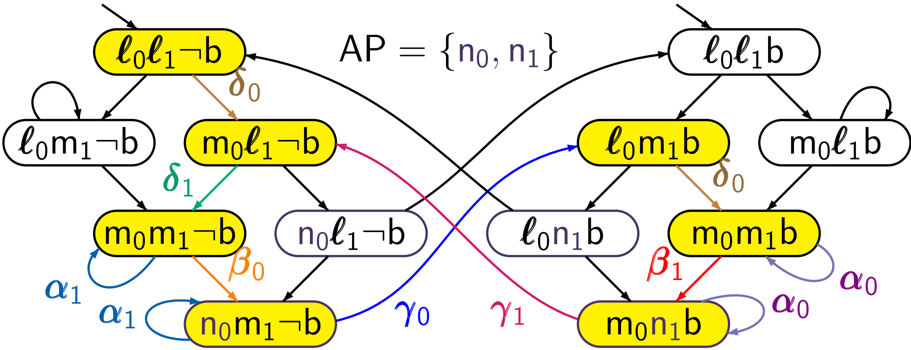

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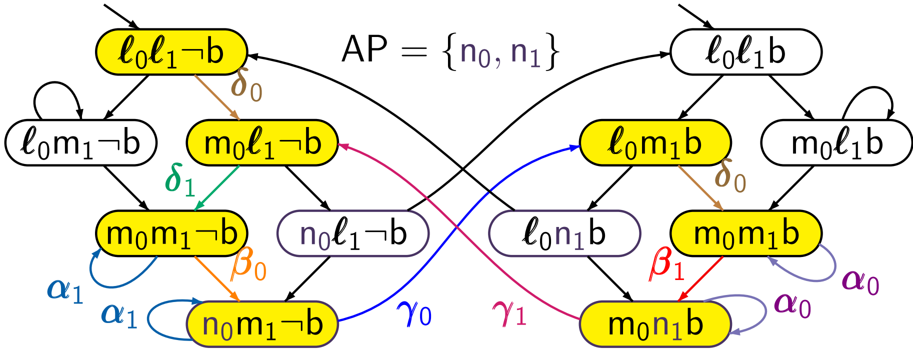

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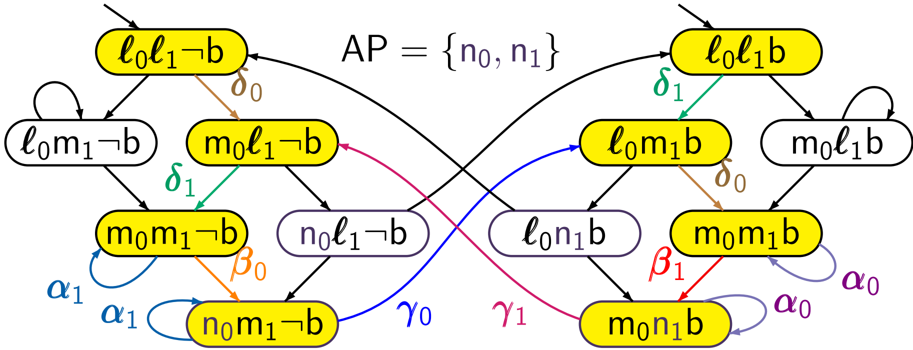

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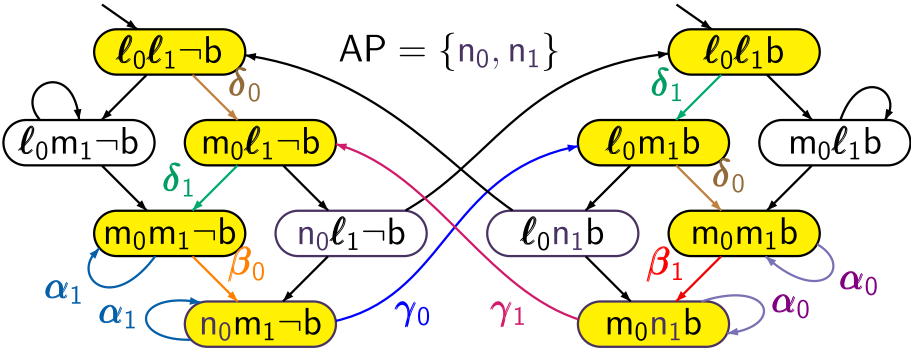
reduction: 8 out of 12 states



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Nested DFS with POR

Nested DFS (standard approach)

remind: nested DFS for checking "$\mathcal{T} \models \Diamond\Box a$?" uses:

outer DFS: visits all reachable states

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CYCLE_CHECK(s)

- is called for each state s that violates the persistence condition a
- must not be started before the outer DFS is finished for s
- early termination, e.g., abort with the answer

  $$\text{CYCLE\_CHECK}(s) = \textit{true}$$

  as soon as the inner DFS visits a state in the DFS-stack of the outer DFS

Nested DFS with POR

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- for $Act(s) = \{\alpha_1, \ldots, \alpha_k\}$:

$$a_i = 1 \text{ iff } \alpha_i \in ample(s)$$

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starting point: syntactic description of the processes $P_1, \ldots, P_n$ of a parallel system

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Computing the ample set for state s

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select a process P_i not considered before

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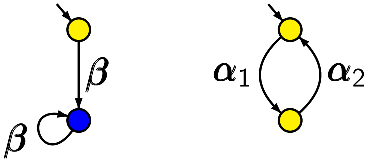
<u>THEN</u> ample(s) := Act(s) **<u>FI</u>**

Example: construction of $\mathcal{T}_{\text{red}}$

process 1 process 2



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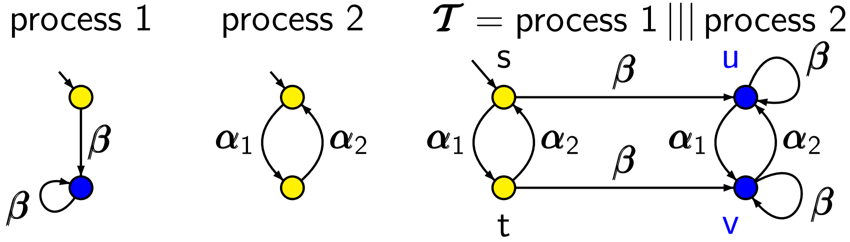

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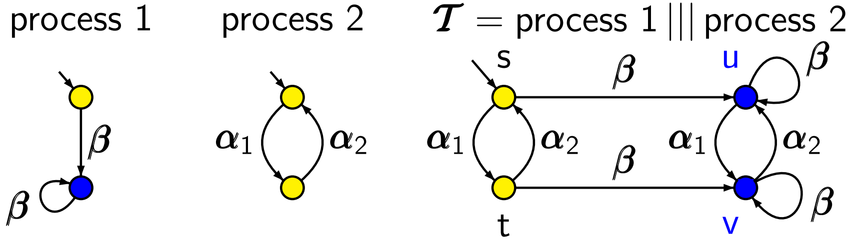

DFS(s)

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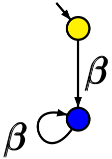
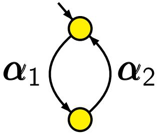
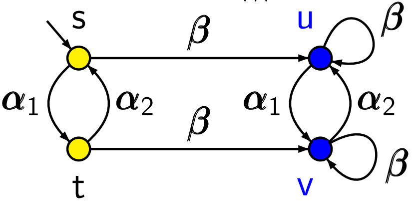


process 2

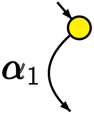


$\mathcal{T}$ = process 1 ||| process 2



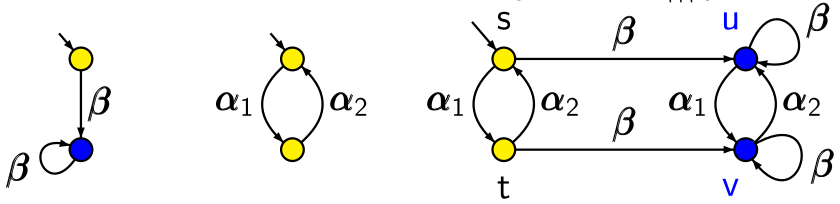
DFS(s)

ample(s) = $\{\alpha_1\}$

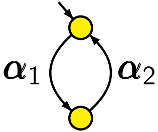


Example: construction of $\mathcal{T}_{\text{red}}$

process 1 process 2 $\mathcal{T}$ = process 1 ||| process 2



DFS(s)
  ample(s) = $\{\alpha_1\}$
  DFS(t)
    ample(t) = $\{\alpha_2\}$



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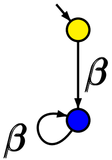
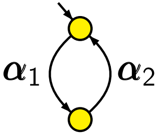


process 2



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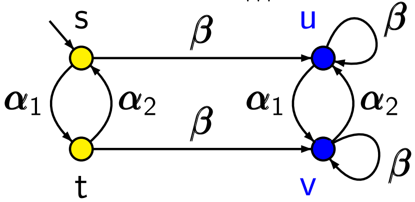


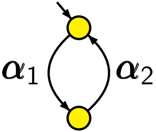
DFS(s)

ample(s) $= \{\alpha_1\}$

DFS(t)

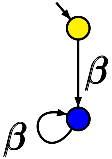
ample(t) $= \{\alpha_2\}$

backward edge t $\longrightarrow$ s

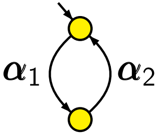


Example: construction of $\mathcal{T}_{\text{red}}$

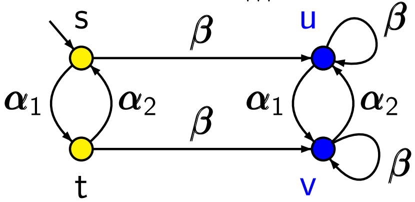
process 1



process 2



$\mathcal{T}$ = process 1 ||| process 2



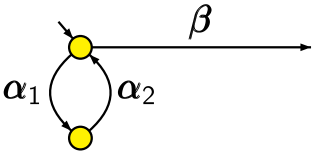
DFS(s)

ample(s) = $\{\alpha_1\} \cup \{\beta\}$

DFS(t)

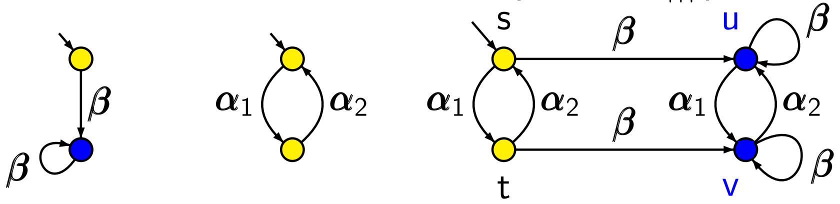
ample(t) = $\{\alpha_2\}$

backward edge t $\longrightarrow$ s



Example: construction of $\mathcal{T}_{\text{red}}$

process 1   process 2 $\mathcal{T}$ = process 1 ||| process 2



DFS(s)

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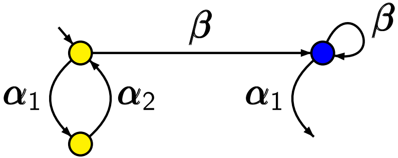
DFS(t)

ample(t) = $\{\alpha_2\}$

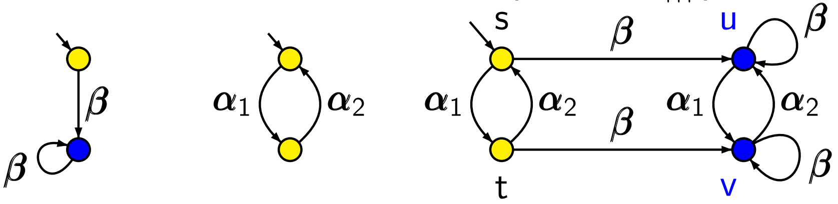
backward edge t $\longrightarrow$ s

DFS(u) ...

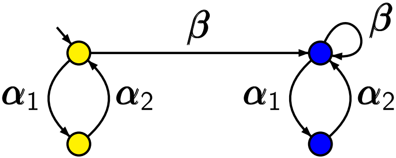


Example: construction of $\mathcal{T}_{red}$

process 1 process 2 $\mathcal{T}$ = process 1 ||| process 2



DFS(s)
- ample(s) $= \{\alpha_1\} \cup \{\beta\}$
- DFS(t)
  - ample(t) $= \{\alpha_2\}$
  - backward edge t $\longrightarrow$ s
- DFS(u) ...
  - DFS(v) ...



Computing the ample set for state s

REPEAT

select a process P_i not considered before

$A :=$ action set of $P_i \cap Act(s)$

IF $A \neq \emptyset$ and $(A2)$ holds

and all actions of A are stutter actions

THEN ample(s) $:= A$ **FI**

UNTIL all processes have been considered

or ample(s) is defined;

IF ample(s) is not yet defined

THEN ample(s) $:= Act(s)$ **FI**

Checking the dependence condition (A2)?

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THEN $\text{ample}(s) := \mathit{Act}(s)$ **FI**

Checking the dependence condition (A2)?

(A1) nonemptiness condition

(A2) dependence condition:

for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from ample(s) there is some $i < n$ with $\beta_i \in$ ample(s)

(A3) stutter condition

(A4) cycle condition

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checking (A2) is as hard as the unreachability problem

given: finite transition system $\mathcal{T}$, $a \in AP$

question: does $\mathcal{T} \not\models \exists\Diamond a$ hold?

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(A2) dependence condition: ⟵ global condition

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Algorithmic difficulty of checking (A2)

show that the unreachability problem

given: finite transition system $\mathcal{T}$
$\mathbf{a} \in \mathsf{AP}$

question: does $\mathcal{T} \not\models \exists\Diamond \mathbf{a}$ hold?

is polynomially reducible to the problem of checking (A2)

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question: does $\mathcal{T} \not\models \exists\Diamond a$ hold?

is polynomially reducible to the problem of checking (A2)

given: finite transition system $\mathcal{T}'$, ample sets for $\mathcal{T}'$

question: does (A2) hold?

i.e., does for each execution fragment in $\mathcal{T}'$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from ample(s)

there is some $i < n$ with $\beta_i \in$ ample(s)?

Algorithmic difficulty of checking $(A2)$

show that the unreachability problem

given: finite transition system $\mathcal{T}$ and initial state s_0
$\mathbf{a} \in AP$

question: does $s_0 \not\models \exists\Diamond \mathbf{a}$ hold?

is polynomially reducible to the problem of checking $(A2)$

given: finite transition system $\mathcal{T}'$, ample sets for $\mathcal{T}'$

question: does $(A2)$ hold?
i.e., does for each execution fragment in $\mathcal{T}'$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

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unreachability problem	$\leq_{\text{poly}}$	problem of checking (A2)

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Algorithmic difficulty of checking (A2)

unreachability problem	$\leq_{\text{poly}}$	problem of checking (A2)

finite TS $\mathcal{T}$ + state s_0 + atomic prop. a $\rightsquigarrow$ finite TS $\mathcal{T}'$ + ample sets

s.t. $s_0 \not\models \exists\Diamond a$ iff (A2) holds

Algorithmic difficulty of checking $(A2)$

unreachability problem	$\leq_{\text{poly}}$	problem of checking $(A2)$

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$\mathcal{T}'$ results from $\mathcal{T}$ by adding two fresh actions α, β s.t.

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finite TS $\mathcal{T}$ + state s_0 + atomic prop. a $\rightsquigarrow$ finite TS $\mathcal{T}'$ + ample sets

s.t. $s_0 \not\models \exists\Diamond a$ iff (A2) holds

$\mathcal{T}'$ results from $\mathcal{T}$ by adding two fresh actions α, β s.t.

- α are β are dependent
- α is independent from all actions in $\mathcal{T}$
- β is enabled exactly in the states t with $t \models a$

	finite TS $\mathcal{T}$ + state s_0 + atomic prop. a	$\rightsquigarrow$	finite TS $\mathcal{T}'$ + ample sets
s.t.	$s_0 \not\models \exists\Diamond a$	iff	(A2) holds

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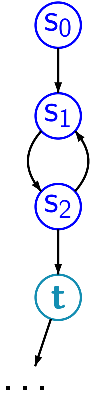

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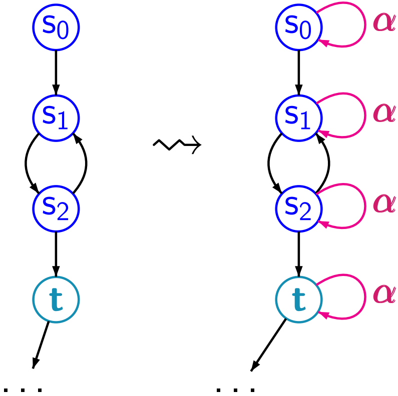

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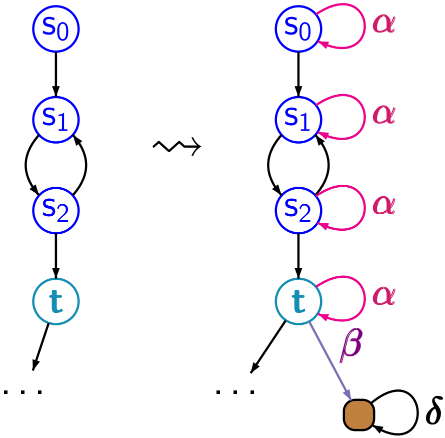

	finite TS $\mathcal{T}$ + state s_0		finite TS $\mathcal{T}'$
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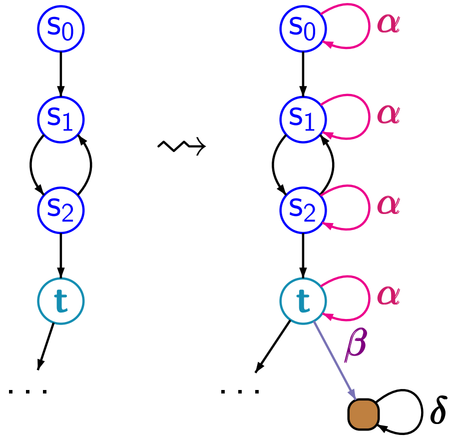

α, β dependent
α independent from all other actions

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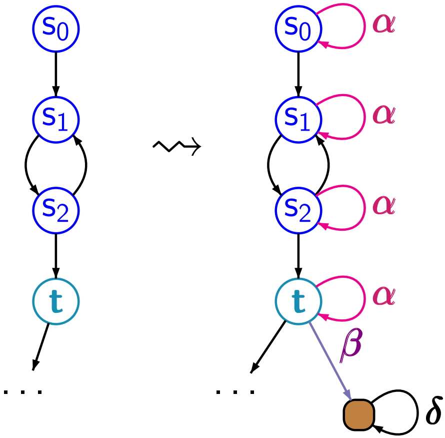

α, β dependent

α independent from all other actions

$\text{ample}(s_0) = \{\alpha\}$

$\text{ample}(u) = Act(u)$ for all other states u

Local criterion for condition $(\mathrm{A2})$

idea: replace the global dependency condition $(\mathrm{A2})$ by a stronger local condition

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e.g., the P_i's are given as program graphs of a channel system. Then: each state s has the form

$$s = \langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$$

where ℓ_i is a location of process P_i, η a variable evaluation, ξ a channel evaluation

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Provide local criteria such that $ample(s) = Act_i(s)$ fulfills the dependency condition (A2)

Local criterion for condition (A2)

Let $s = \langle \boldsymbol{\ell}_1, \ldots, \boldsymbol{\ell}_{i-1}, \boldsymbol{\ell}_i, \boldsymbol{\ell}_{i+1}, \ldots, \boldsymbol{\ell}_n, \ldots \rangle$.
Suppose that

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(A2.2) there is <u>no</u> action γ of a process P_j where $j \neq i$ s.t. γ can enable an action $\beta \in Act_i \setminus Act(s)$ from some state s' with location ℓ_i for process P_i

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$$\langle \ldots h_j \ldots \ell_i \ldots \rangle \xrightarrow{\gamma} \langle \ldots k_j \ldots \ell_i \ldots \rangle \xrightarrow{\beta}$$
$$\beta \not\downarrow$$

for some $\beta \in Act_i \setminus Act(s)$

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Then (A2) holds for $ample(s) = Act_i(s)$.

Heuristics for condition (A2)

⋮ expansion of state $s = \langle \ldots \ell_j \ldots \ell_i \ldots \rangle$

Heuristics for condition (A2)

LTL3.4-45

$\vdots$ expansion of state $s = \langle \ldots \ell_j \ldots \ell_i \ldots \rangle$

$A := Act_i(s)$

Heuristics for condition (A2)

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check if for all other processes P_j the following holds:

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if yes then set $ample(s) := A$

$\vdots$

Correct or wrong?

Let $\mathcal{T}_1, \mathcal{T}_2$ be transition systems with $\mathcal{T}_1 \triangleq \mathcal{T}_2$, and let fair be an **LTL** fairness assumption.

Remind: $\triangleq$ denotes stutter trace equivalence.
E.g., $\mathcal{T}_1 = \mathcal{T}$, $\mathcal{T}_2 = \mathcal{T}_{\text{red}}$

Then, for all $\textbf{LTL}_{\setminus\bigcirc}$ formulas φ:

$$\mathcal{T}_1 \models_{\text{fair}} \varphi \quad \text{iff} \quad \mathcal{T}_2 \models_{\text{fair}} \varphi$$

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