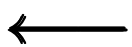
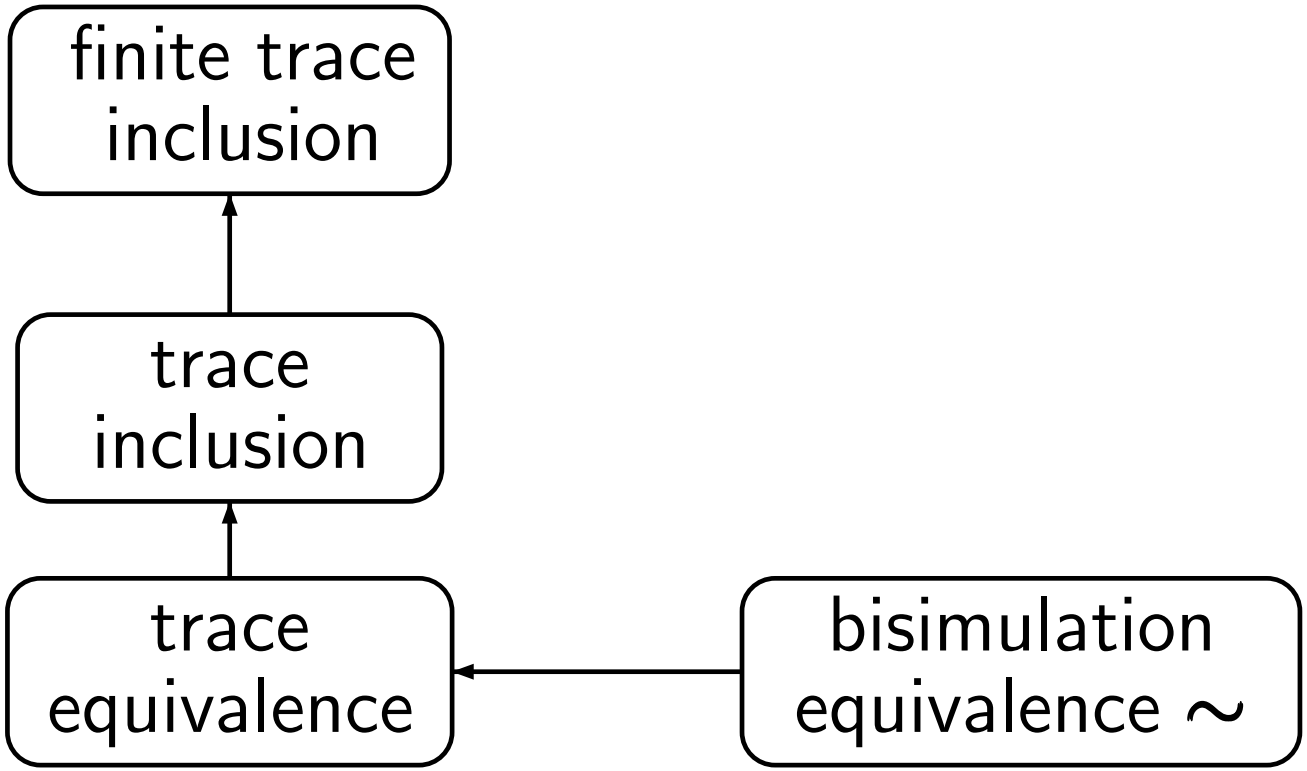
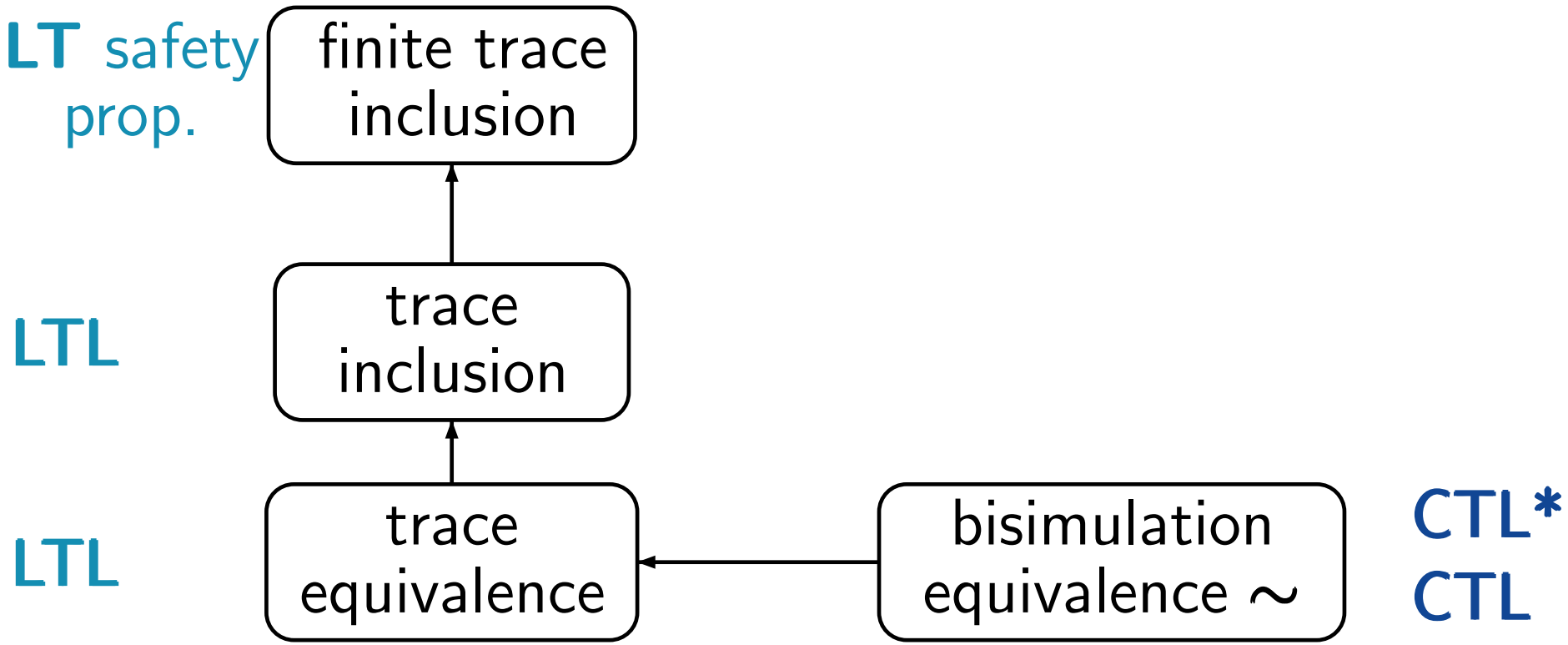
Overview



Equivalences and preorders



Logical characterizations



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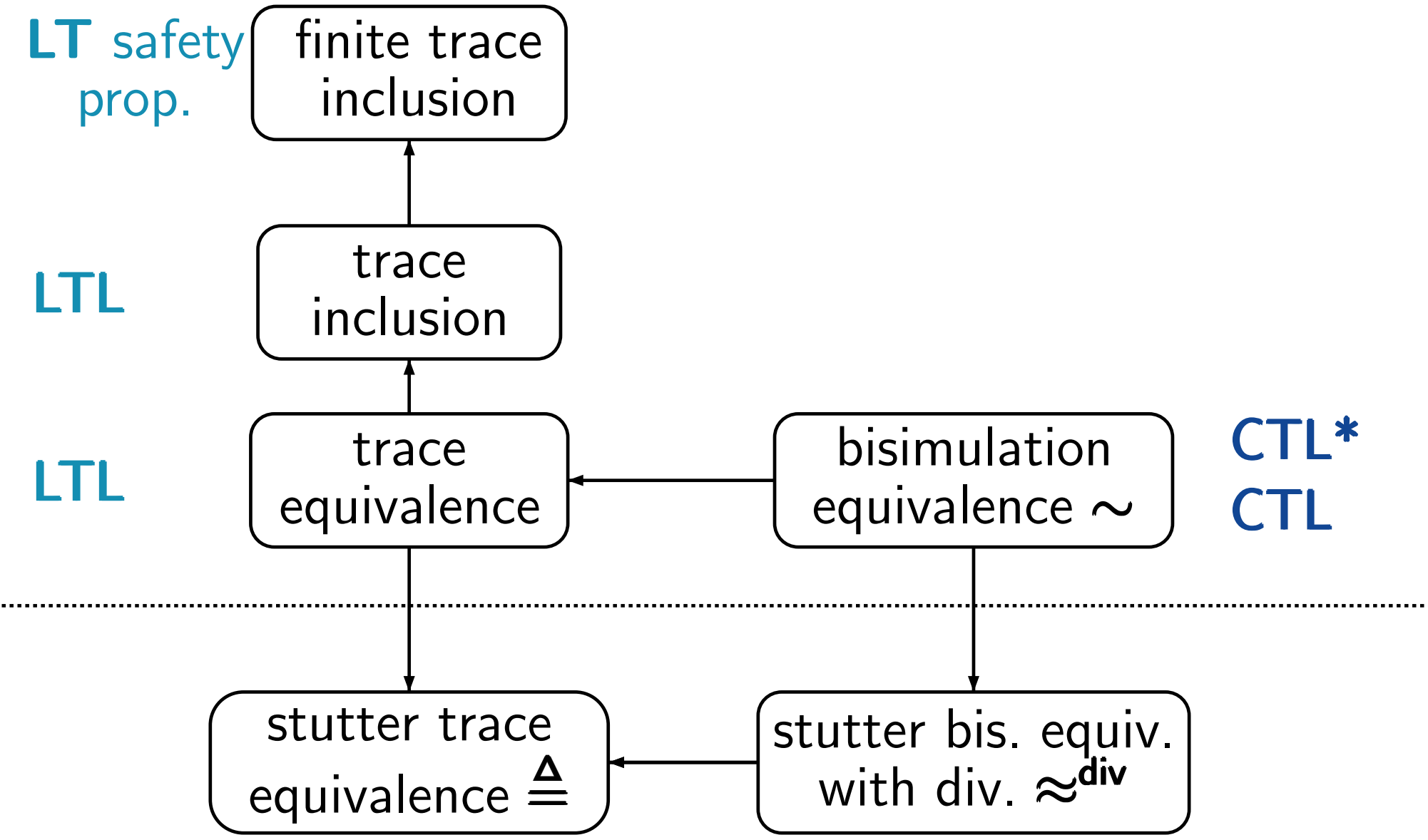

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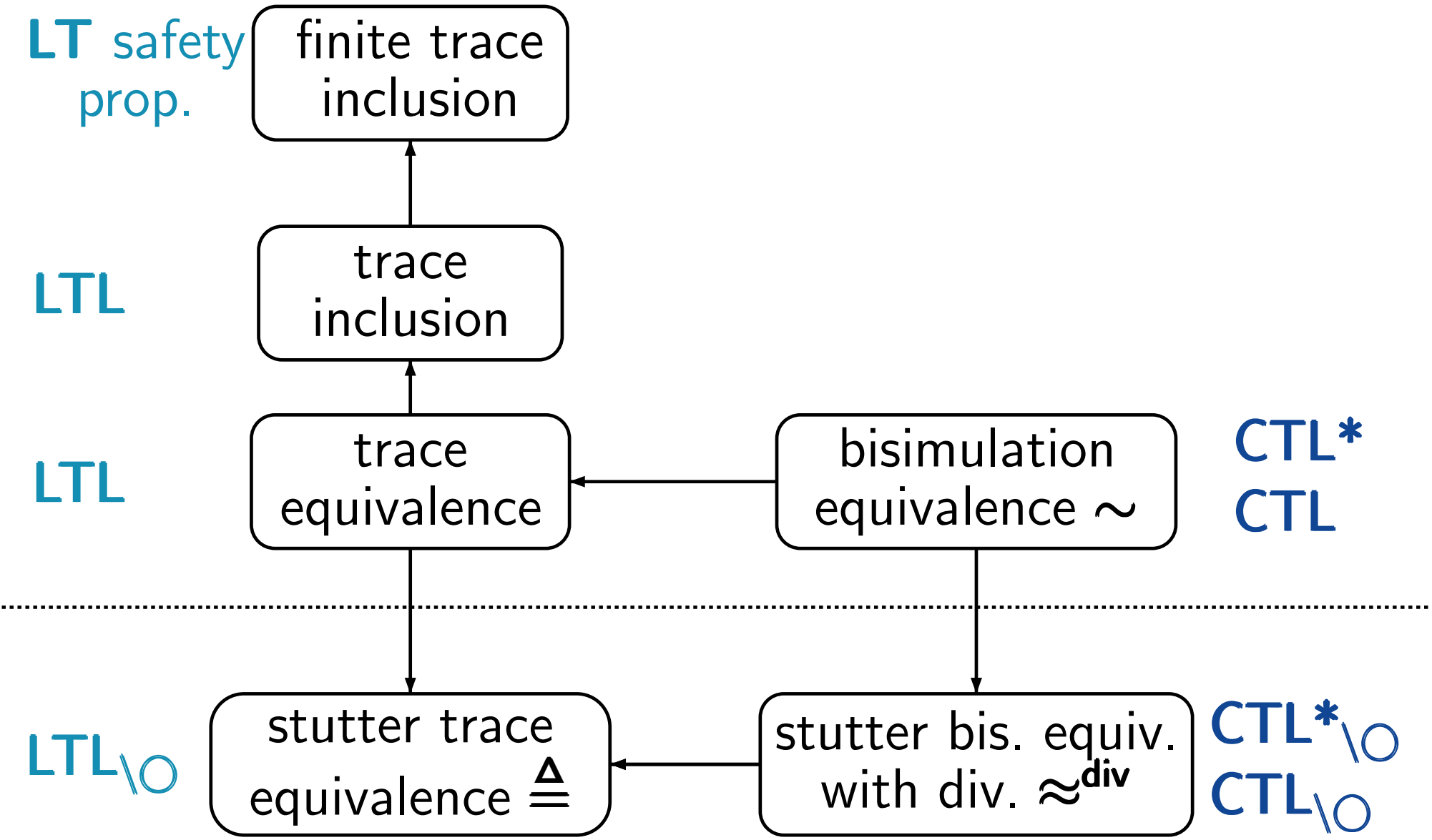

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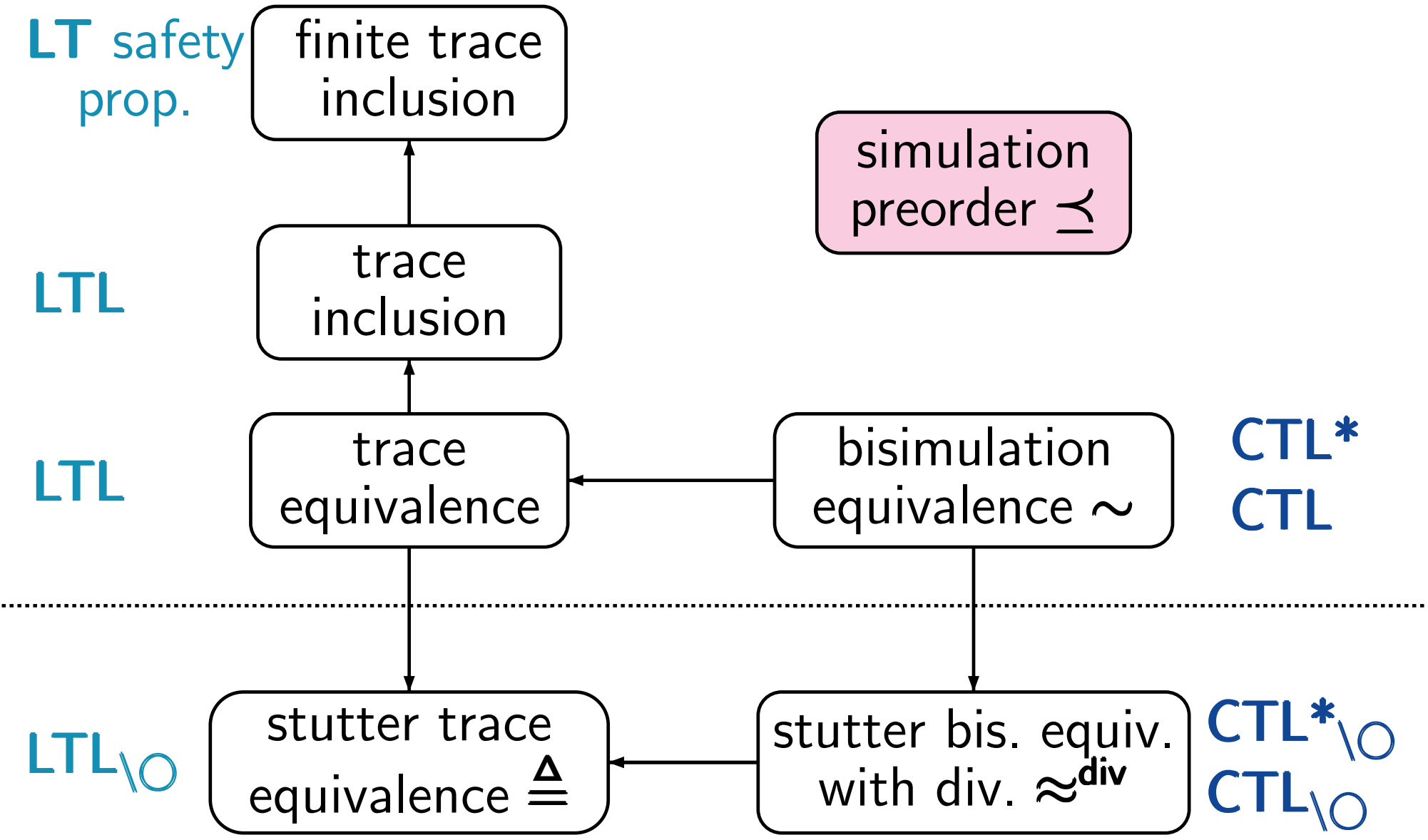

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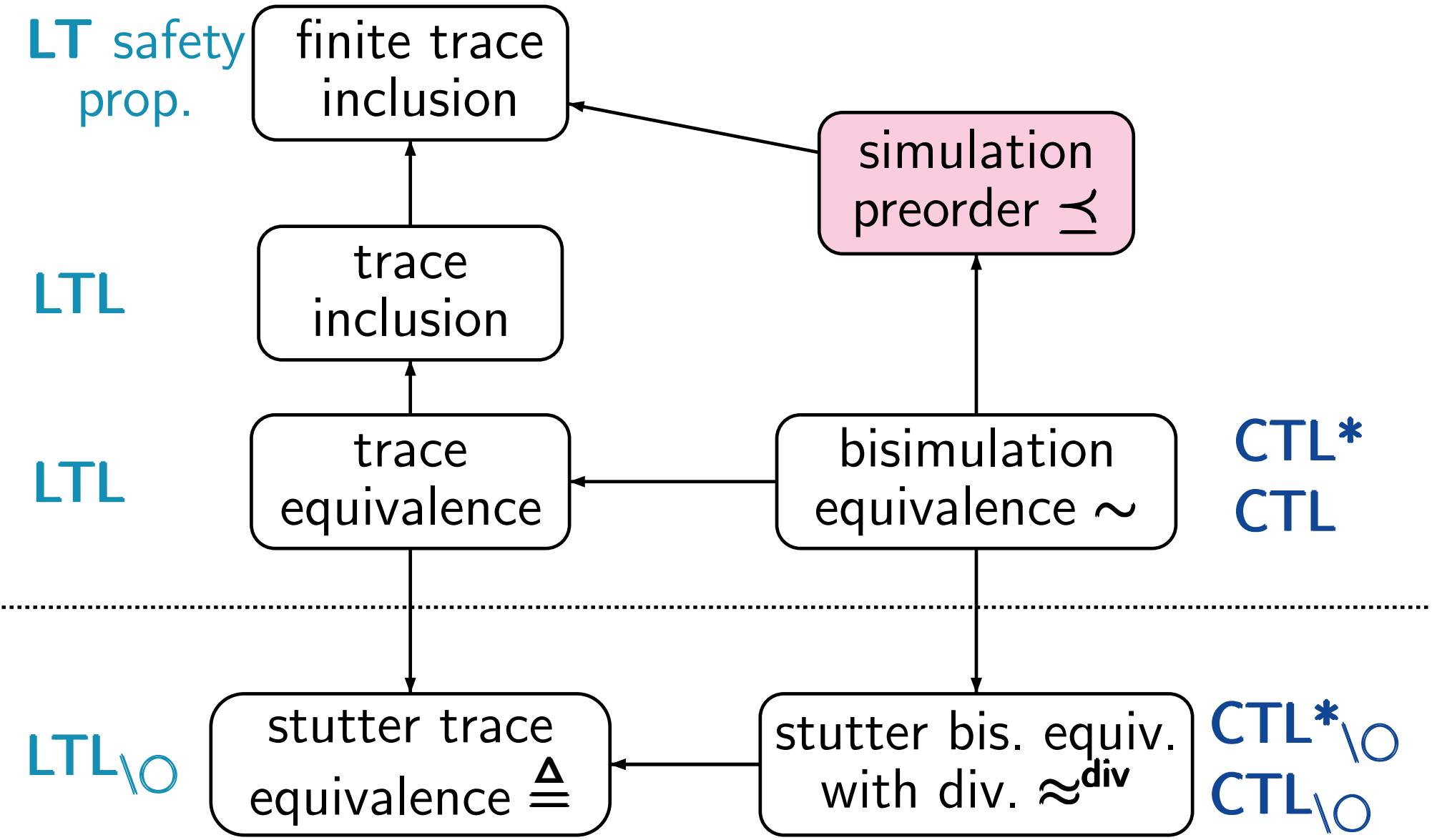

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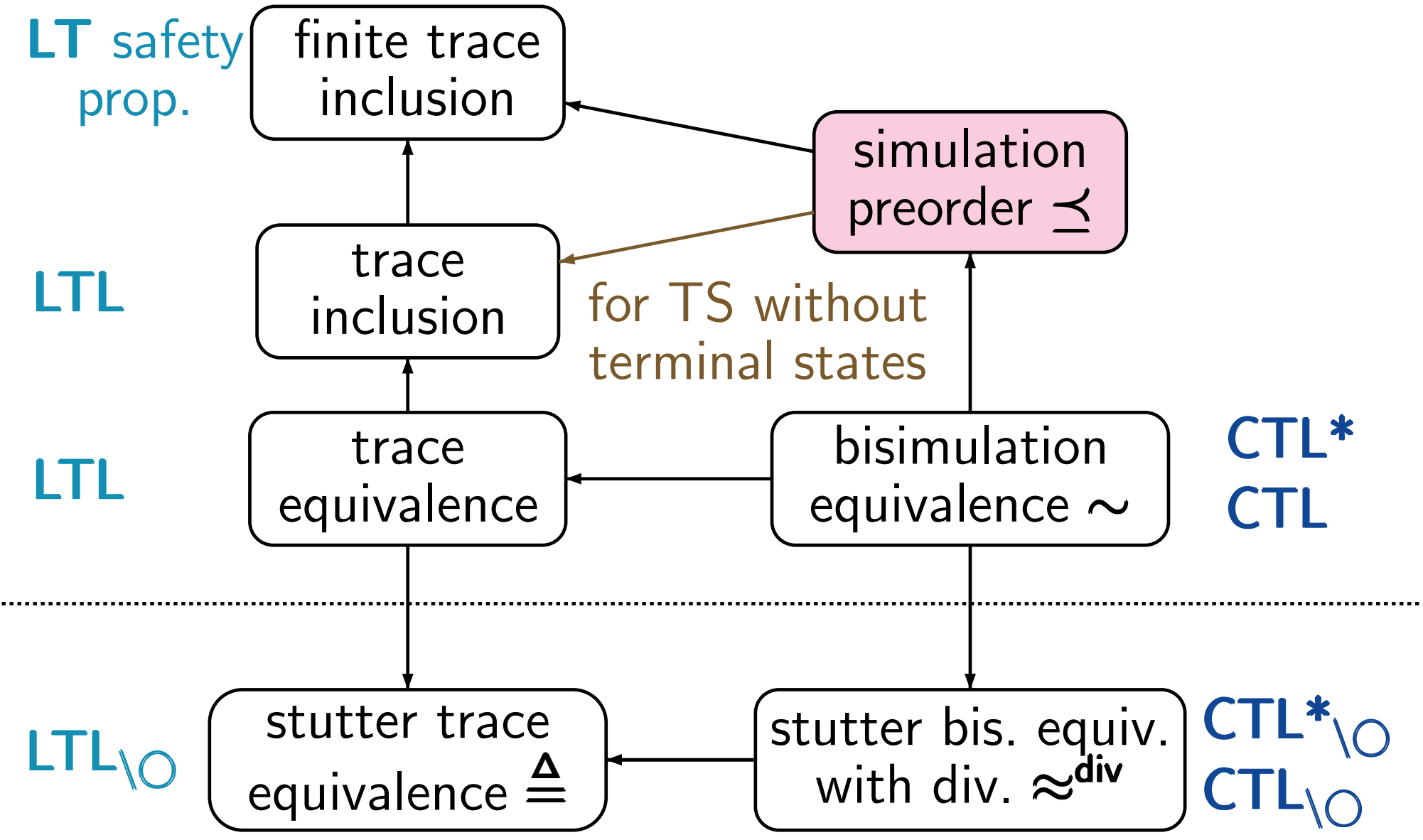

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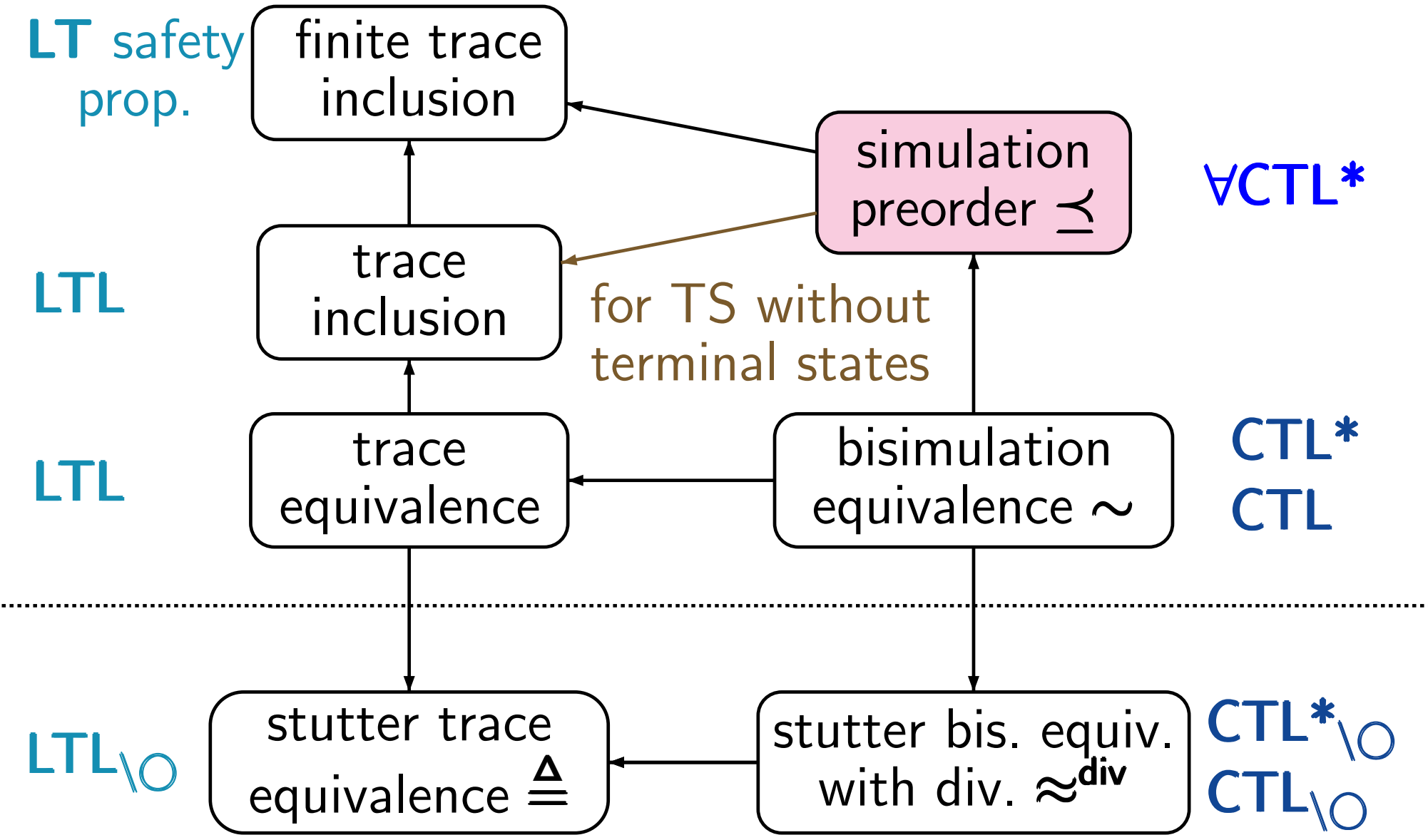

Logical characterization

for bisimulation equivalence $\sim_{\mathcal{T}}$:

$s_1 \sim_{\mathcal{T}} s_2$ iff s_1, s_2 satisfy the same **CTL*** formulas

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observation: $\mathbb{L}$ cannot be closed under negation

The universal fragment ∀CTL* of CTL*

CTL* formulas in positive normal form, without ∃

Syntax of ∀CTL*

∀CTL* state formulas:

$$\Phi ::= \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid \forall\varphi$$

∀CTL* path formulas:

$$\varphi ::= \Phi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2$$

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always: $\Box\varphi \stackrel{\text{def}}{=} \varphi\,\mathbf{W}\,\mathit{false}$

Embedding of LTL in ∀CTL*

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but $\forall\Diamond\forall\Box a$ cannot be expressed in **LTL**

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∀**CTL**: sublogic of ∀**CTL***

- no Boolean operators for paths formulas
- the arguments of the temporal modalities $\bigcirc$, **U** and **W** are state formulas

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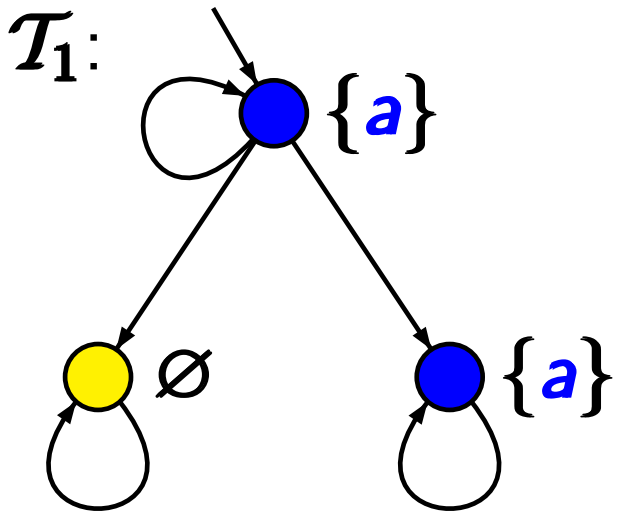
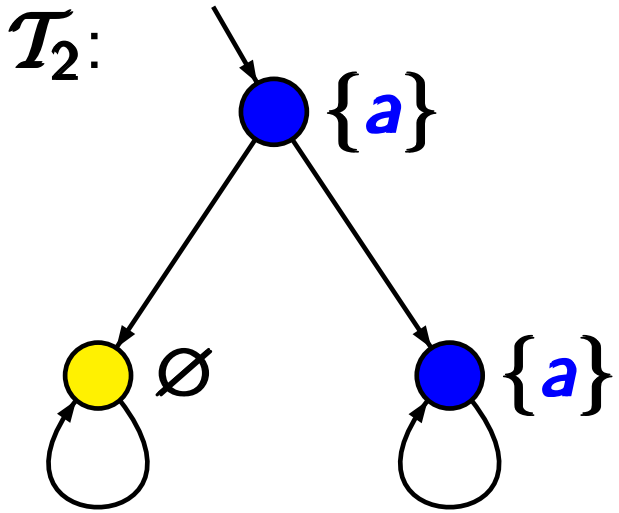
∀CTL and simulation





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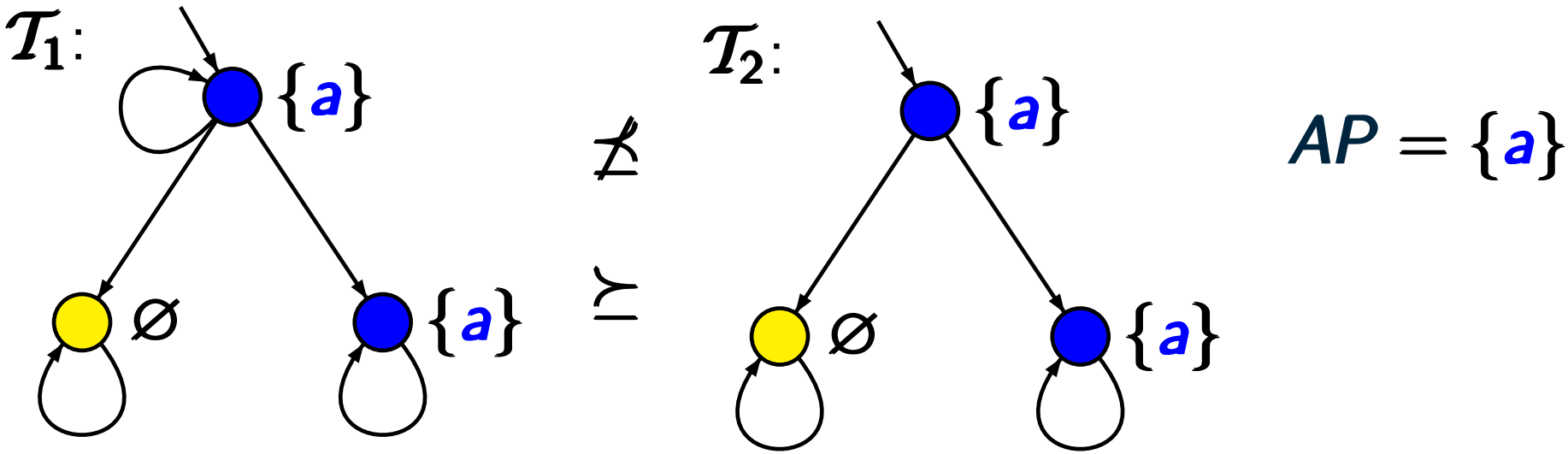

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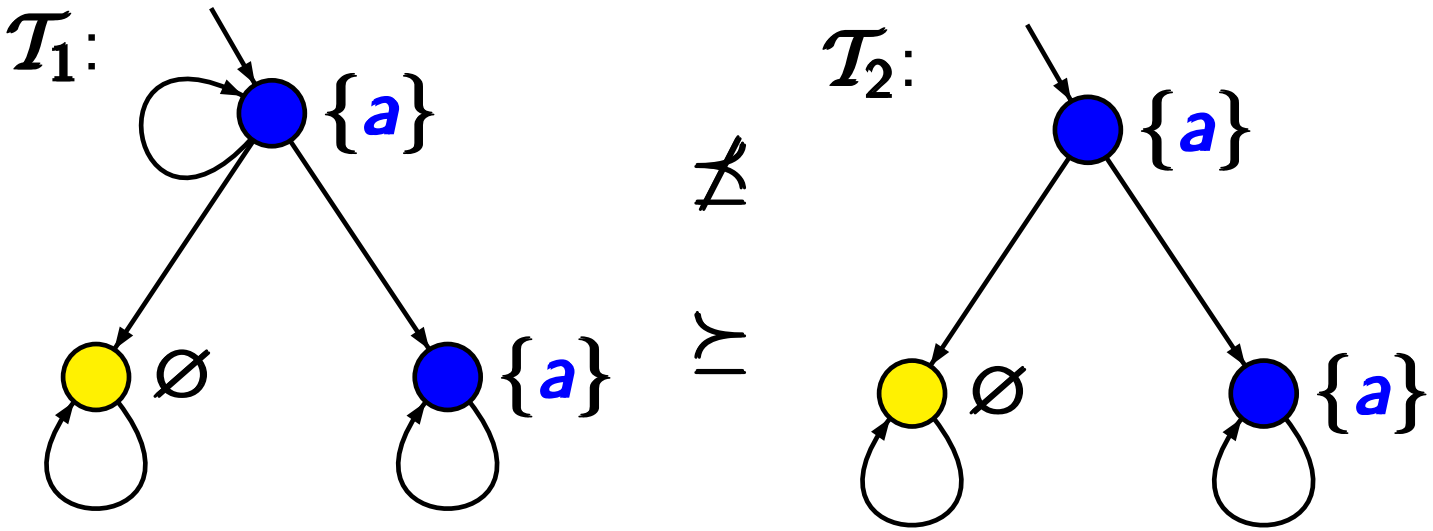

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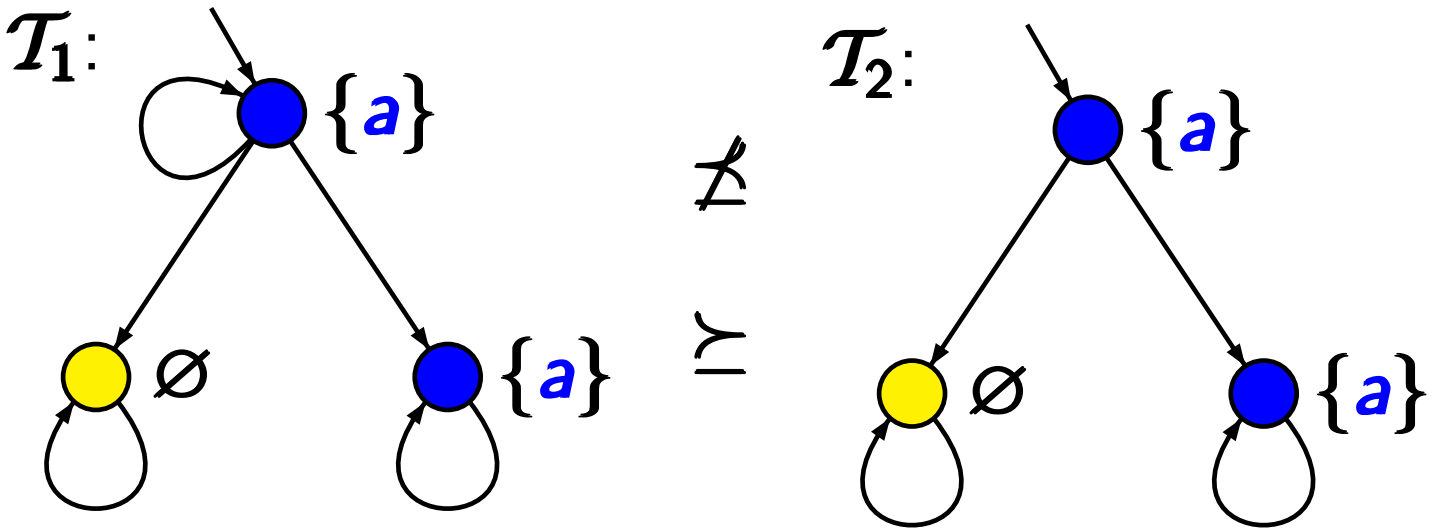

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∀CTL/∀CTL* and the simulation preorder

For finite TS without terminal states, the following statements are equivalent:

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proof by structural induction

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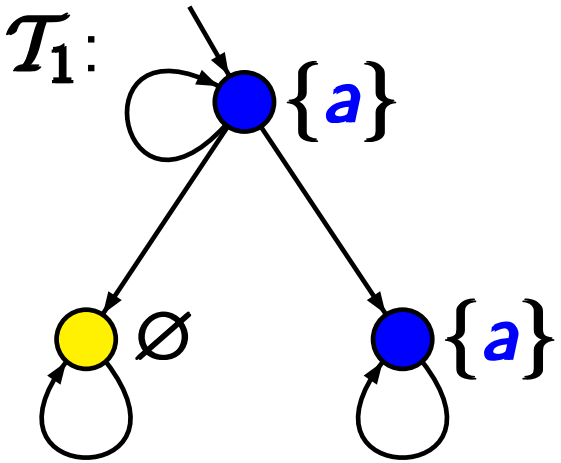
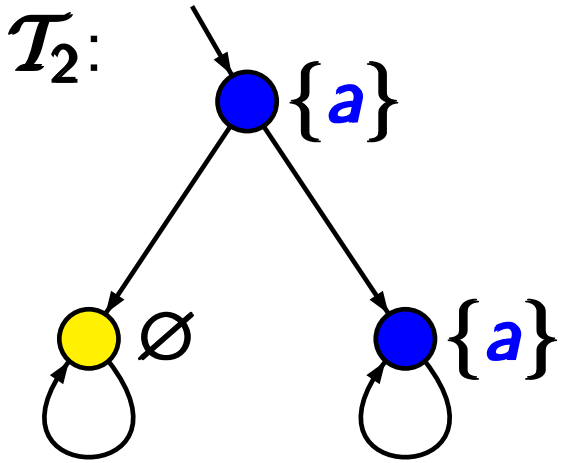
Example: ∀CTL/∃CTL and simulation





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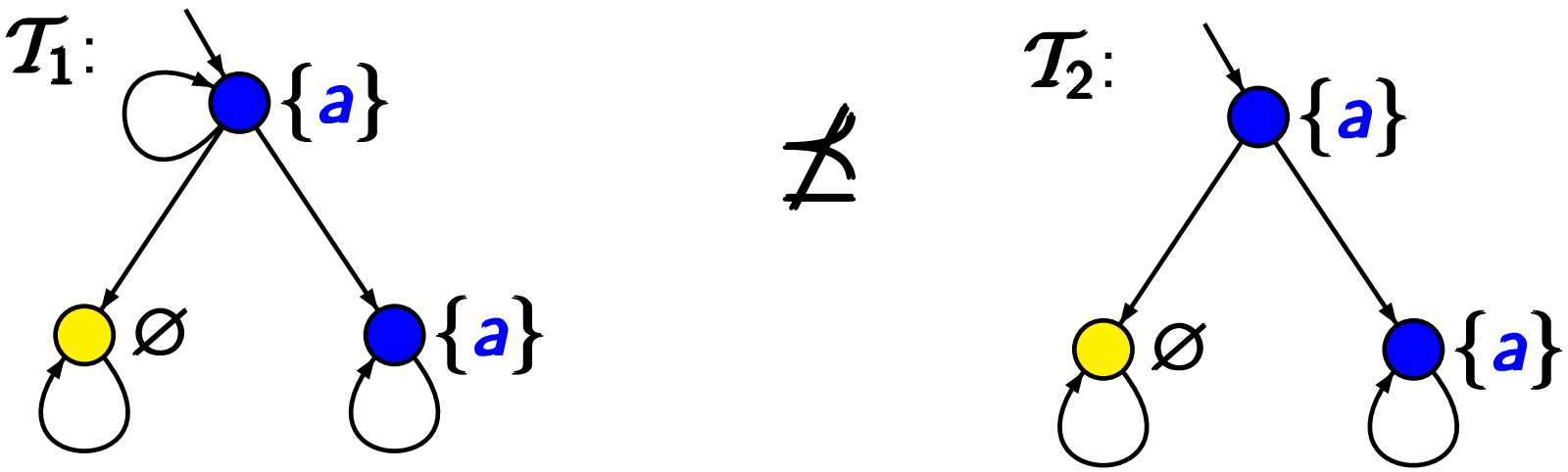

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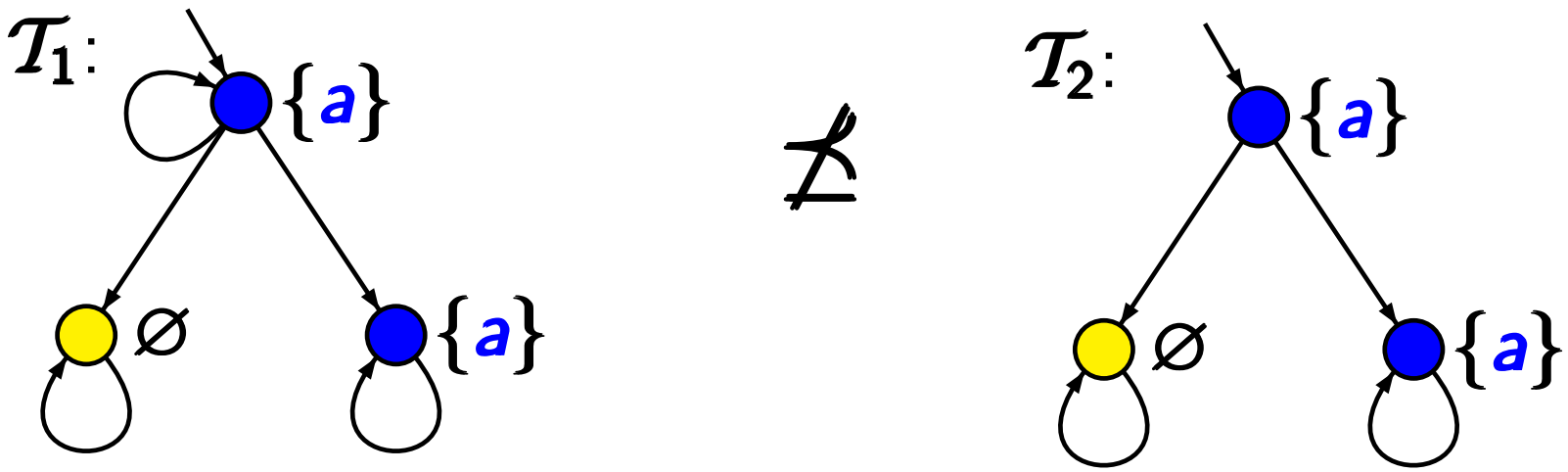

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∀CTL formula

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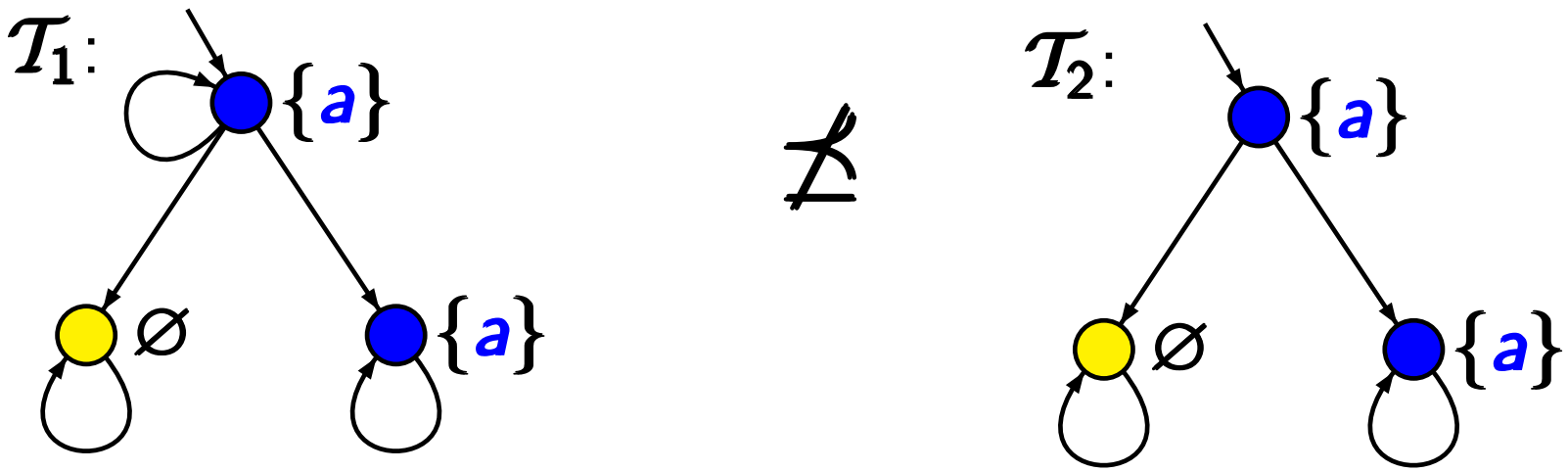

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∃CTL formula

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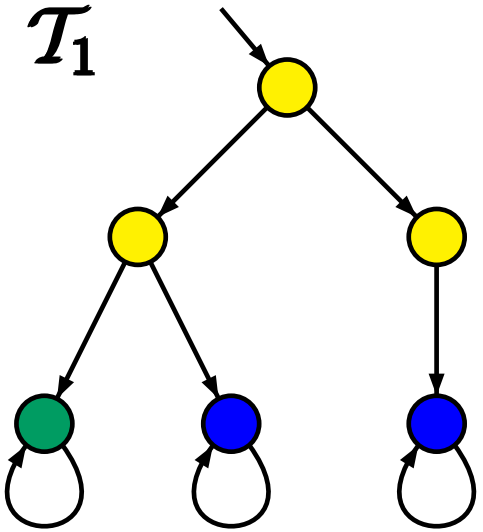
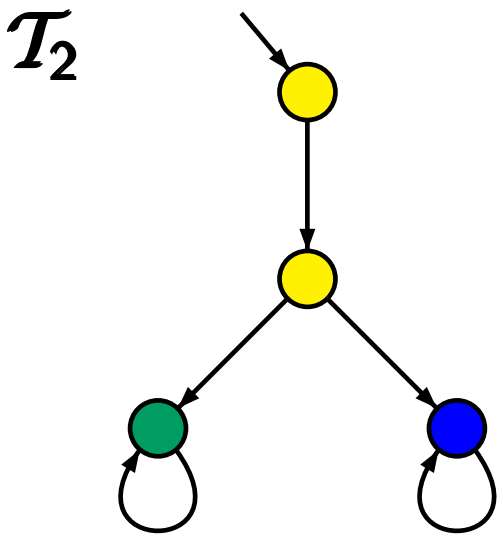
iff $\mathcal{T}_1, \mathcal{T}_2$ satisfy the same $\exists$CTL formulas

... even holds for $\forall\text{CTL}^*_{\setminus U,W}$, $\forall\text{CTL}_{\setminus U,W}$, $\exists\text{CTL}^*_{\setminus U,W}$, $\exists\text{CTL}_{\setminus U,W}$

Simulation equivalence

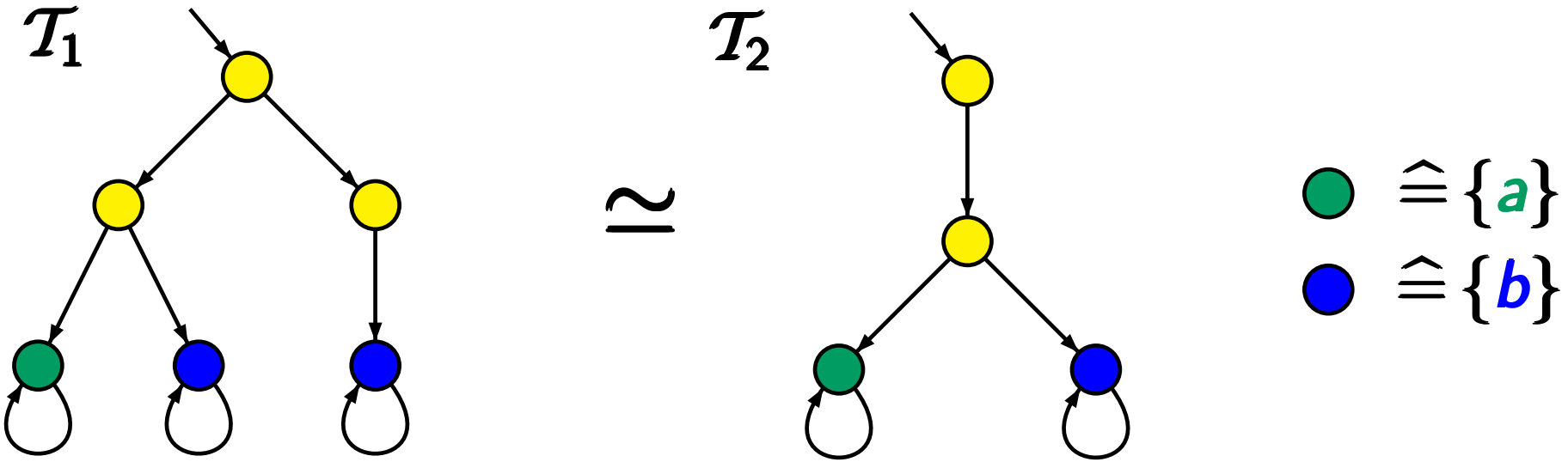




$\widehat{=} \{a\}$

$\widehat{=} \{b\}$

Simulation equivalence



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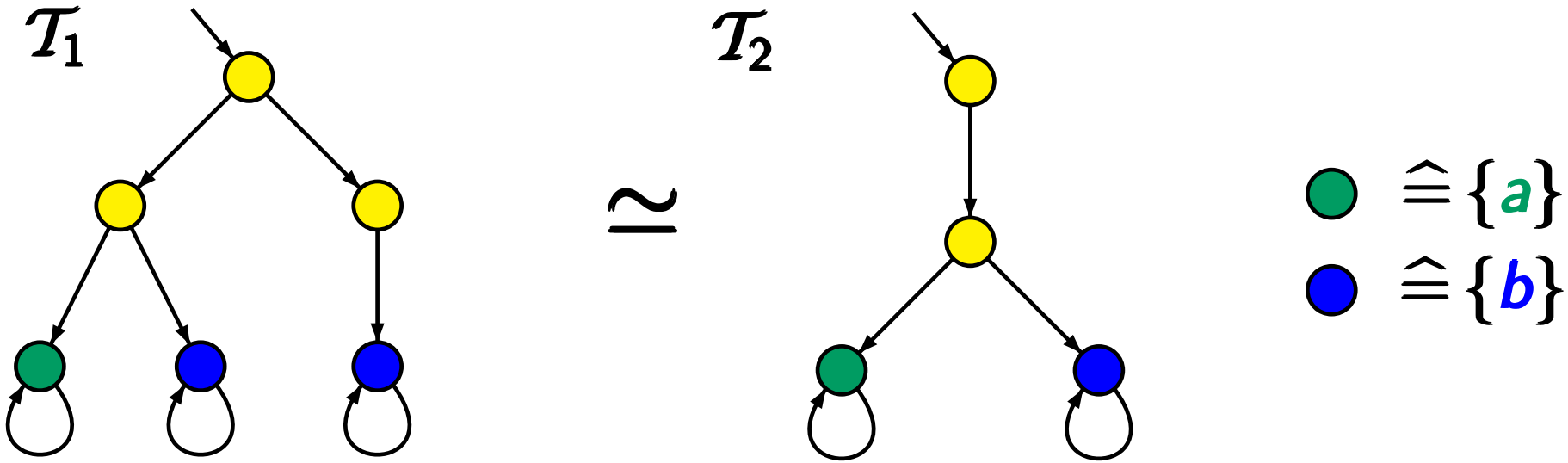

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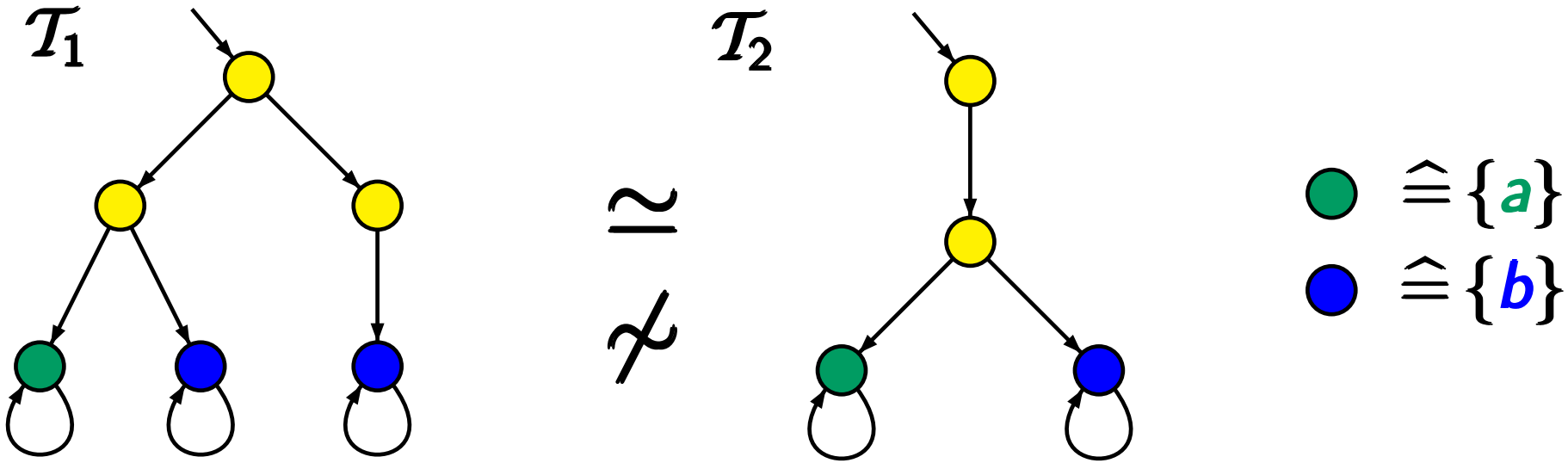

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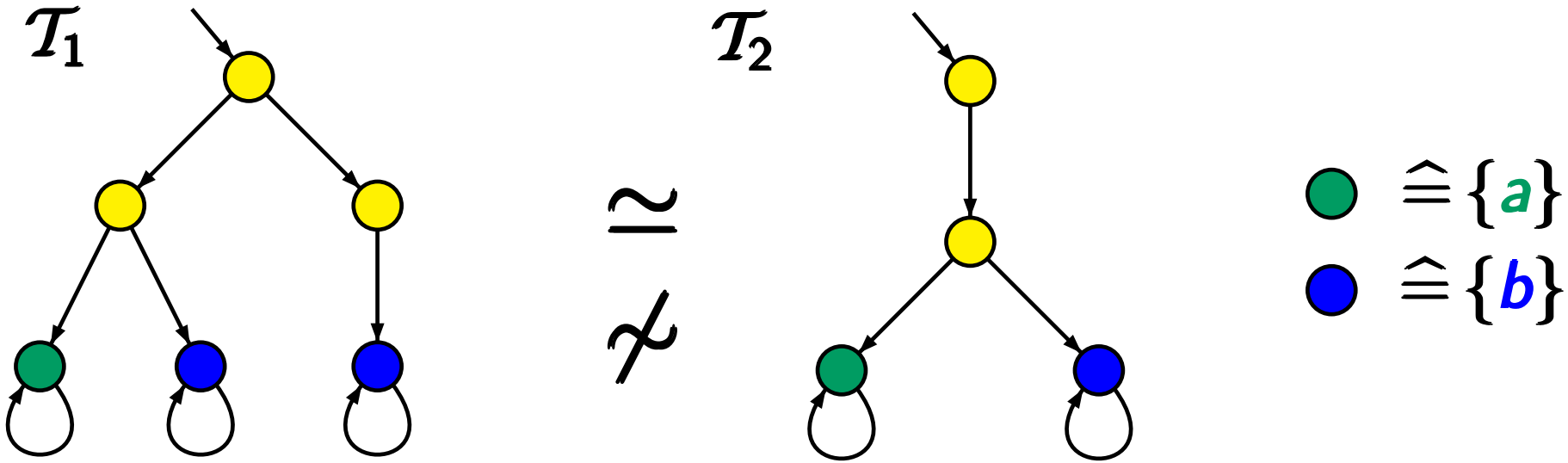

$\mathcal{T}_1$, $\mathcal{T}_2$ cannot be distinguished by the temporal logics
$\forall$**CTL**, $\forall$**CTL***, $\exists$**CTL**, or $\exists$**CTL***,

but by **CTL**:

$$\mathcal{T}_1 \not\models \forall\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$

$$\mathcal{T}_2 \models \forall\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$

Correct or wrong?

In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

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$\exists$CTL equivalence

$=$ simulation equivalence

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correct

$\exists$CTL equivalence

$=$ simulation equivalence

$=$ $\forall$CTL* equivalence

and LTL is a sublogic of $\forall$CTL*

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correct

> If s_1, s_2 satisfy the same LTL formulas
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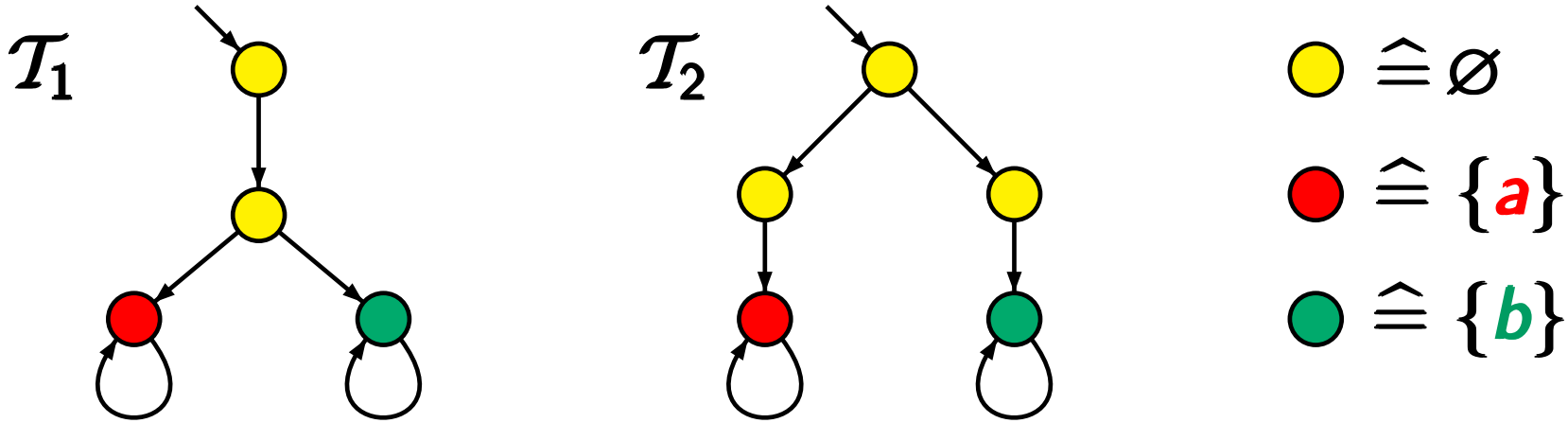
wrong, as trace equivalence does not imply simulation equivalence

Does there exist ...?



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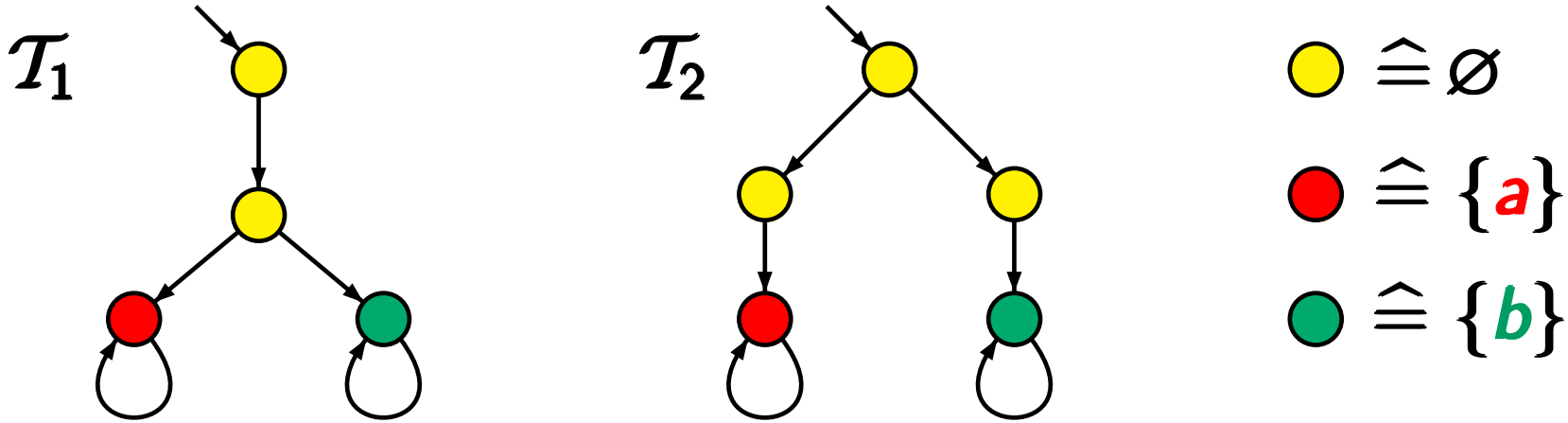

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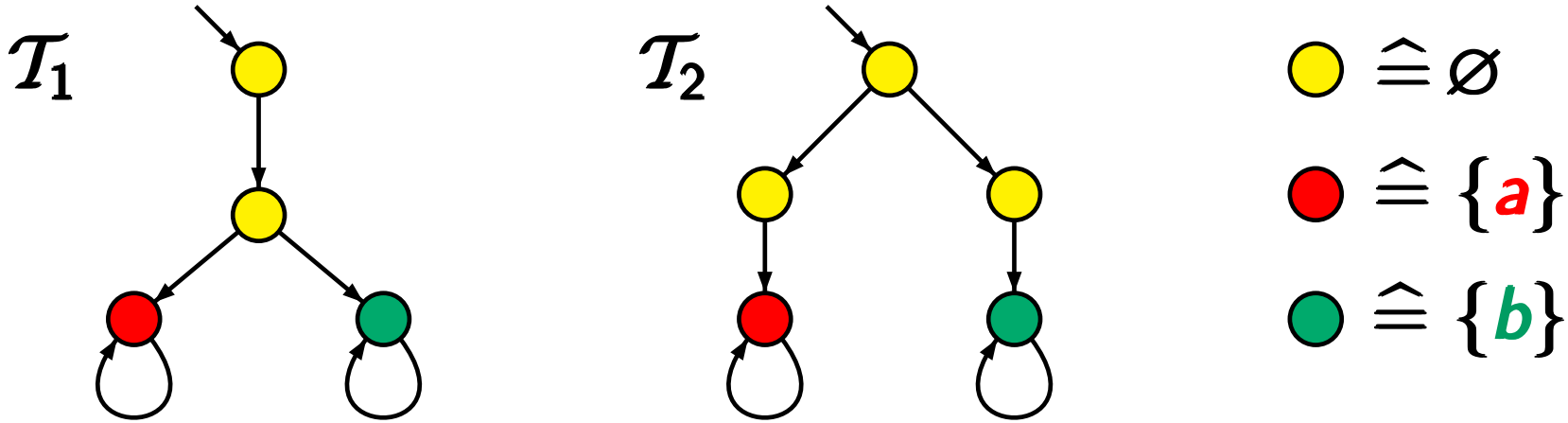

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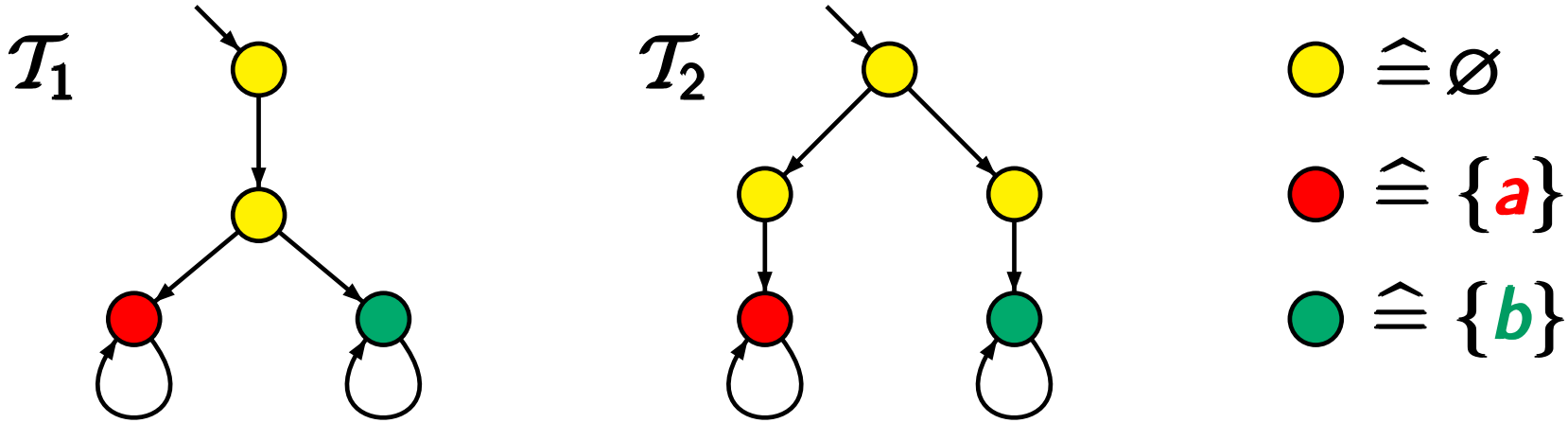

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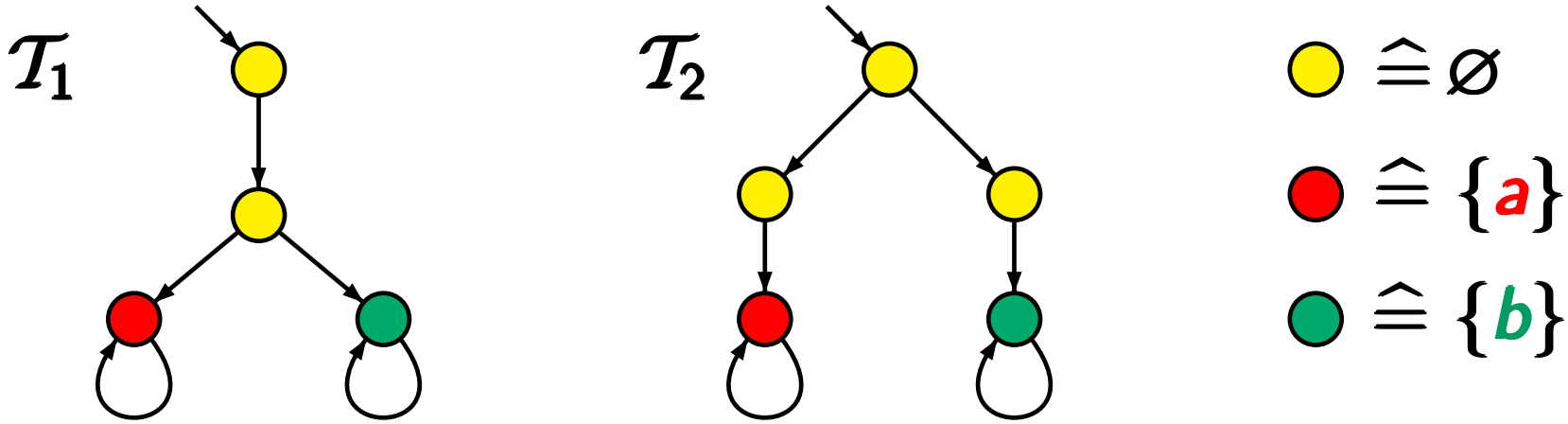

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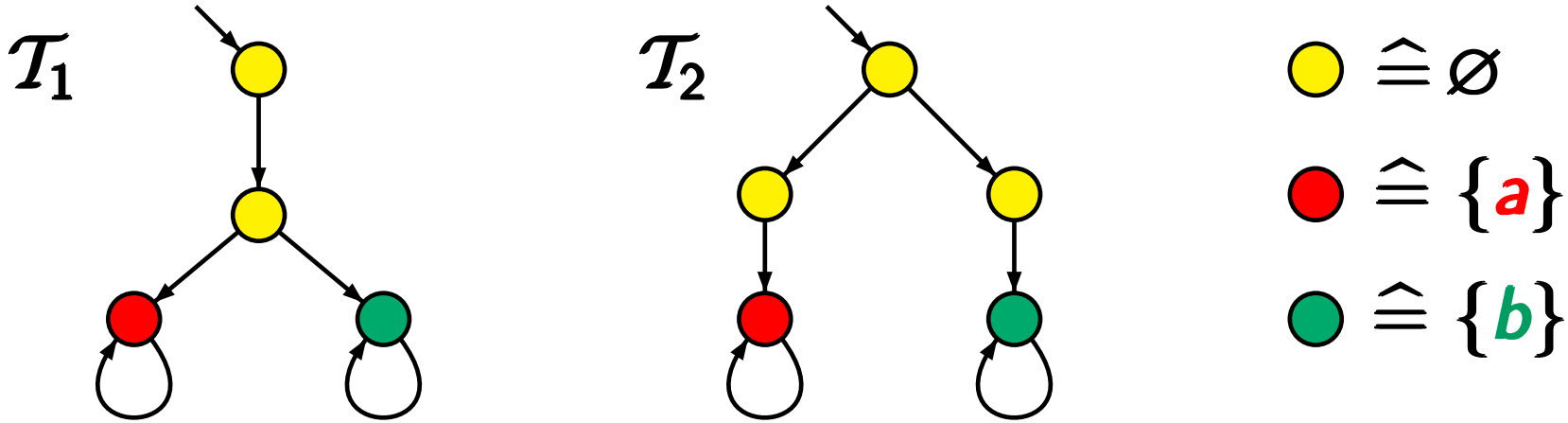

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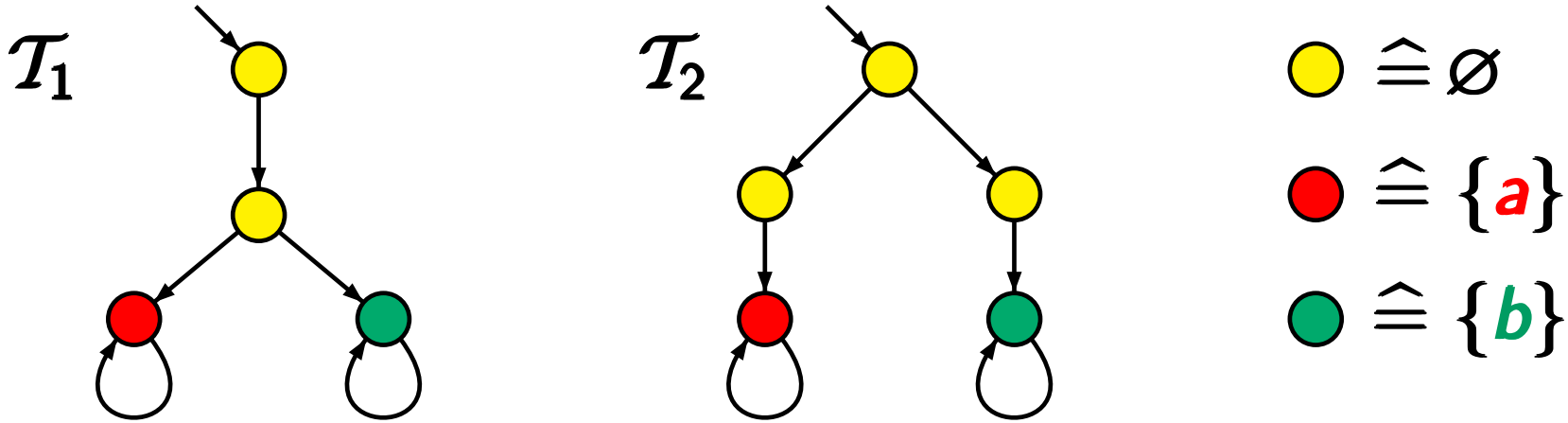

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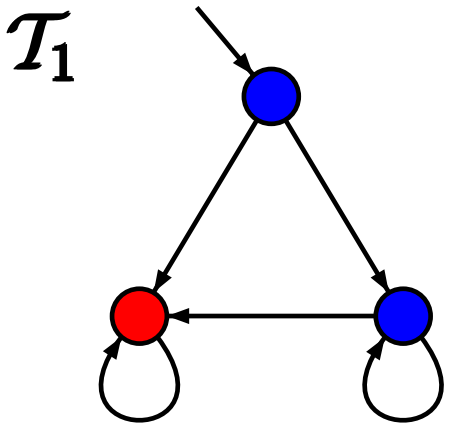

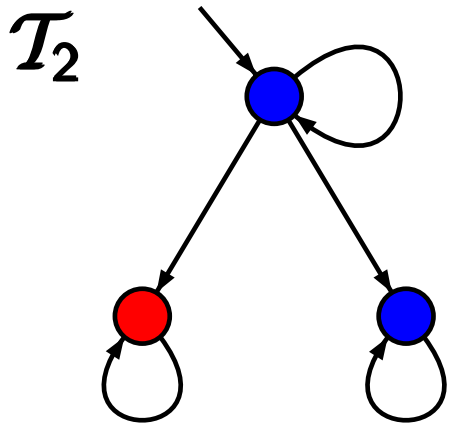

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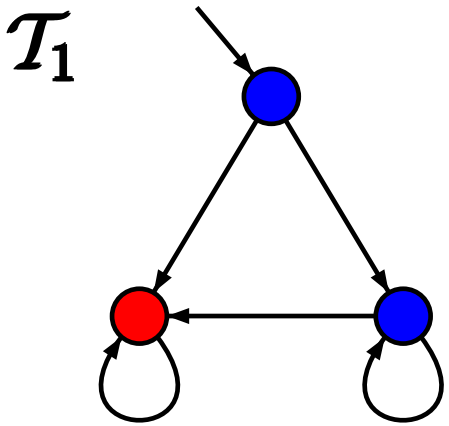

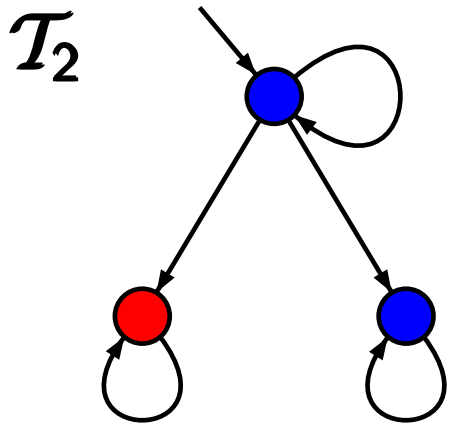

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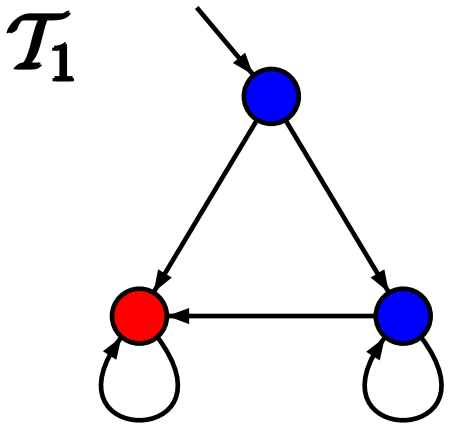

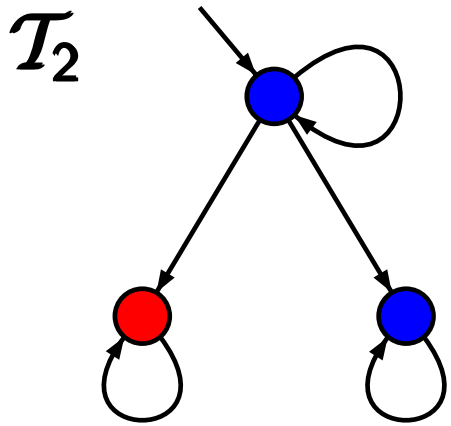

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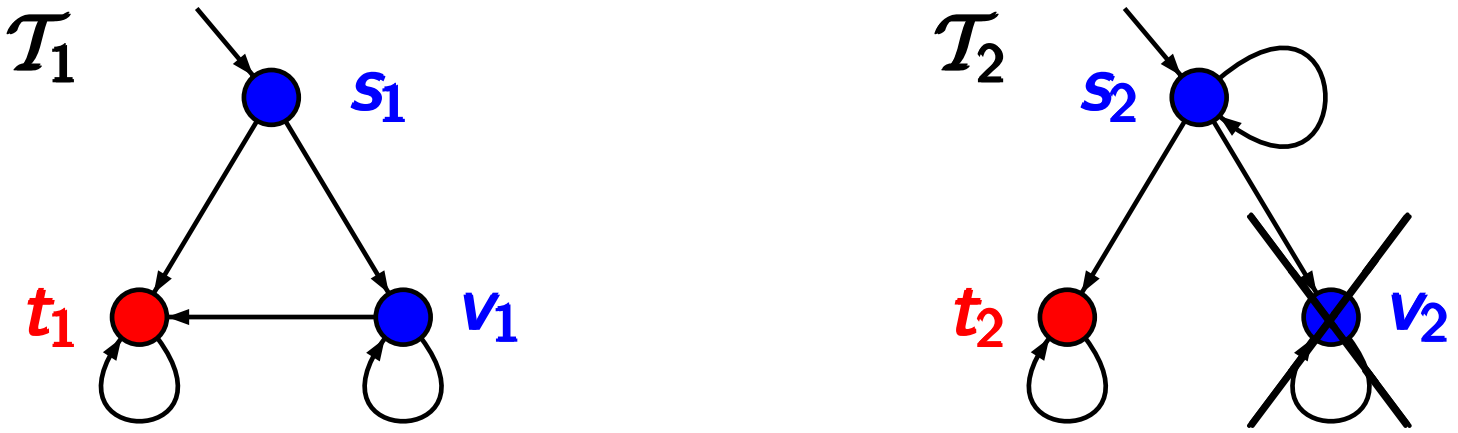

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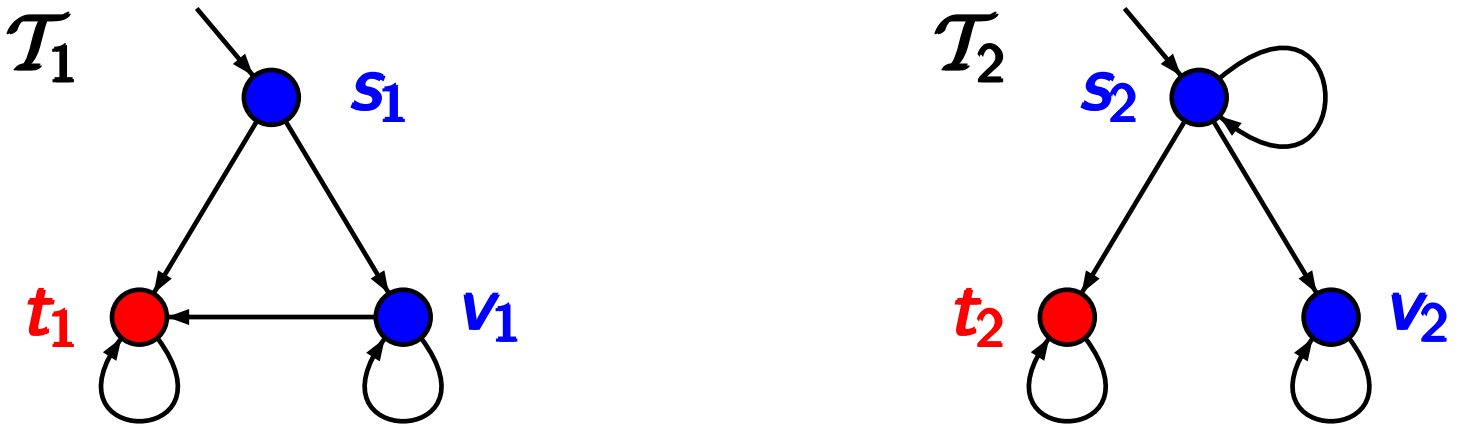

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simulation for $(\mathcal{T}_2, \mathcal{T}_1)$:

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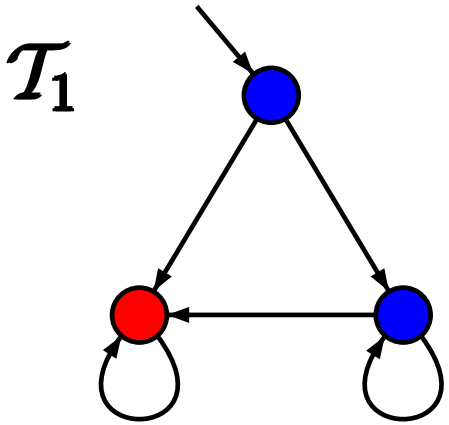

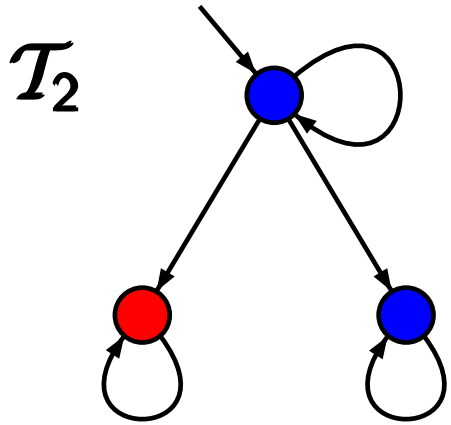

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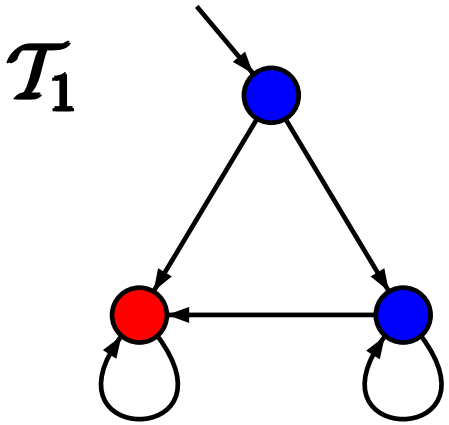

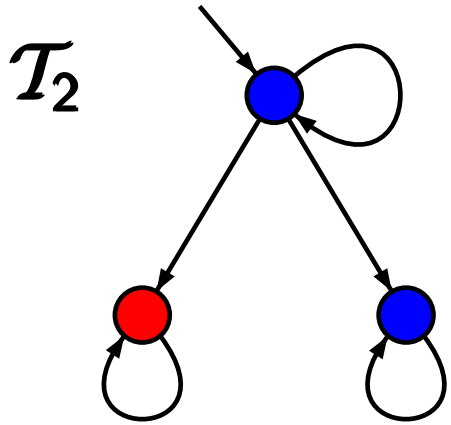

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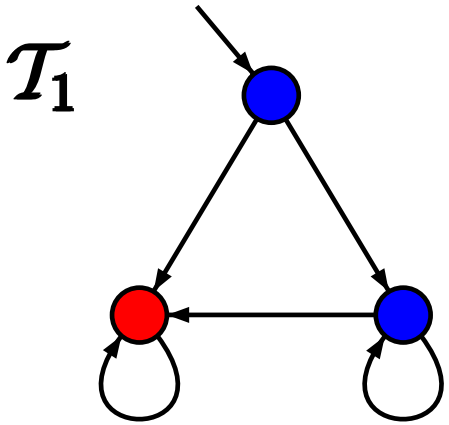

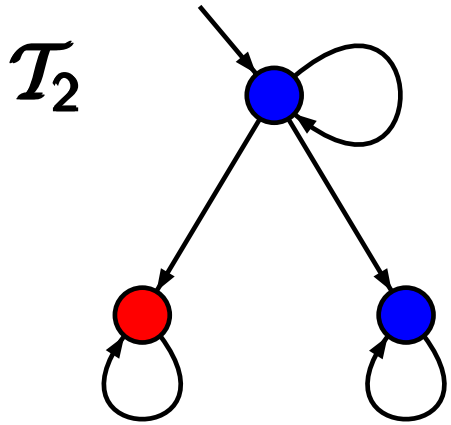

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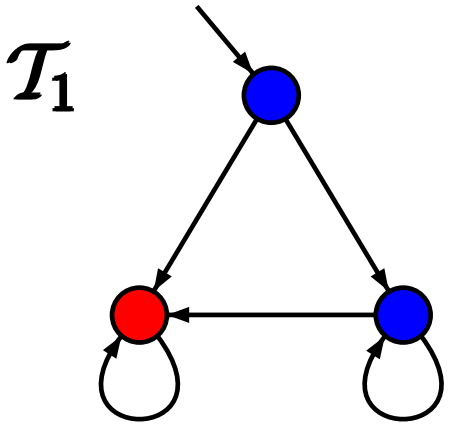

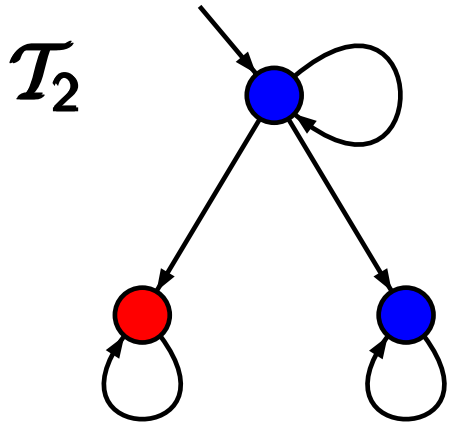

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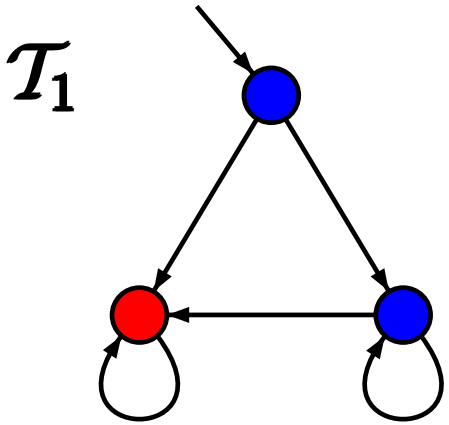
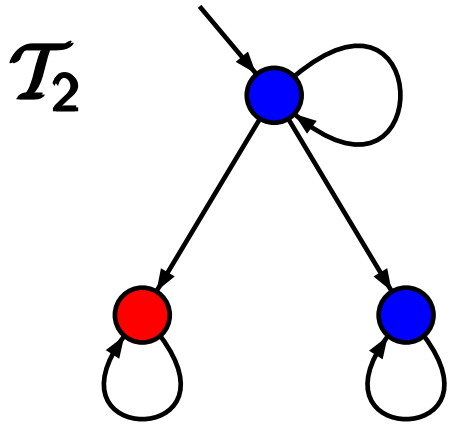
Does there exist ...?





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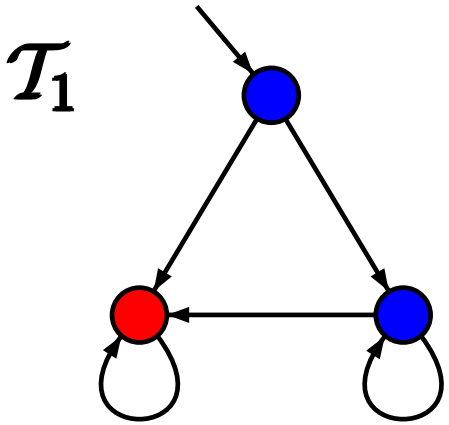
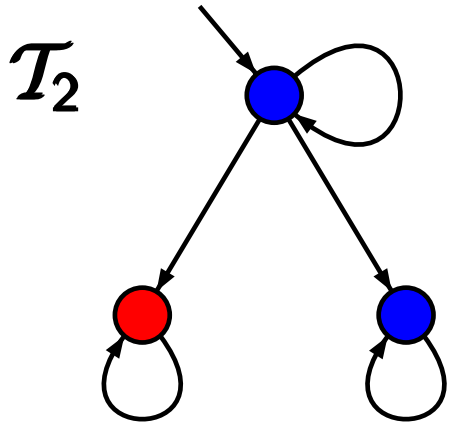
Does there exist a **LTL** formula φ s.t.

$\mathcal{T}_1 \not\models \varphi$ and $\mathcal{T}_2 \models \varphi$?

Does there exist ...?





Does there exist a **CTL** formula Φ s.t.

$\mathcal{T}_1 \not\models \Phi$ and $\mathcal{T}_2 \models \Phi$?

yes, as $\mathcal{T}_1 \not\sim \mathcal{T}_2$, e.g., $\Phi = \exists\bigcirc\forall\Box \textit{blue}$

Does there exist a **LTL** formula φ s.t.

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no

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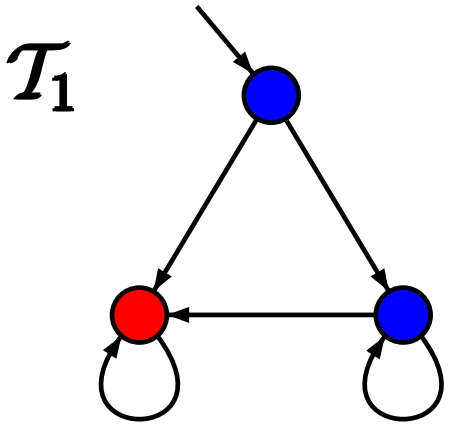

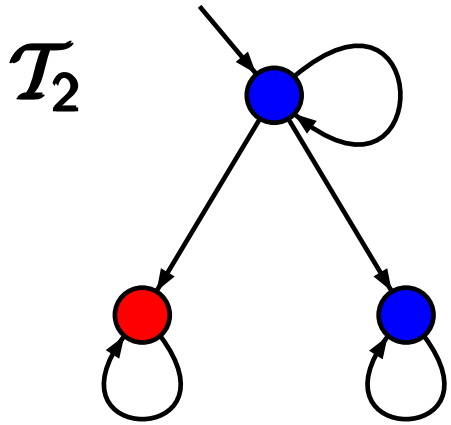

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no, as $\mathcal{T}_1$, $\mathcal{T}_2$ are simulation equivalent

Simulation quotient

Simulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

simulation quotient $\mathcal{T}/\simeq$:

transition system that arises from $\mathcal{T}$ by collapsing all simulation equivalent states

Simulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\simeq \stackrel{\text{def}}{=} (S/\simeq, Act', \rightarrow_{\simeq}, S'_0, AP', L')$$

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$$[s] = \{s' \in S : s \simeq_{\mathcal{T}} s'\}$$

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- labeling: $AP' = AP$ and $L'([s]) = L(s)$

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  action labels: irrelevant

Similarity of $\mathcal{T}$ and $\mathcal{T}/\simeq$

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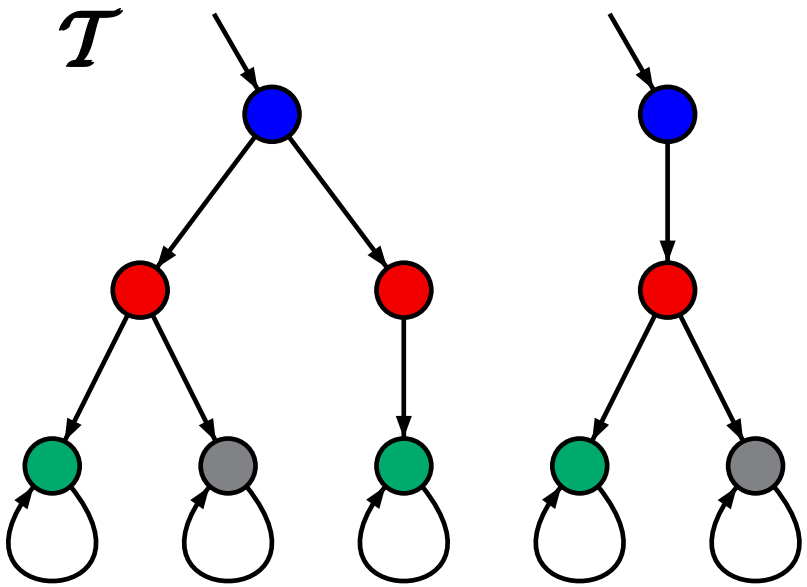
Example: simulation quotient



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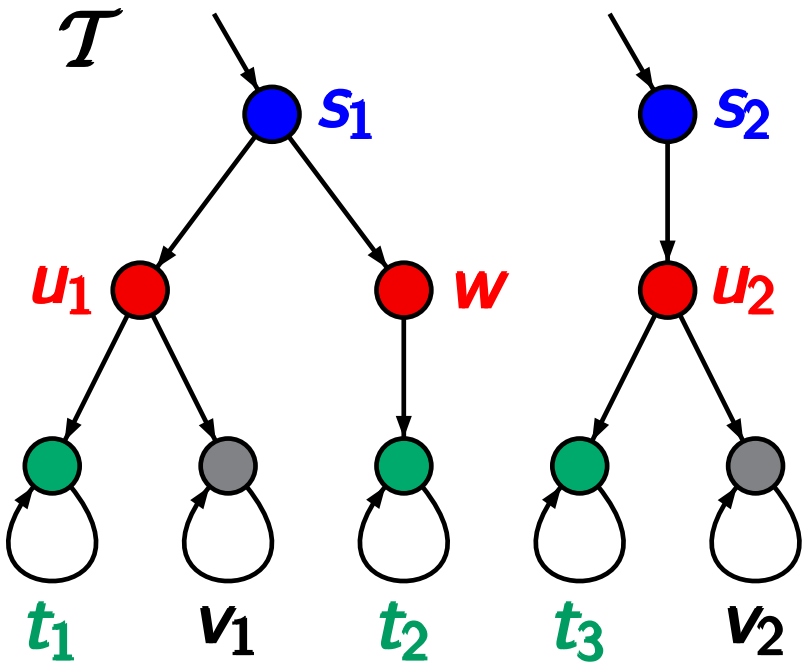
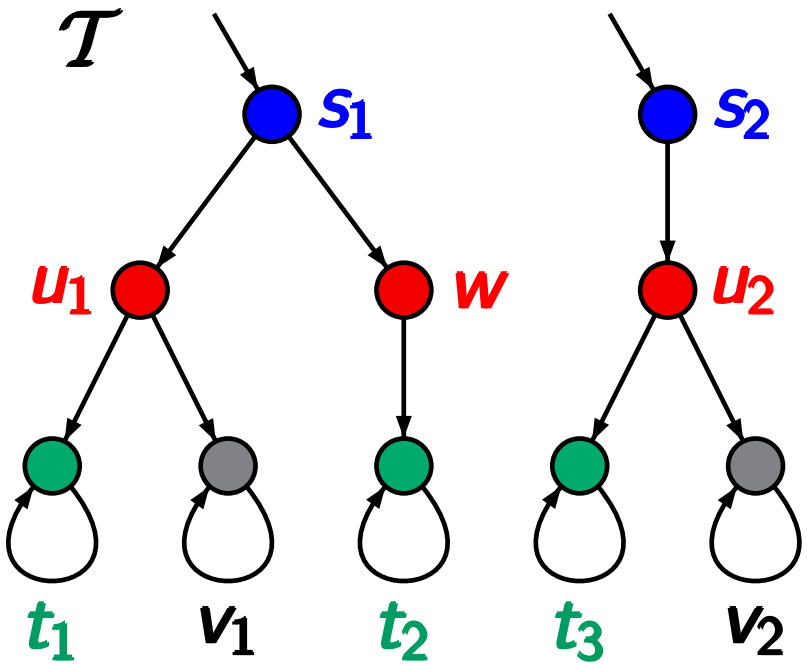


t_1, t_2, t_3 are simulation equivalent

v_1, v_2 are simulation equivalent

Example: simulation quotient



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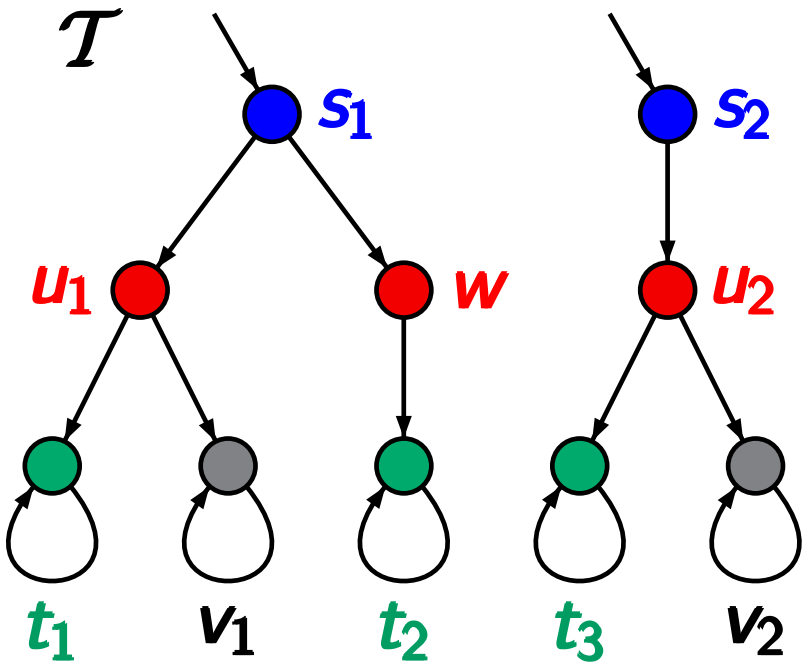

t_1, t_2, t_3 are simulation equivalent

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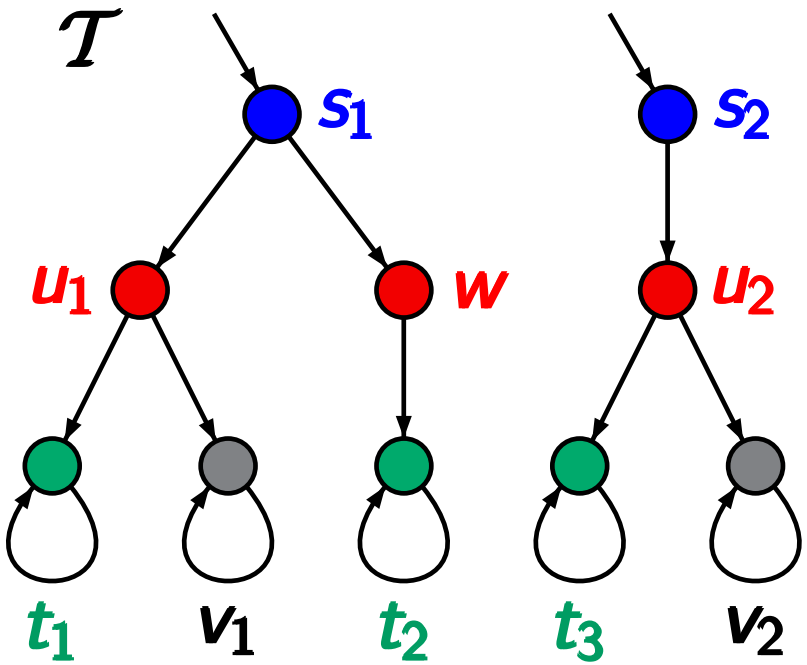

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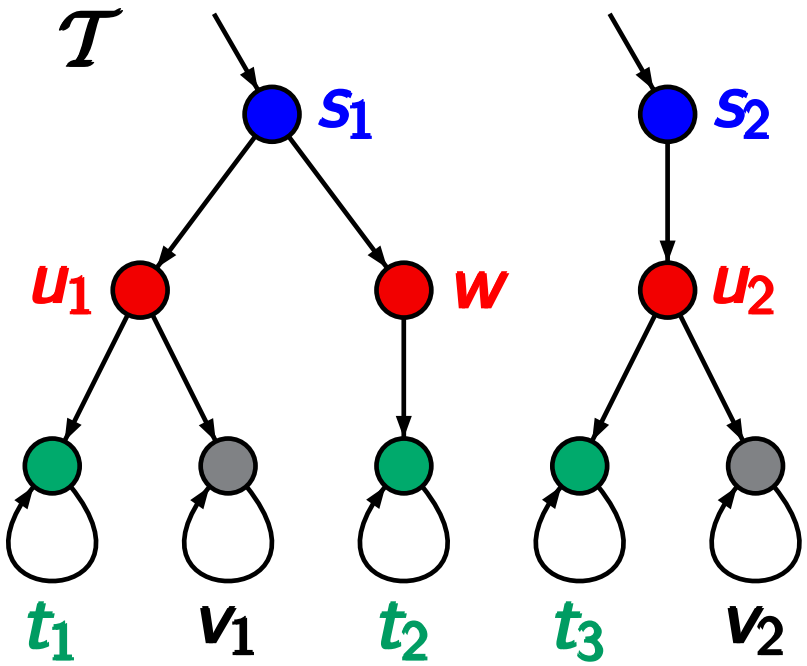

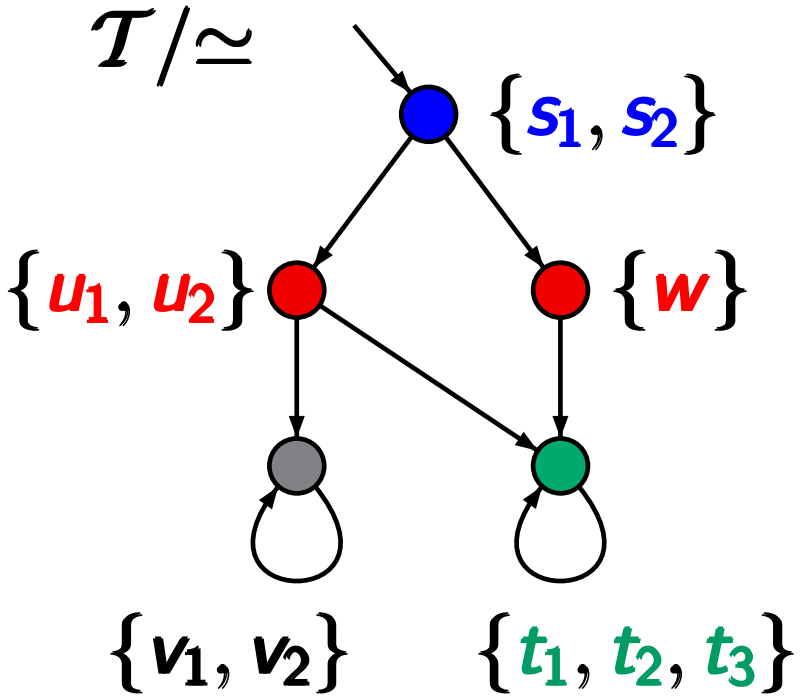

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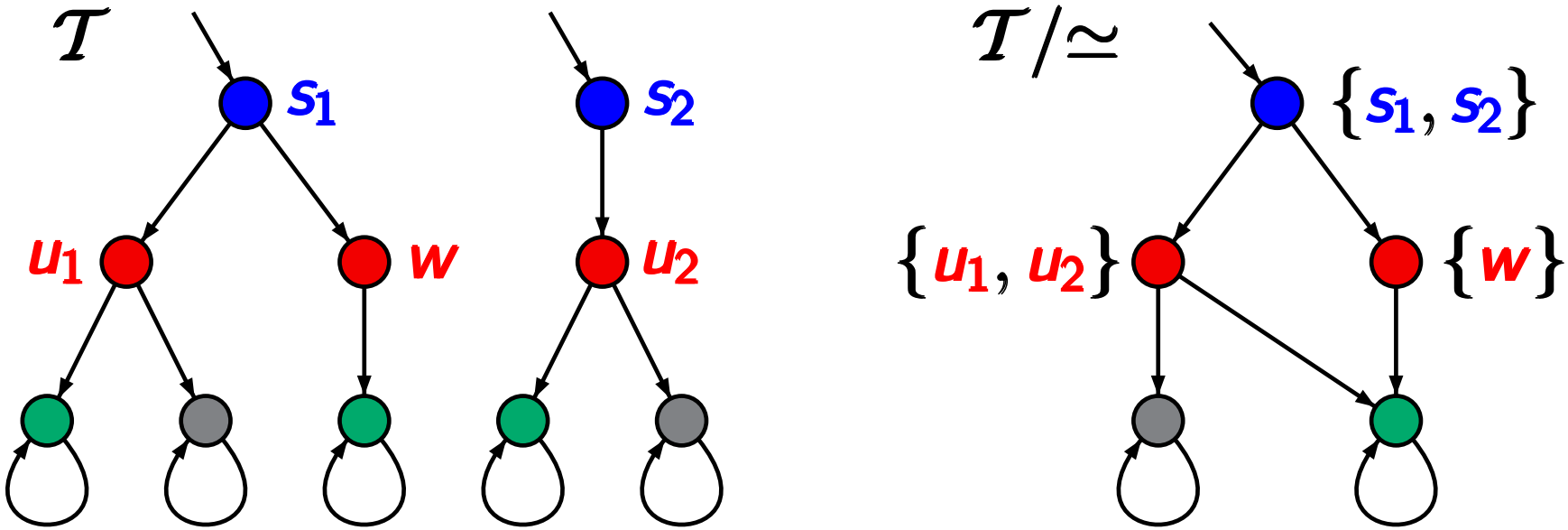
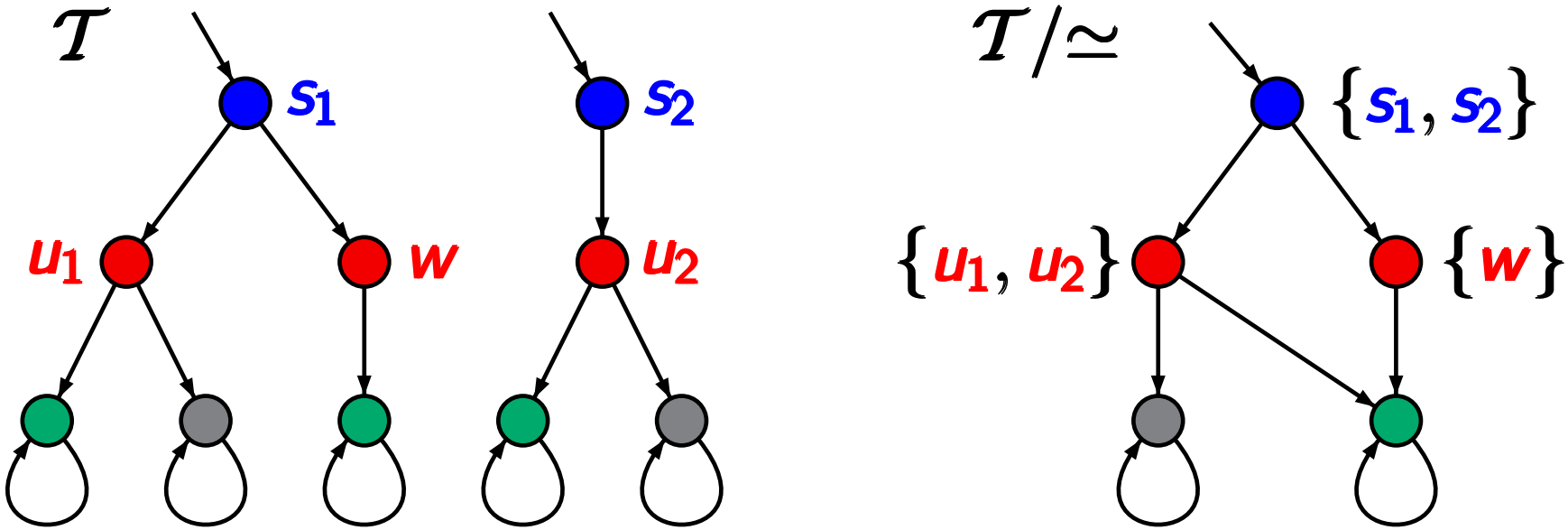


simulation for $(\mathcal{T}, \mathcal{T}/\simeq)$:

$$\{(s, [s]) : s \text{ is a state in } \mathcal{T}\}$$

Example: simulation quotient



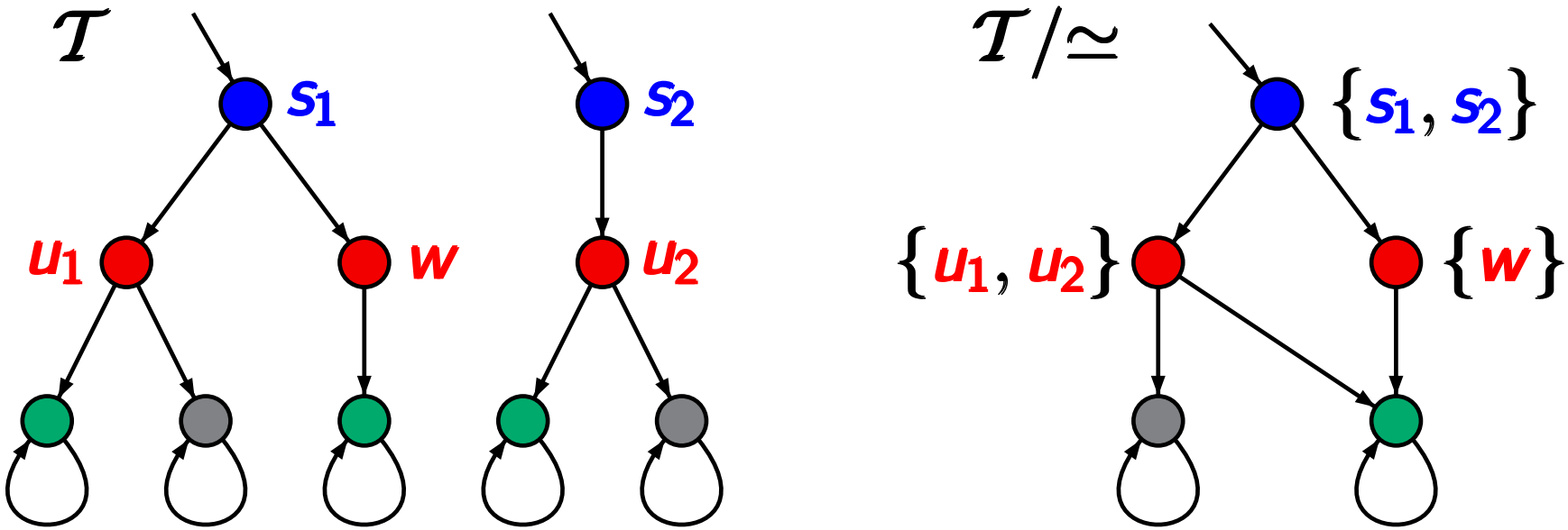
simulation for $(\mathcal{T}, \mathcal{T}/\simeq)$:

$$\{(s, [s]) : s \text{ is a state in } \mathcal{T}\}$$

but $\{([s], s) : s \text{ is a state in } \mathcal{T}\}$

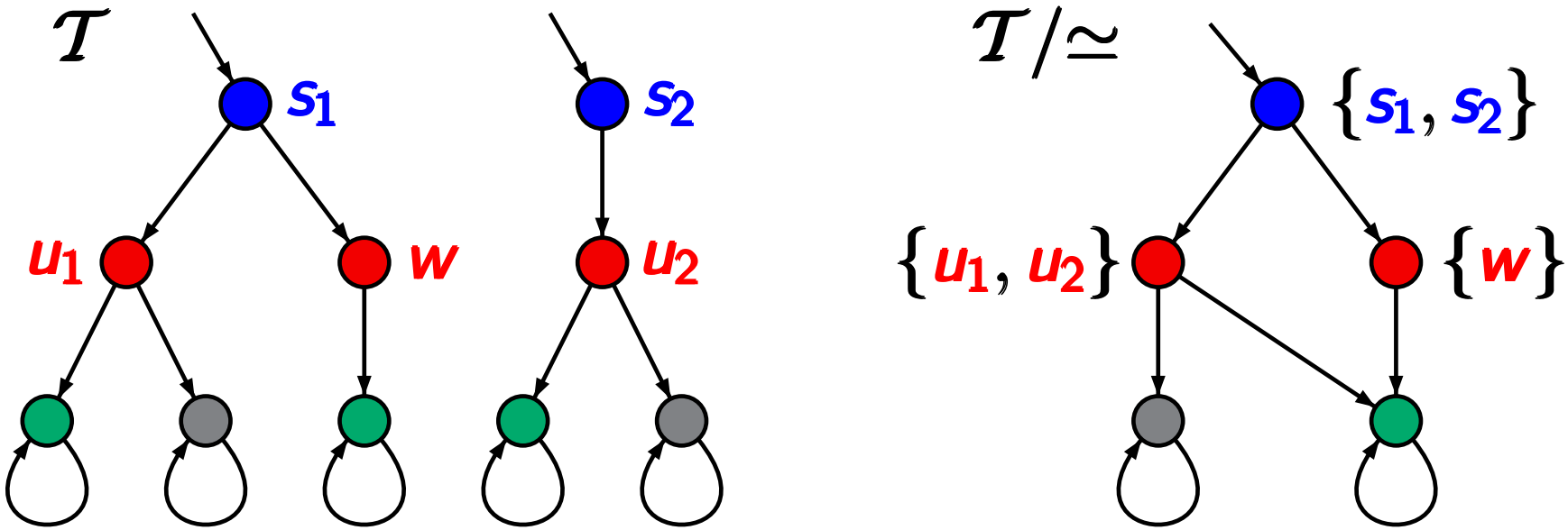
is not a simulation for $(\mathcal{T}/\simeq, \mathcal{T})$

Example: simulation quotient



show that $\mathcal{R} = \{([s], s) : s \text{ is a state in } \mathcal{T}\}$
is not a simulation for $(\mathcal{T}/\simeq, \mathcal{T})$

Example: simulation quotient



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regard $(\{s_1, s_2\}, s_2) \in \mathcal{R}$

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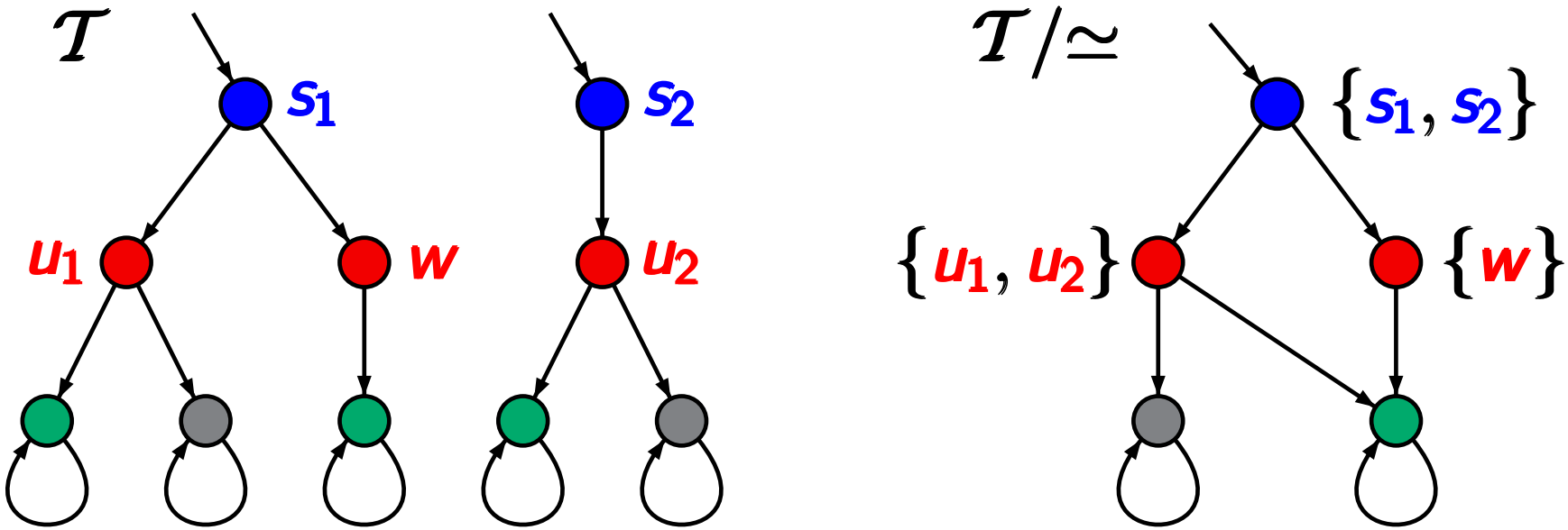

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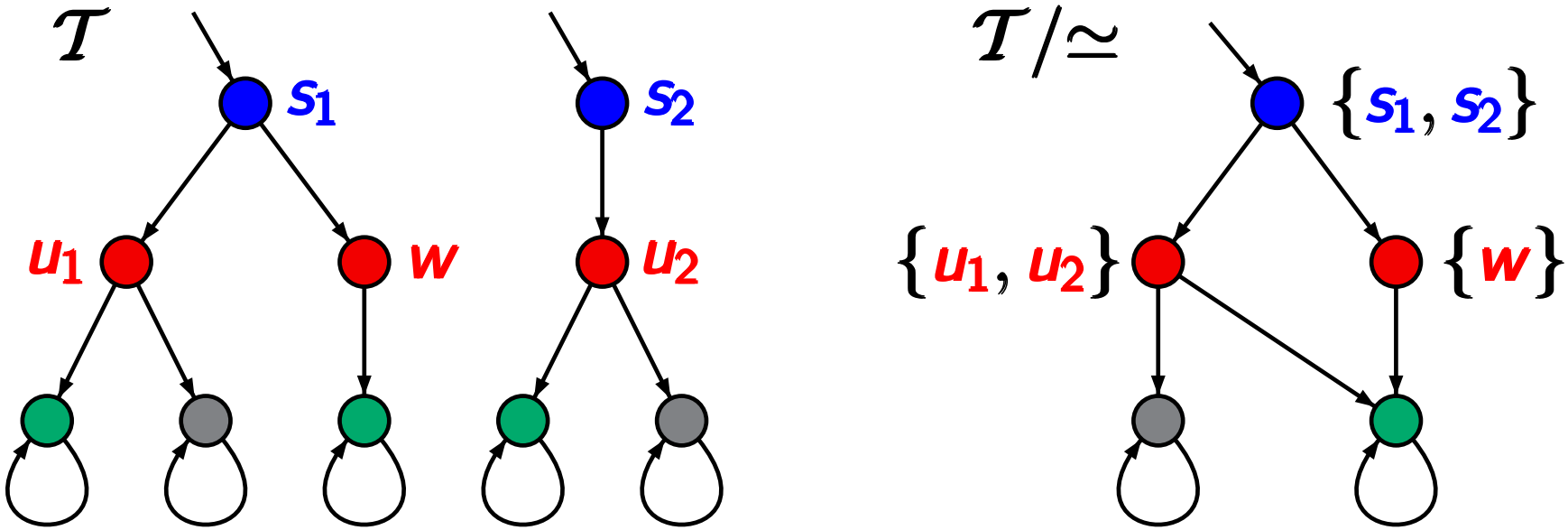

show that $\mathcal{R} = \{([s], s) : s \text{ is a state in } \mathcal{T}\}$
is not a simulation for $(\mathcal{T}/\simeq, \mathcal{T})$

regard $(\{s_1, s_2\}, s_2) \in \mathcal{R}$ and $\{s_1, s_2\} \rightarrow_{\simeq} \{w\}$

there is no transition $s_2 \rightarrow w'$ in $\mathcal{T}$ s.t. $(\{w\}, w') \in \mathcal{R}$

Similarity of $\mathcal{T}$ and $\mathcal{T}/\simeq$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

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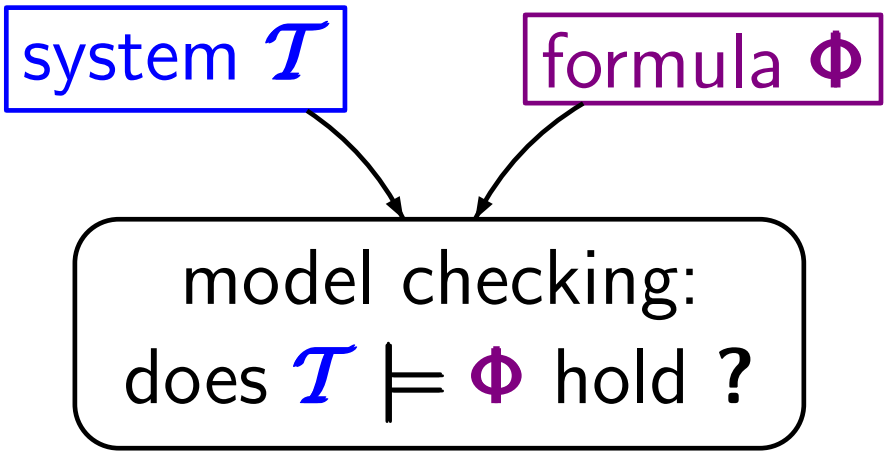
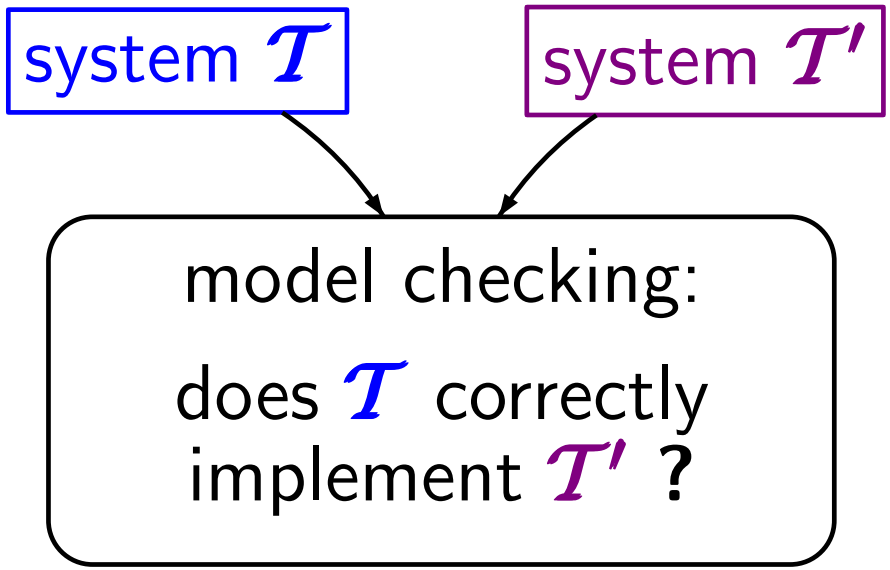
simulation for $(\mathcal{T}/\simeq, \mathcal{T})$: $\{([s], t) : s \preceq_{\mathcal{T}} t\}$

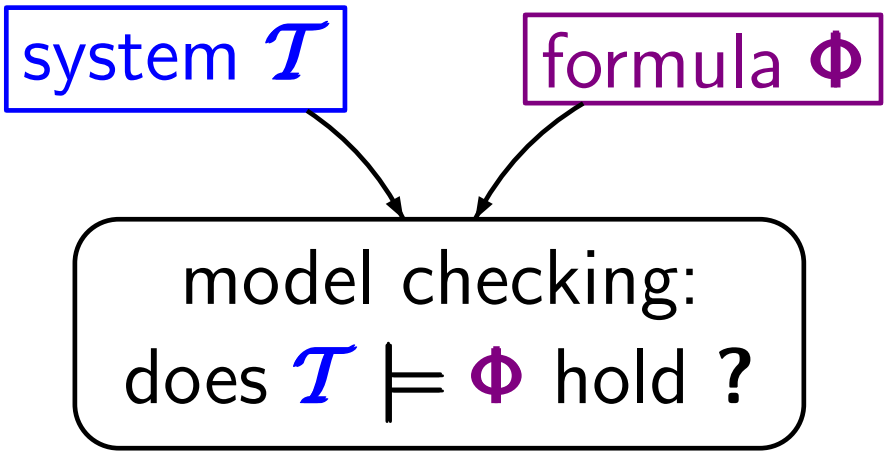
Heterogeneous/homogeneous model checking GRM5.5-30

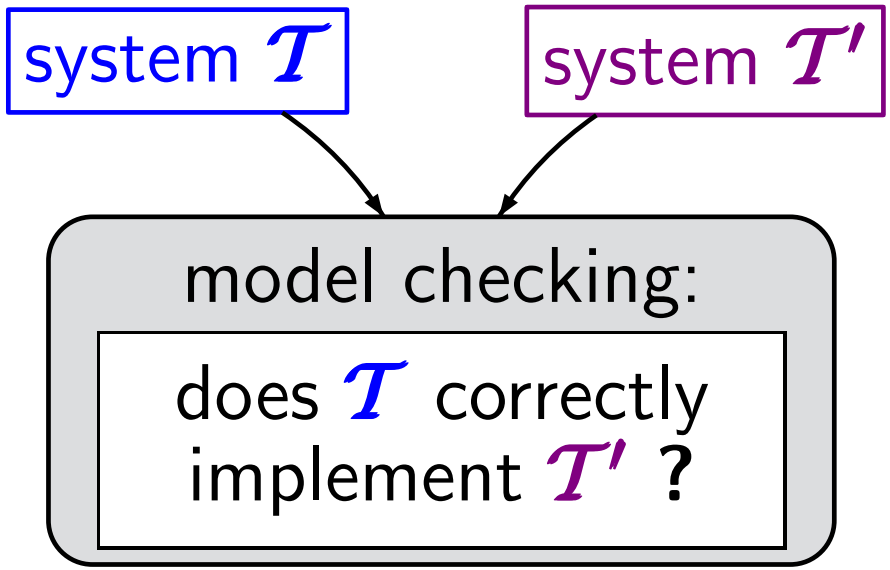
Heterogeneous/homogeneous model checking





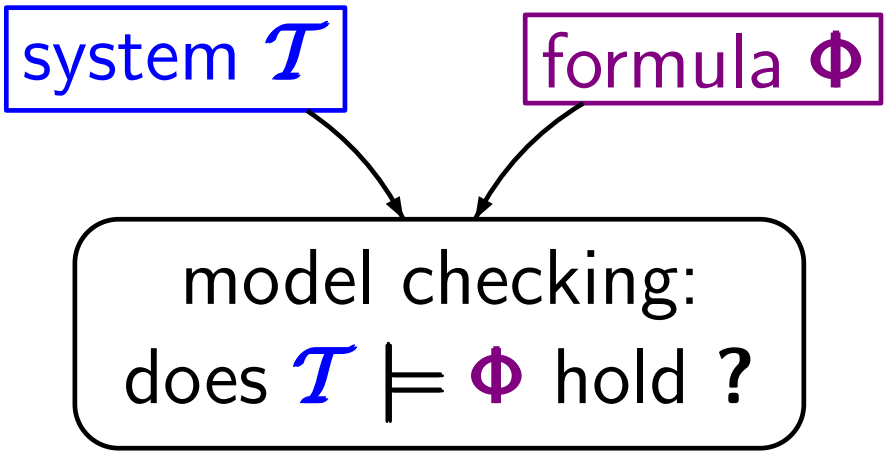
Heterogeneous/homogeneous model checking

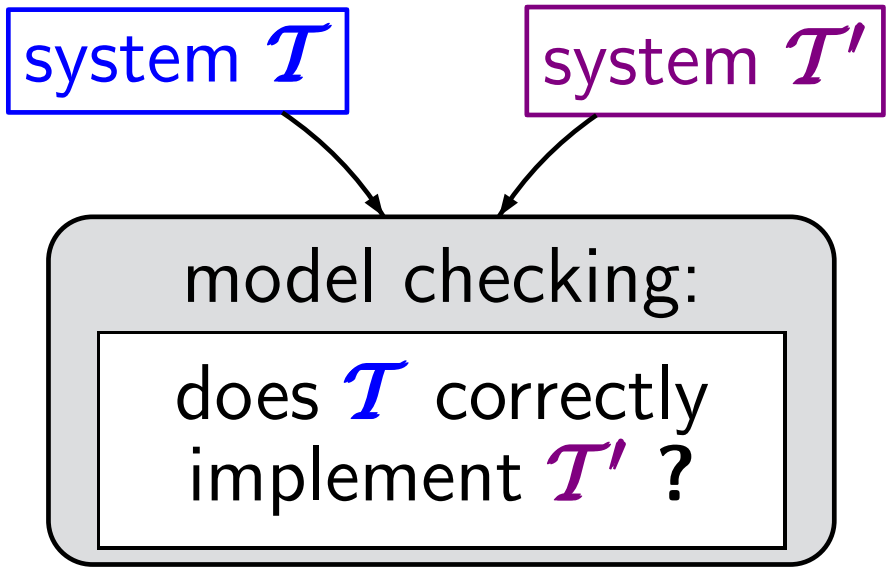




trace inclusion checking
trace equivalence checking

Heterogeneous/homogeneous model checking

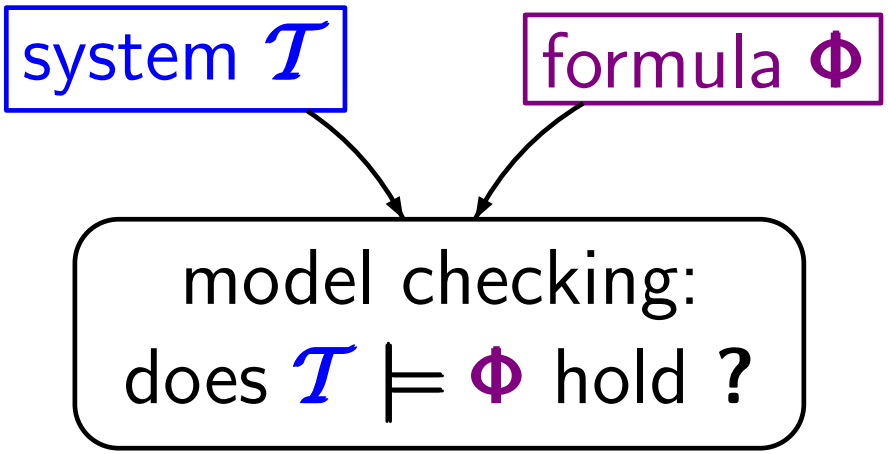


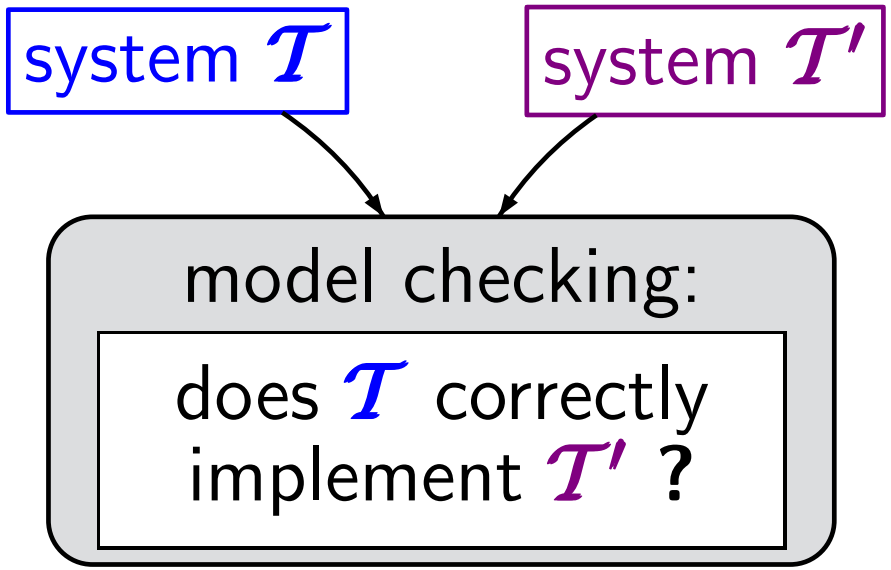


trace inclusion checking
trace equivalence checking

$\longleftarrow$ ***PSPACE***-complete

Heterogeneous/homogeneous model checking





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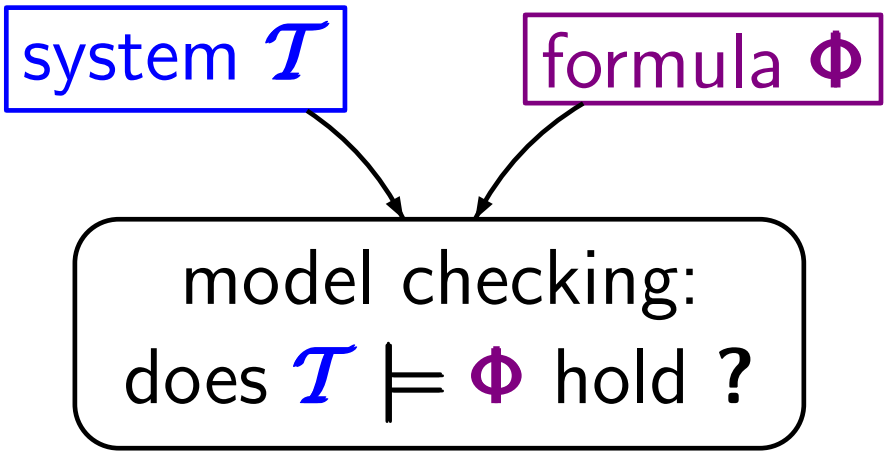
trace equivalence checking

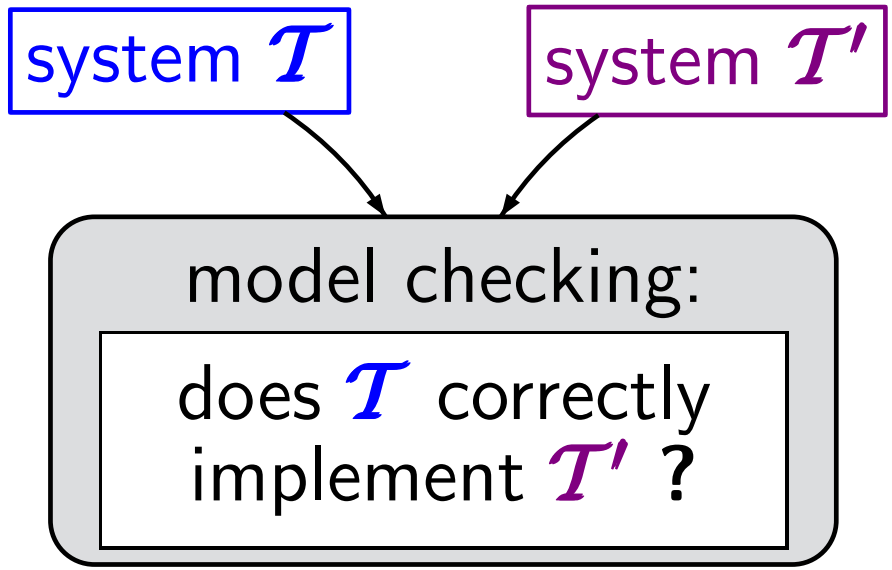
⟵ ***PSPACE***-complete

bisimulation equivalence checking

"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

Heterogeneous/homogeneous model checking



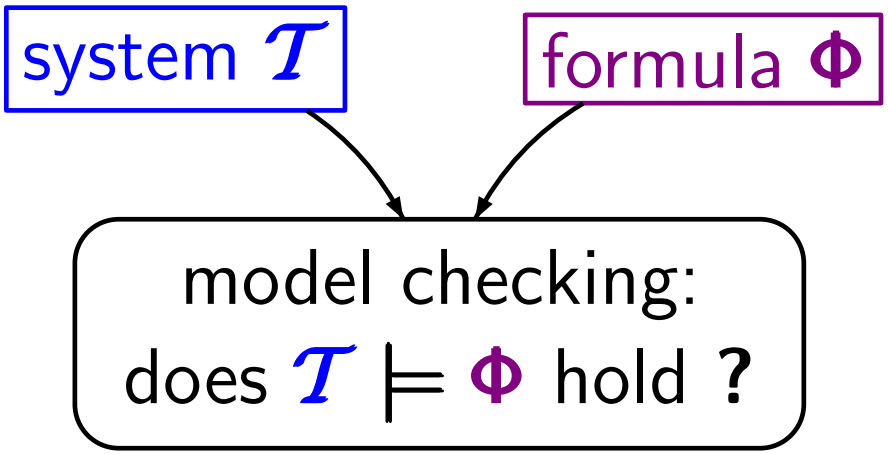


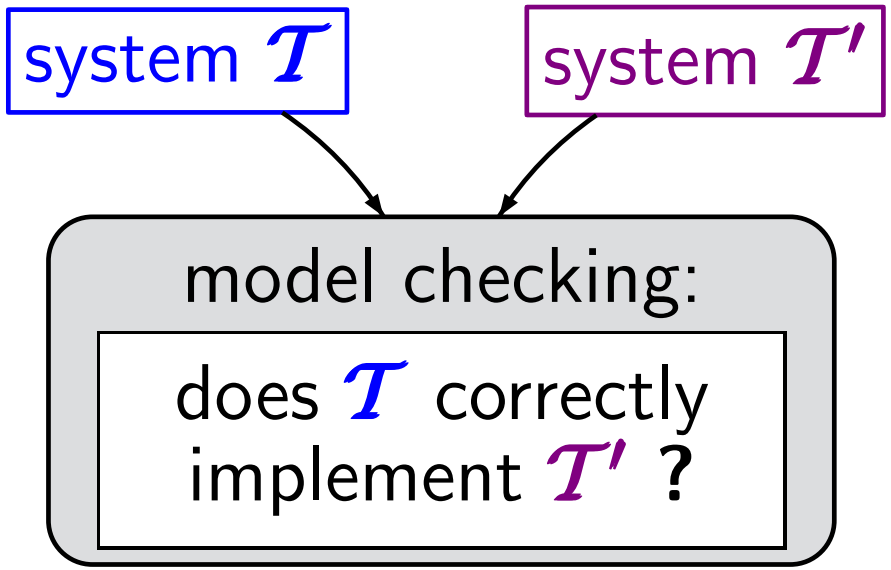
trace inclusion checking
trace equivalence checking
$\longleftarrow$ ***PSPACE***-complete

bisimulation equivalence checking
"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"
$\longleftarrow$ $\mathcal{O}(m \cdot \log n)$

n = #states
m = #transitions

Heterogeneous/homogeneous model checking





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trace equivalence checking
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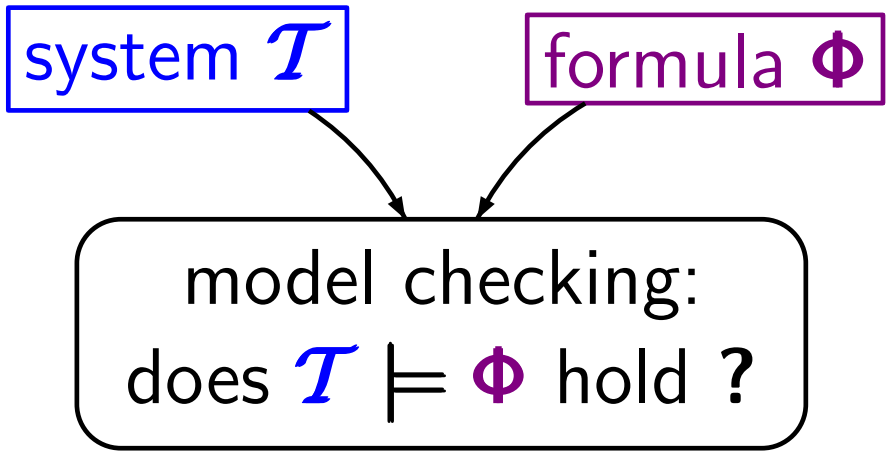
bisimulation equivalence checking
$\longleftarrow$ $\mathcal{O}(m \cdot \log n)$

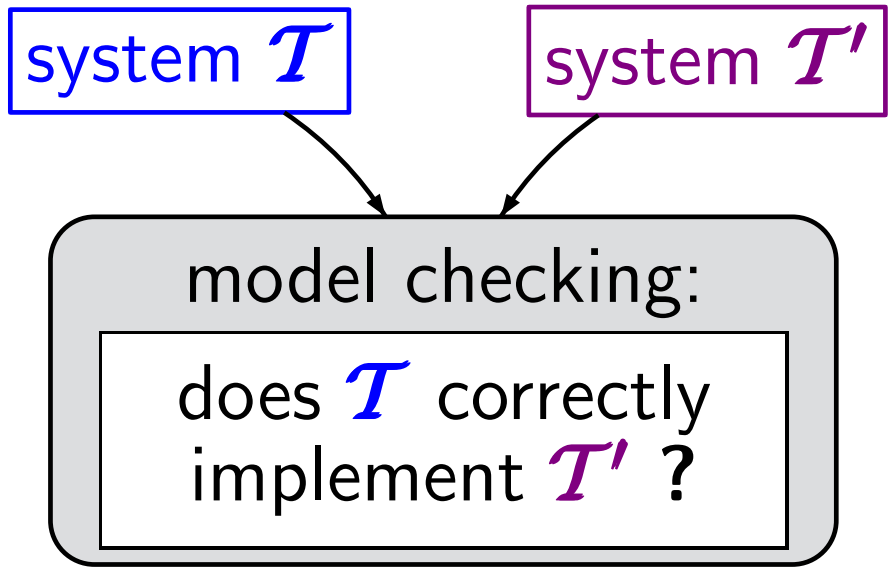
"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

refinement checking via simulation

"does $\mathcal{T} \preceq \mathcal{T}'$ hold ?"

Heterogeneous/homogeneous model checking





trace inclusion checking
trace equivalence checking
$\longleftarrow$ ***PSPACE***-complete

bisimulation equivalence checking
"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"
$\longleftarrow$ $\mathcal{O}(m \cdot \log n)$

refinement checking via simulation
"does $\mathcal{T} \preceq \mathcal{T}'$ hold ?"
$\longleftarrow$ $\mathcal{O}(m \cdot n)$

Refinement checking via simulation

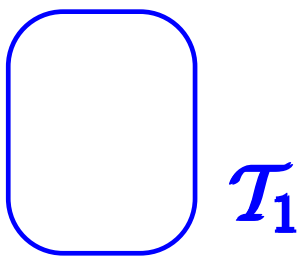
given: **2** finite transition system $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set of propositions AP

question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold **?**

Refinement checking via simulation

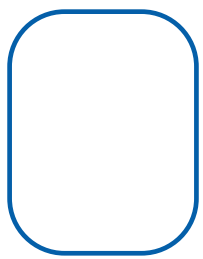
given: 2 finite transition system $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set of propositions AP

question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold ?



$\mathcal{T}_1$

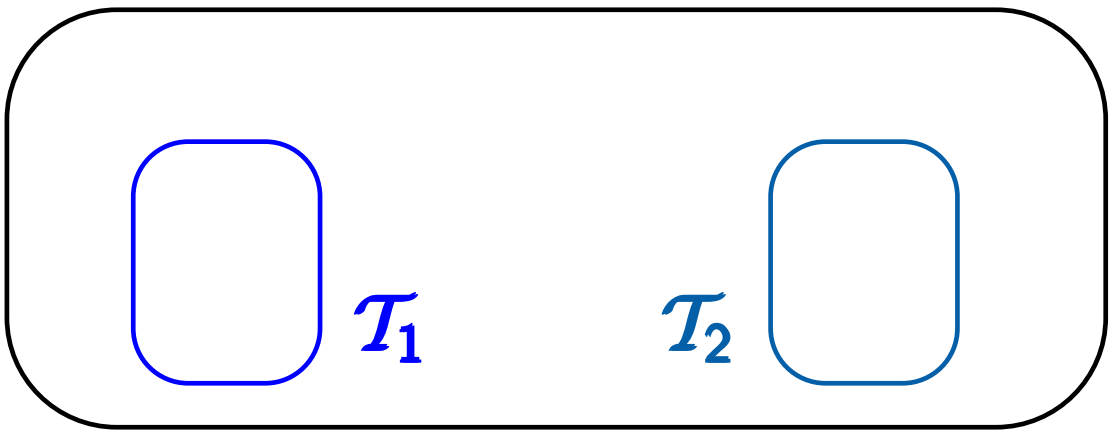
$\mathcal{T}_2$



Refinement checking via simulation

given: 2 finite transition system $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set of propositions AP

question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold ?



composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

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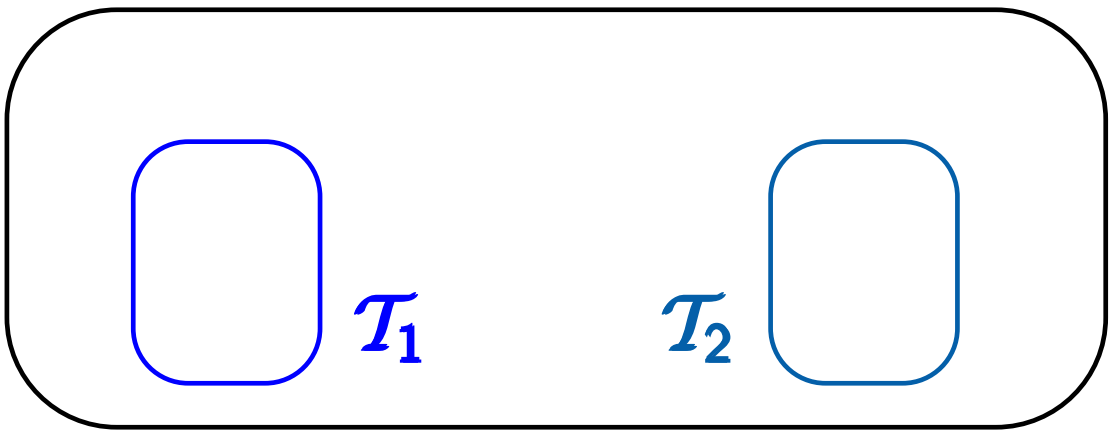

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- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$

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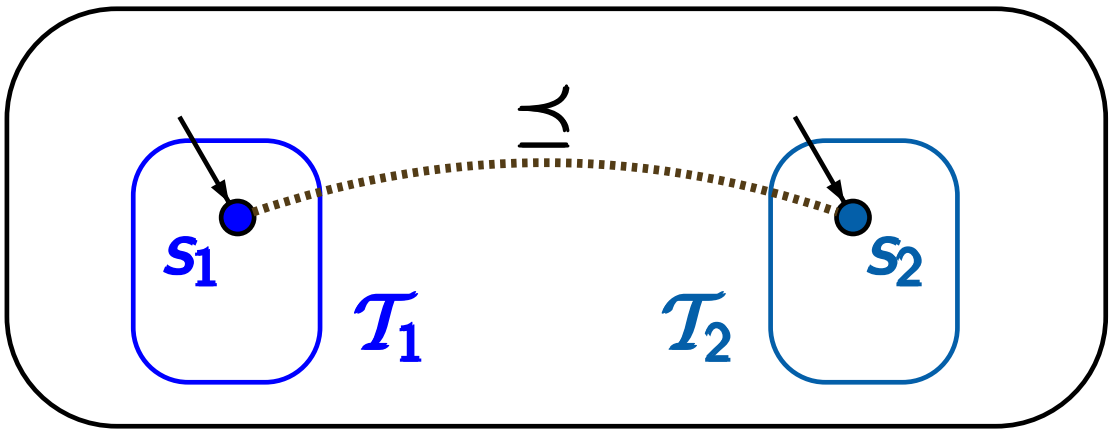

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$
- check whether for all initial states s_1 of $\mathcal{T}_1$ there is an initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \preceq_{\mathcal{T}} s_2$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
possibly with terminal states

goal: compute the simulation preorder $\preceq_{\mathcal{T}}$

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$\rightsquigarrow$ simulation equivalence classes

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method: iterative refinement of relation $\mathcal{R} \subseteq S \times S$

Computing the simulation preorder $\preceq_T$

$\mathcal{R} := \{ (s_1, s_2) \in S \times S : L(s_1) = L(s_2) \}$;

WHILE $\mathcal{R}$ is no simulation DO

choose $(s_1, s_2) \in \mathcal{R}$ s.t. $s_1 \rightarrow s_1'$, but there is no transition $s_2 \rightarrow s_2'$ with $(s_1', s_2') \in \mathcal{R}$

$\mathcal{R} := \mathcal{R} \setminus \{(s_1, s_2)\}$

OD

return $\mathcal{R}$

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OD

return $\mathcal{R}$ ⟵

$\mathcal{R}$ is the coarsest simulation on $\mathcal{T}$ and therefore $\mathcal{R} = \preceq_{\mathcal{T}}$

Computing the simulation preorder $\preceq_T$

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#iterations: $\mathcal{O}(|S|^2)$

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OD

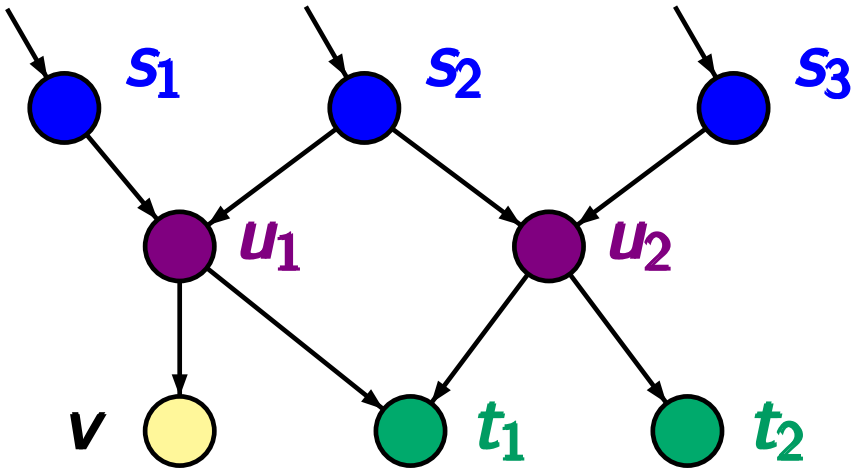
return $\mathcal{R}$

#iterations: $\mathcal{O}(|S|^2)$

representation of $\mathcal{R}$ by simulator sets

$$Sim_{\mathcal{R}}(s_1) = \{ s_2 \in S : (s_1, s_2) \in \mathcal{R} \}$$

Example: computation of $\preceq$



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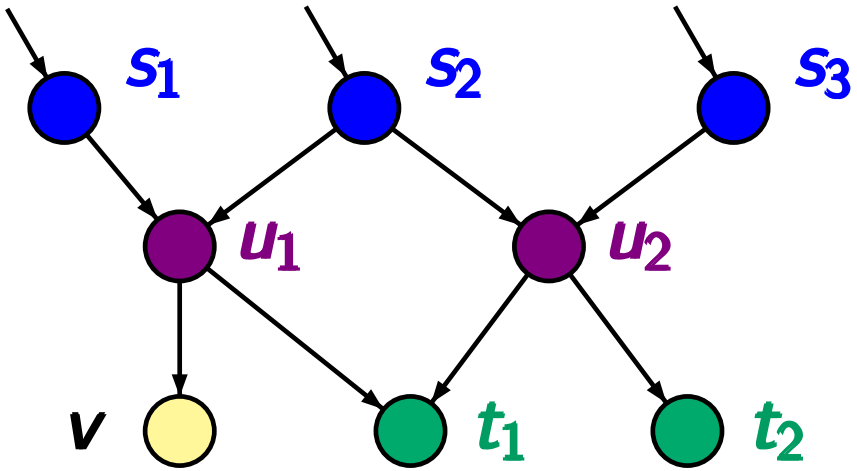

initially:

$$Sim(s_i) = \{s_1, s_2, s_3\}$$
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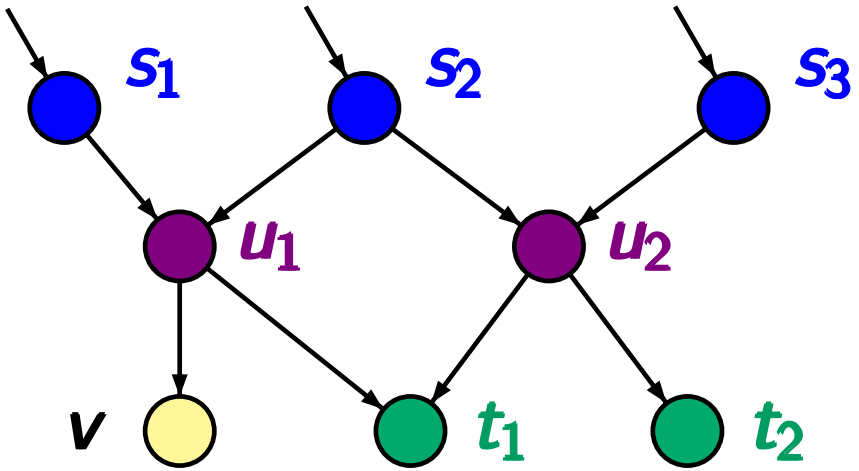

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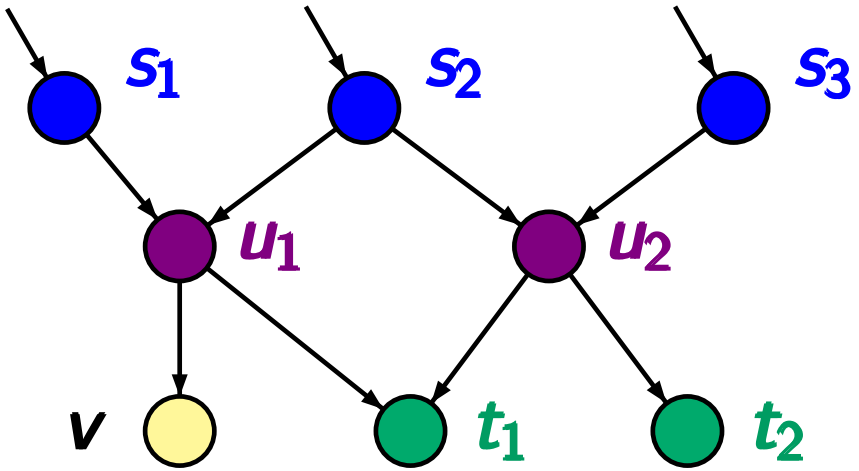

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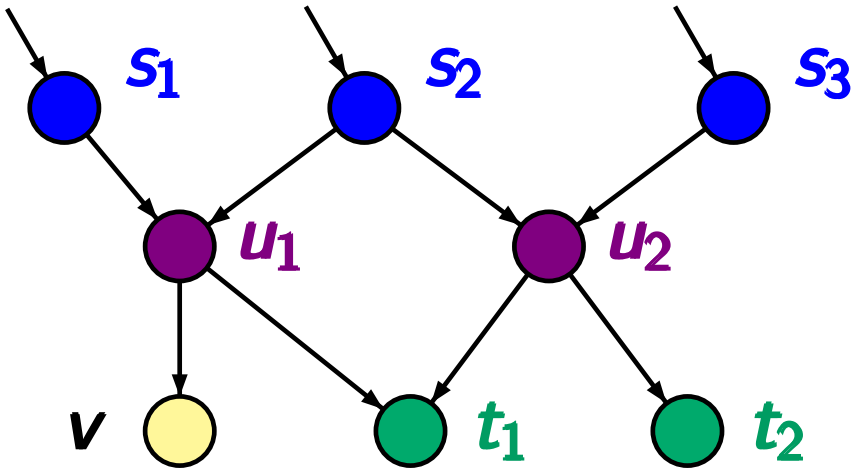

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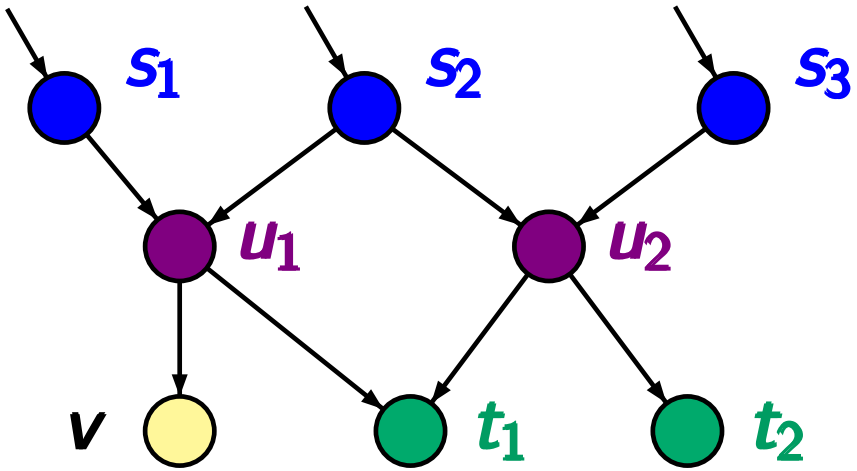

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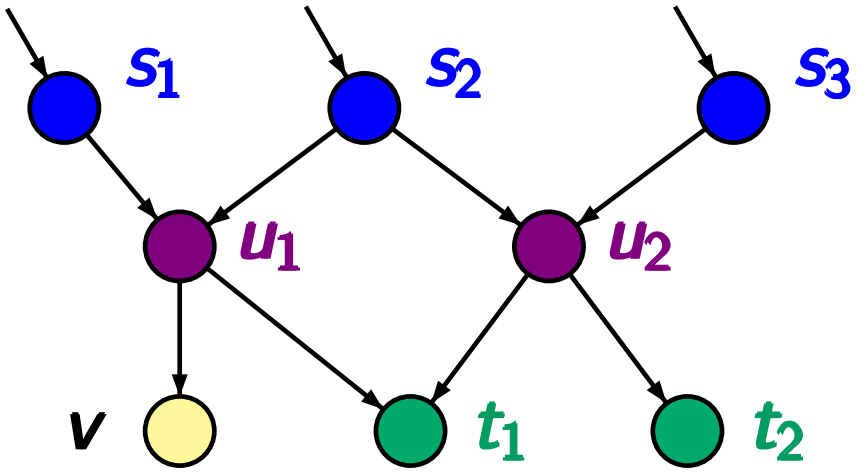

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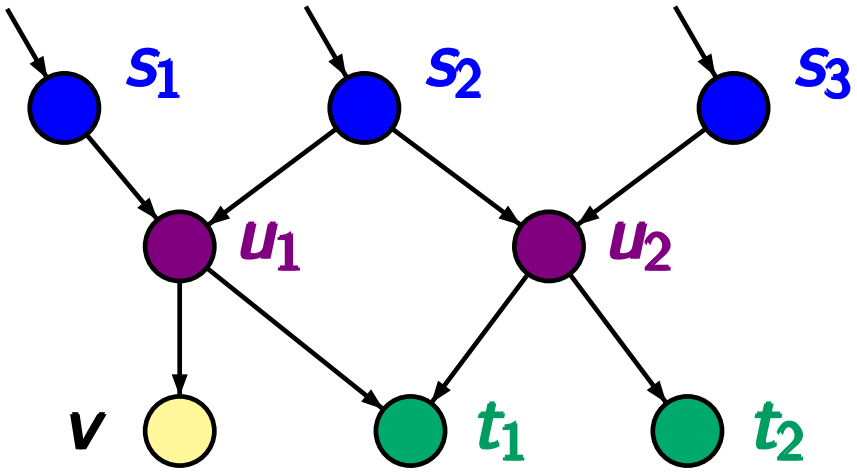
$s_2 \not\preceq s_3$, as $s_2 \rightarrow u_1$, $s_3 \not\rightarrow Sim(u_1)$

Example: computation of $\preceq$



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Computing the simulation preorder $\preceq_{\mathcal{T}}$

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FOR ALL $s_1 \in S$ DO

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OD

WHILE $\exists s_1 \in S\ \exists s_2 \in Sim(s_1)\ \exists s_1' \in Post(s_1)$

  s.t. $Post(s_2) \cap Sim(s_1') = \emptyset$ DO

  choose such states s_1, s_2

  $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

OD

return $\{(s_1, s_2) : s_2 \in Sim(s_1)\}$

Computing the simulation preorder $\preceq_T$

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$s_1 \longrightarrow s_1'$

s_2

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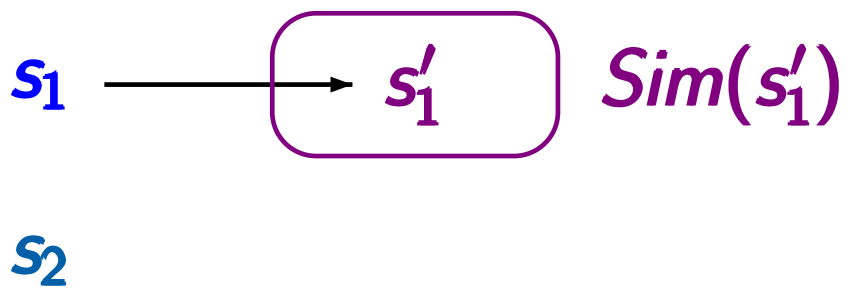

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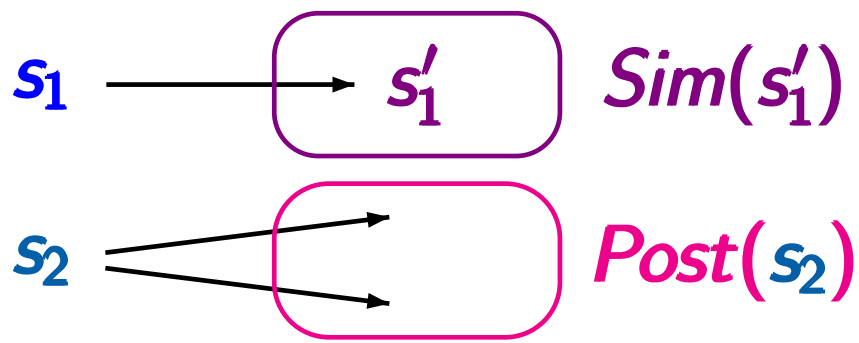

Computing the simulation preorder $\preceq_T$

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OD

WHILE $\exists s_1 \in S\ \exists s_2 \in Sim(s_1)\ \exists s_1' \in Post(s_1)$

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  $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$ $\longleftarrow$ $s_1 \not\preceq_T s_2$

OD

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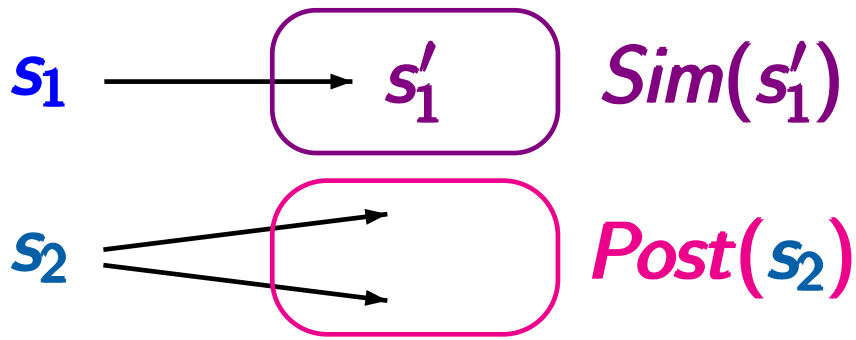

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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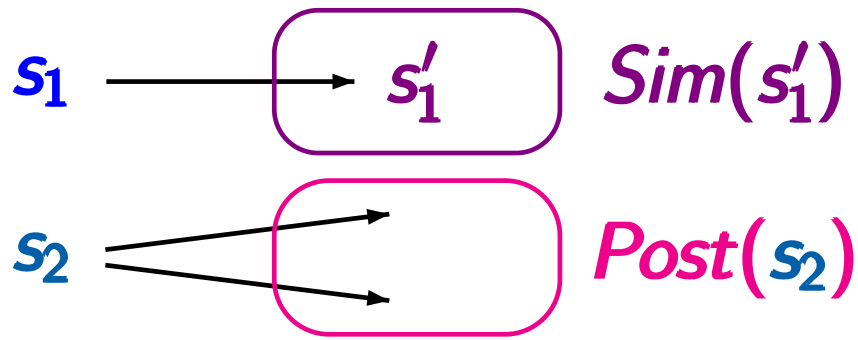
complexity:
$\mathcal{O}(m \cdot |S|^3)$



$m = \#\text{edges}$
$\geqslant |S|$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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OD

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complexity:
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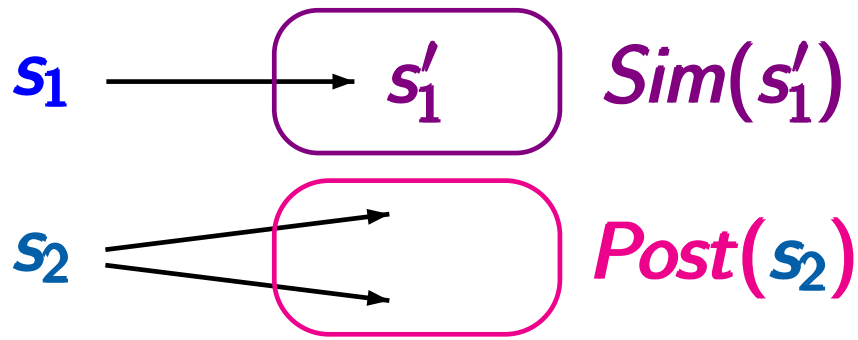

$m = \#\text{edges}$
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$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_{\mathcal{T}}$

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reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$
by means of

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- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_{\mathcal{T}}$

reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$ by means of

- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$
- a set V that organizes all pairs (s_1', s_2) where $\delta(s_1', s_2) = 0$

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

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FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

OD

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WHILE $V \neq \varnothing$ DO

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

WHILE $V \neq \varnothing$ DO

choose $(s_1', s_2) \in V$ and remove (s_1', s_2) from V

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

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  OD

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FOR ALL $u_2 \in Pre(s_2)$ DO

OD

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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FOR ALL $s_1 \in Pre(s_1')$ with $s_2 \in Sim(s_1)$ DO

$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

FOR ALL $u_2 \in Pre(s_2)$ DO

$\delta(s_1, u_2) := \delta(s_1, u_2) - 1$

OD

OD

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FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

WHILE $V \neq \varnothing$ DO

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    FOR ALL $u_2 \in Pre(s_2)$ DO

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in total:
$\mathcal{O}(m \cdot |S|)$

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cost per iteration
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WHILE $V \neq \varnothing$ DO ⟵ #iterations $\leqslant |S|^2$

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An $\mathcal{O}(m \cdot |S|)$-algorithm for computing $\preceq_{\mathcal{T}}$

... algorithm by Henzinger, Henzinger, and Kopke

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Then: if $\mathit{Post}(s_2) \cap \mathit{Sim}(s_1') = \{s_2'\}$ then $s_1 \not\preceq s_2$ and s_2 can be removed from $\mathit{Sim}(s_1)$.

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Then: if $\mathit{Post}(s_2) \cap \mathit{Sim}(s_1') = \{s_2'\}$ then $s_1 \not\preceq s_2$ and s_2 can be removed from $\mathit{Sim}(s_1)$.

idea: collect all such states s_2 in $\mathit{Remove}(s_1')$

Idea of the HHK-algorithm

If s_2' is removed from $Sim(s_1')$ then regard all direct predecessors s_1 of s_1' and remove all states in

$$Remove(s_1') \;=\; Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$$

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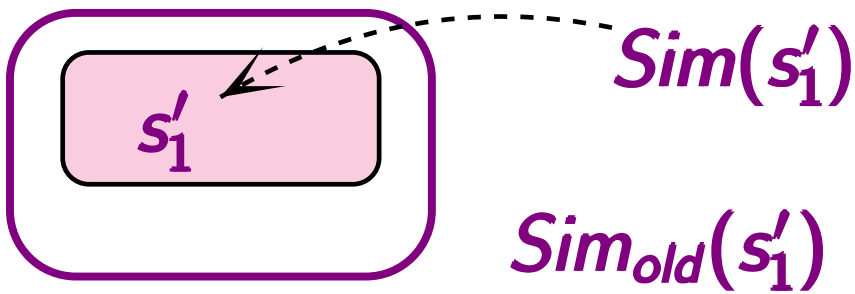

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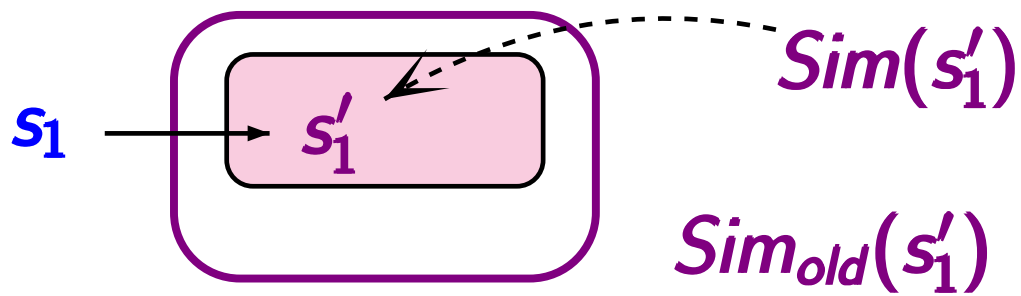

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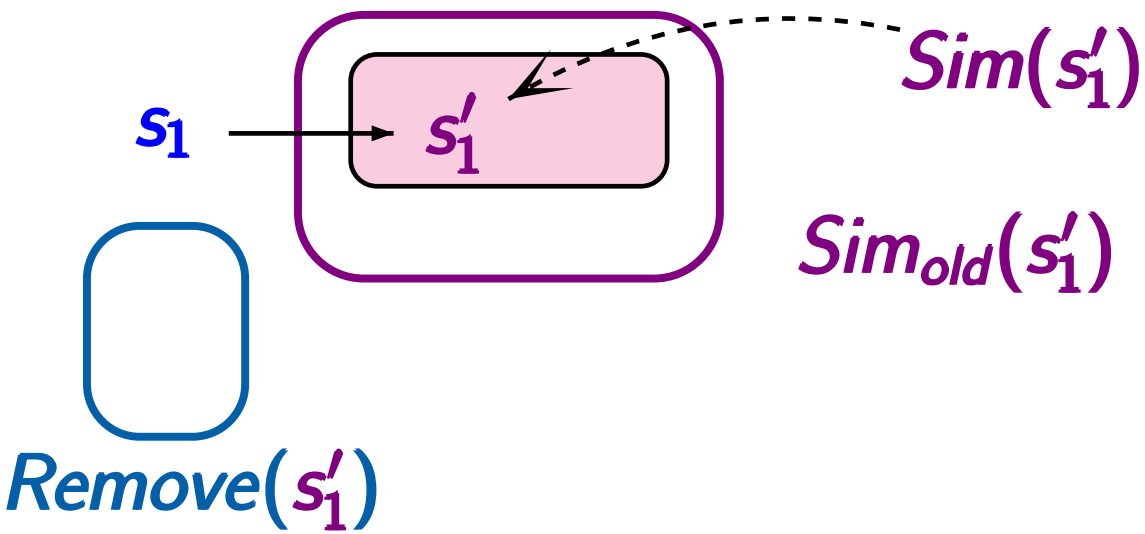

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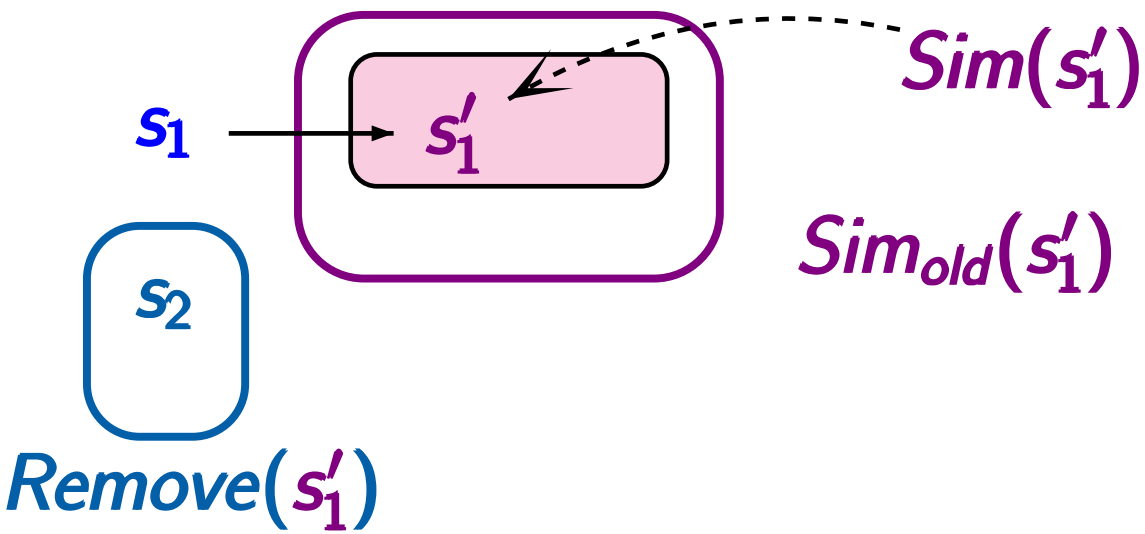

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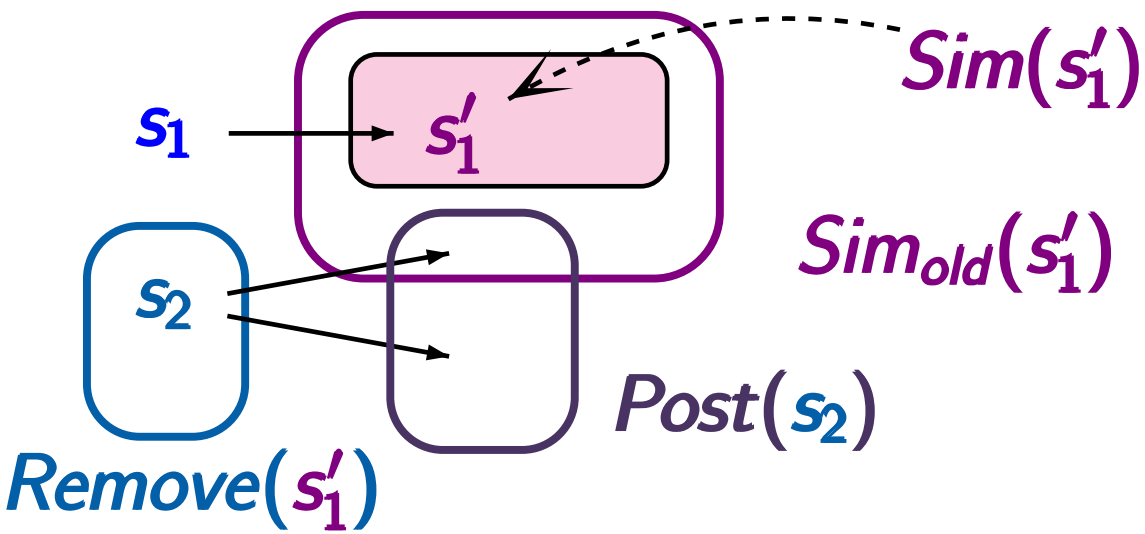

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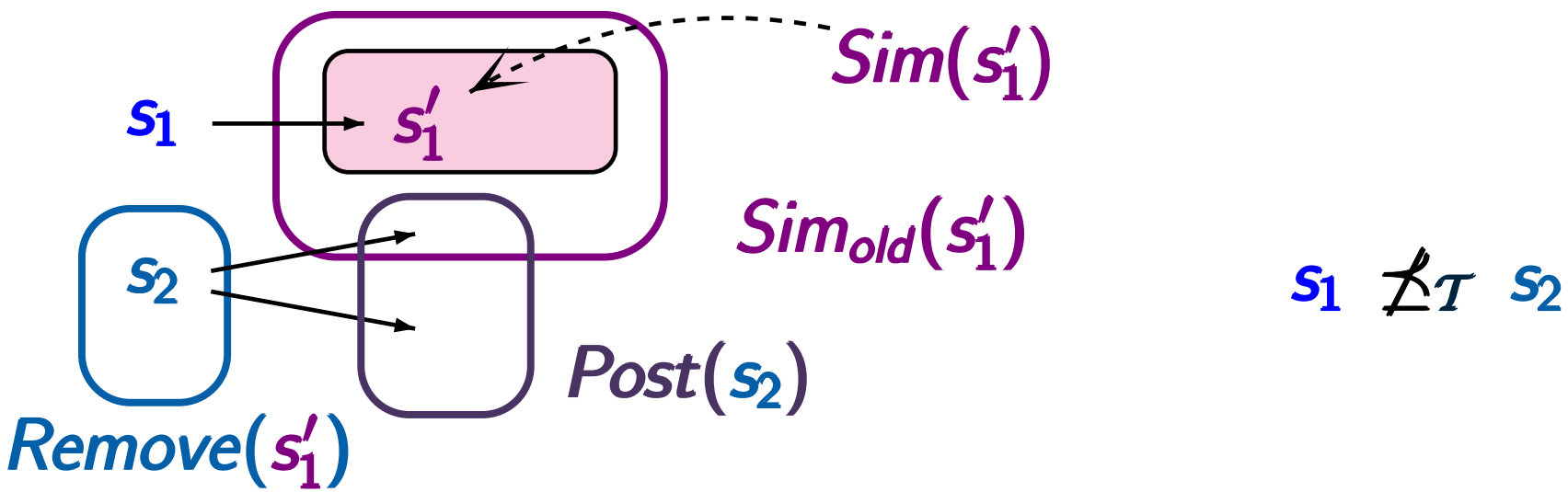

HHK-algorithm (first version)

FOR ALL states s_1' DO

$Sim_{old}(s_1') := S$

$Sim(s_1') := \{ s_2' \in S : L(s_1') = L(s_2') \}$

OD

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OD ;

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OD

return $\{(s_1, s_2') : s_2' \in Sim(s_1')\}$

HHK-algorithm (first version)

if s_2' is terminal then so is s_1'

FOR ALL states s_1' DO

$Sim_{old}(s_1') := S$

$Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2') \text{ and } \ldots\}$

OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

choose such a state s_1';

$Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$;

FOR ALL $s_1 \in Pre(s_1')$ DO

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return $\{(s_1, s_2') : s_2' \in Sim(s_1')\}$

HHK-algorithm (first version)

FOR ALL states s_1' DO

$Sim_{old}(s_1') :=$ "undefined"

$Sim(s_1') := \{ s_2' \in s : L(s_1') = L(s_2') \text{ and } \ldots \}$

OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

choose such a state s_1'

IF $Sim_{old}(s_1') =$ "undefined"

THEN $Remove(s_1') := S \setminus Pre(Sim(s_1'))$

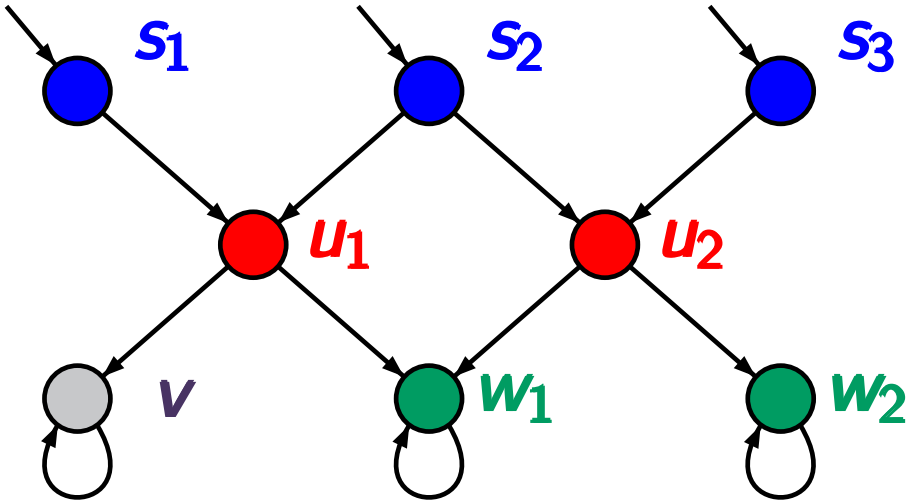
ELSE $Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$

FI

FOR ALL $s_1 \in Pre(s_1')$ DO

...

Example: HHK-algorithm



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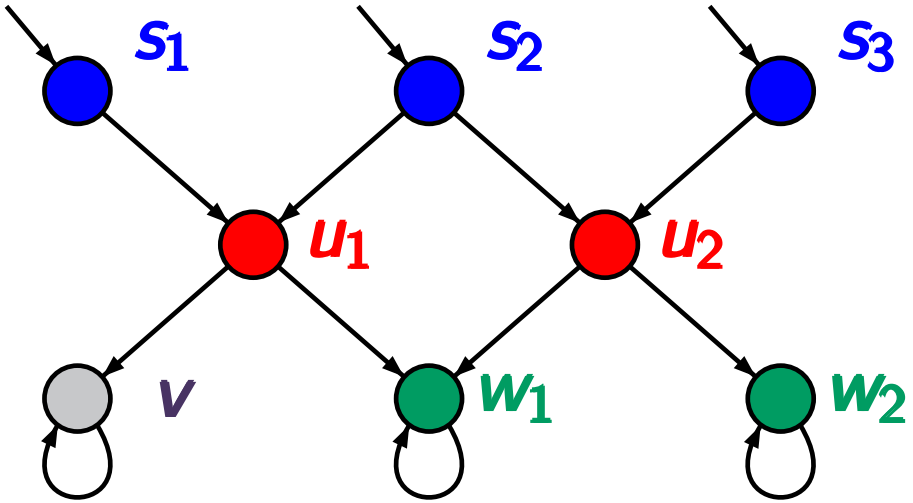

initially:

$Sim_{old}(t) = \bot$ for all states t

$Sim(s_1) = \{s_1, s_2, s_3\}$

$\vdots$

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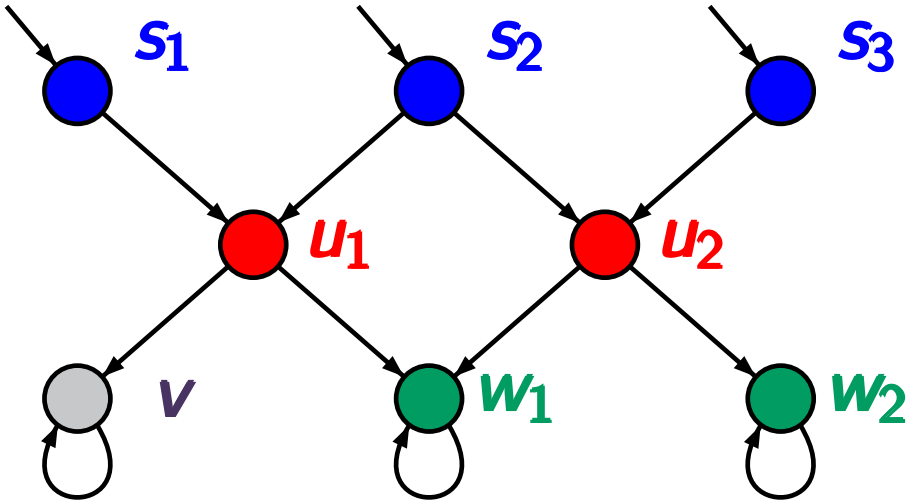

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choose state $s_1' = v$ with $Sim_{old}(v) \neq Sim(v)$:

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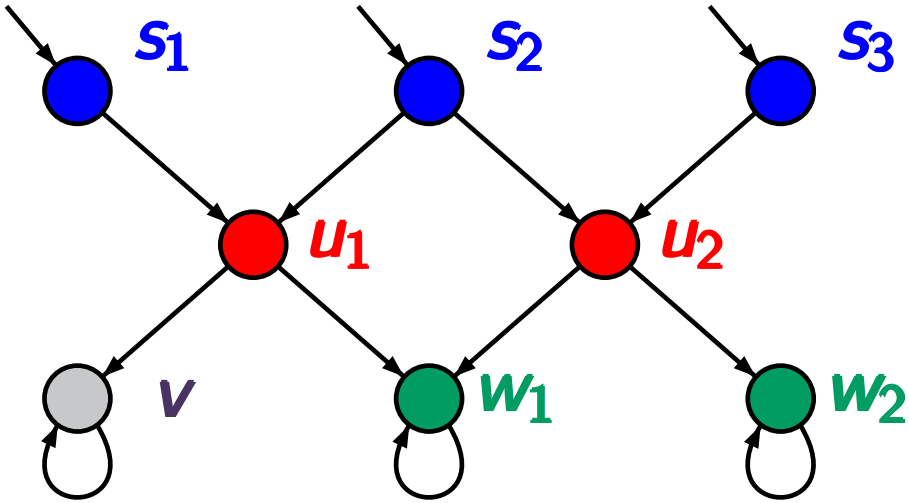

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choose state $s_1' = v$ with $Sim_{old}(v) \neq Sim(v)$:

$$Remove(v) = S \setminus Pre(Sim(v)) = S \setminus \{u_1\}$$

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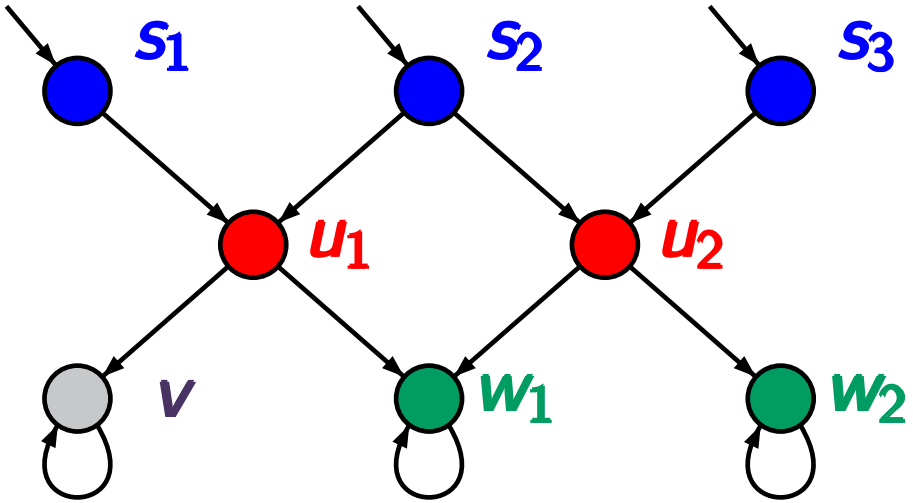

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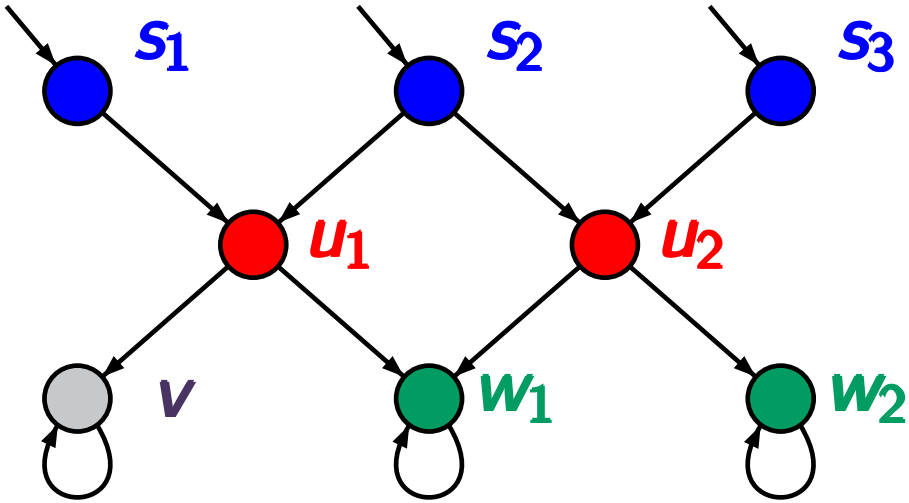

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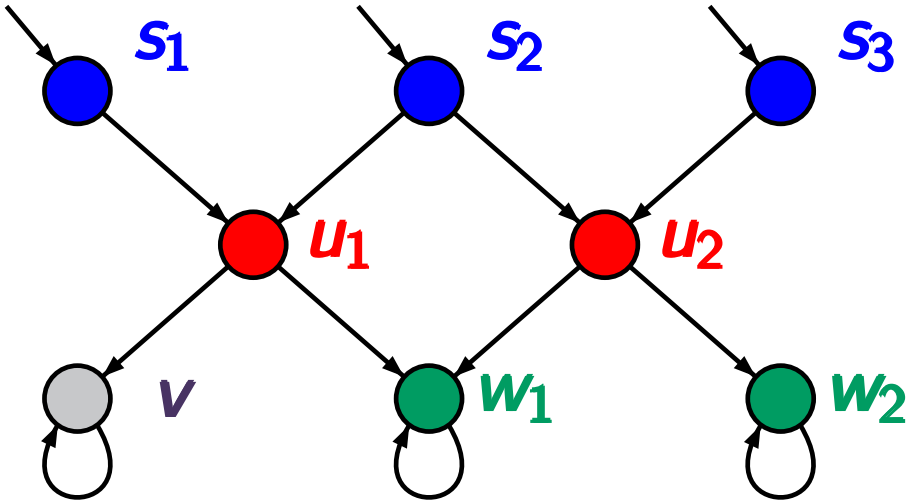
$$Remove(v) = S \setminus Pre(Sim(v)) = S \setminus \{u_1\}$$

$$Sim(u_1) := Sim(u_1) \setminus Remove(v) = \{u_1\}$$

$u_1 \longrightarrow v$ can't be simulated by any of the states in $Remove(v)$

Example: HHK-algorithm



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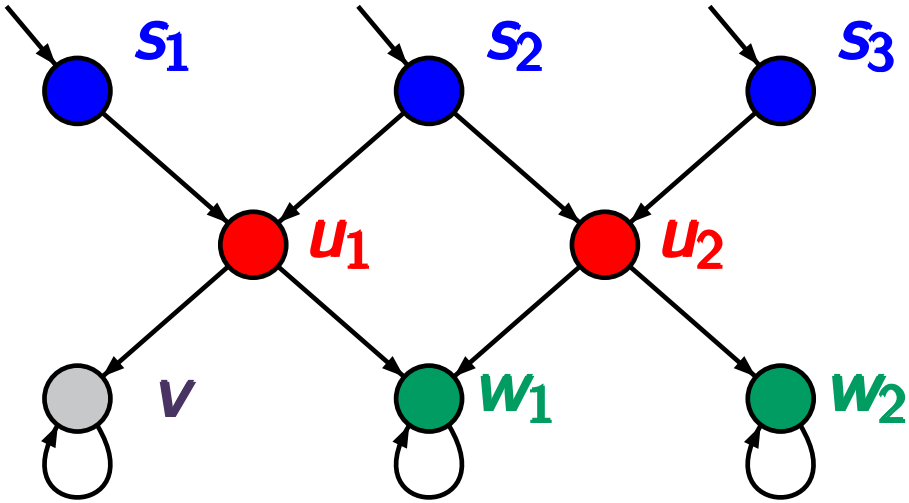

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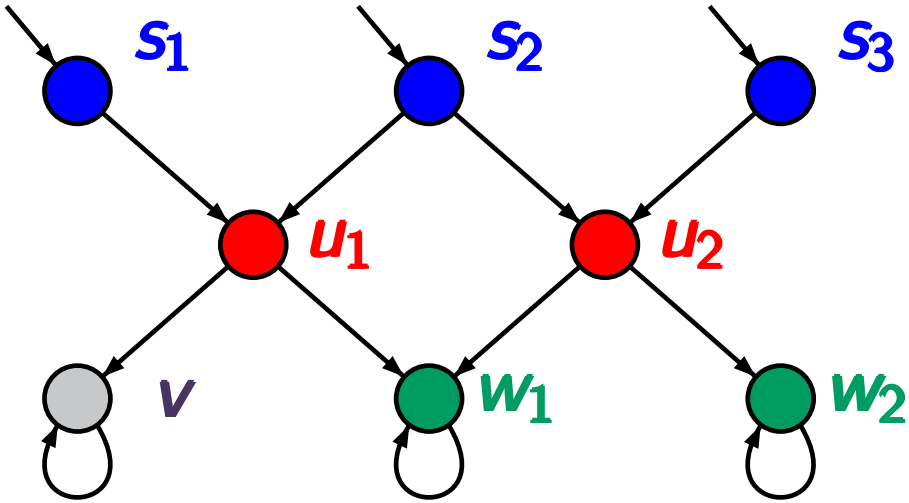

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no change in $Sim(\ldots)$, as $Pre(s_1) = \varnothing$

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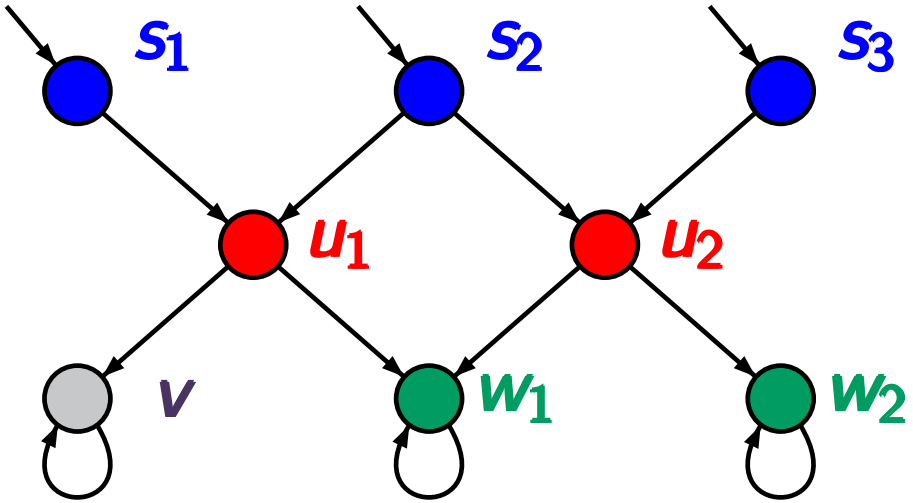

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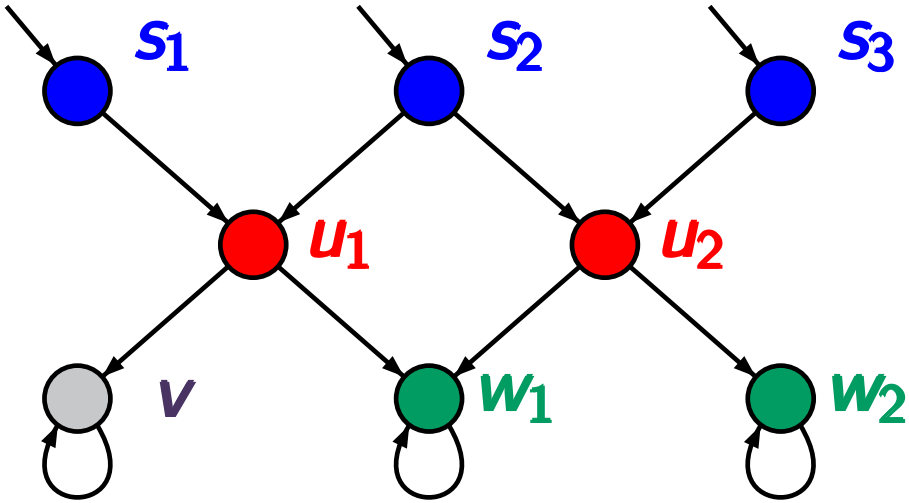

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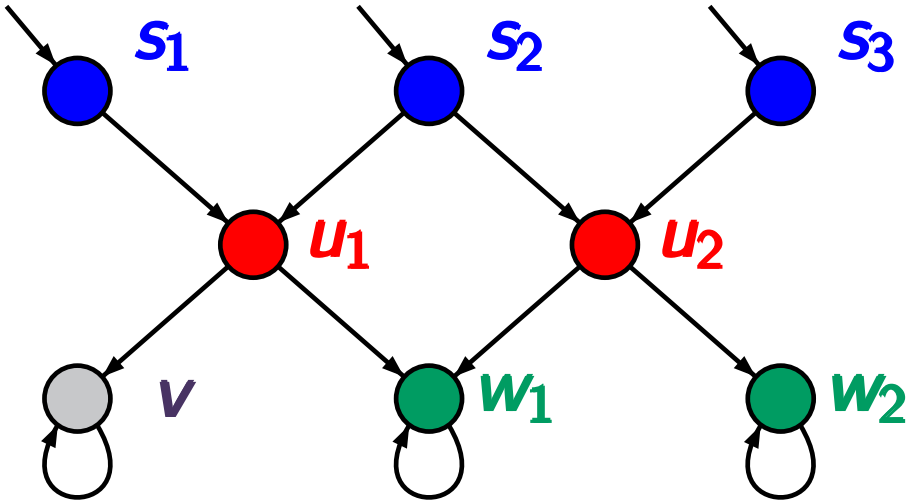

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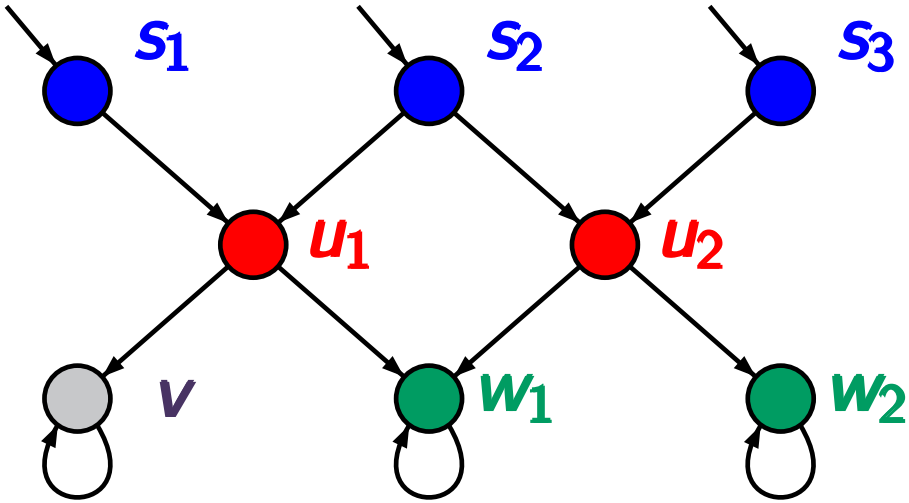

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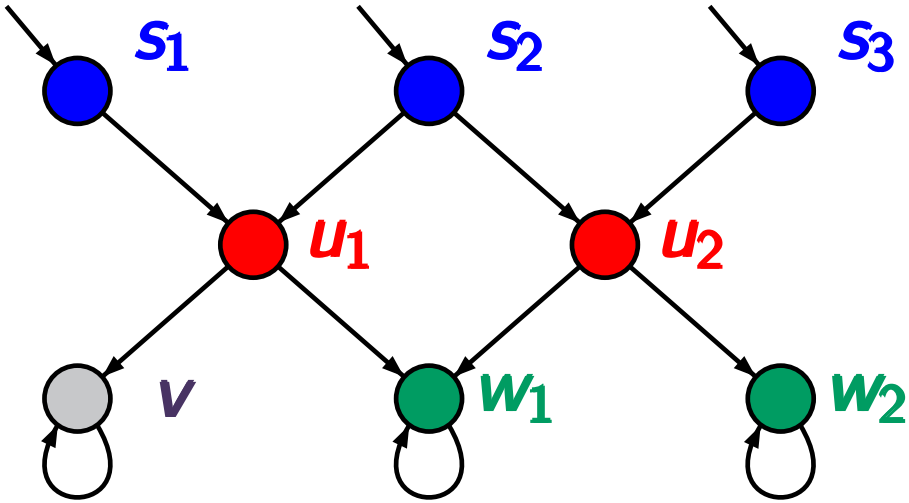

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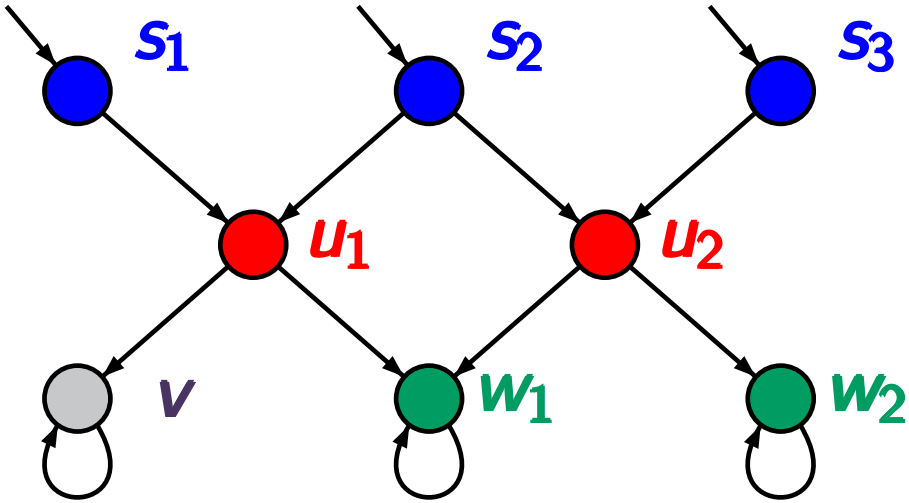

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$s_1 \to u_1$ can't be simulated by any state $t \in Remove(u_1)$

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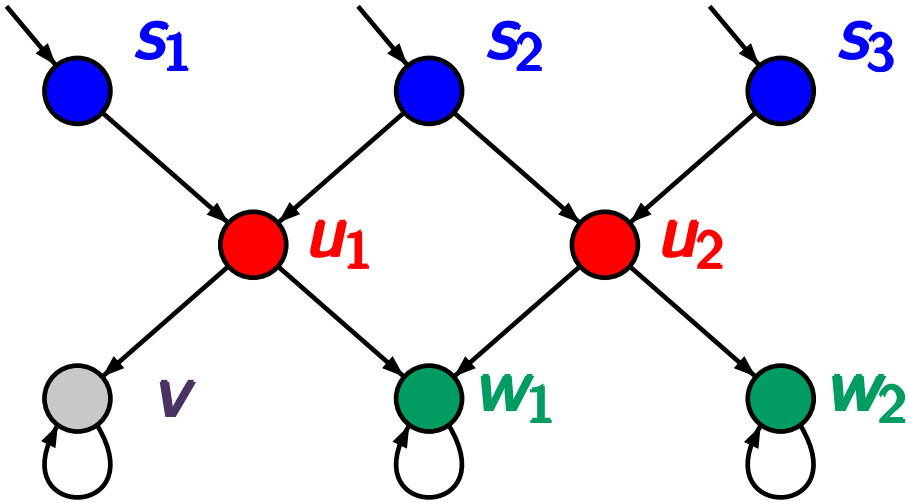

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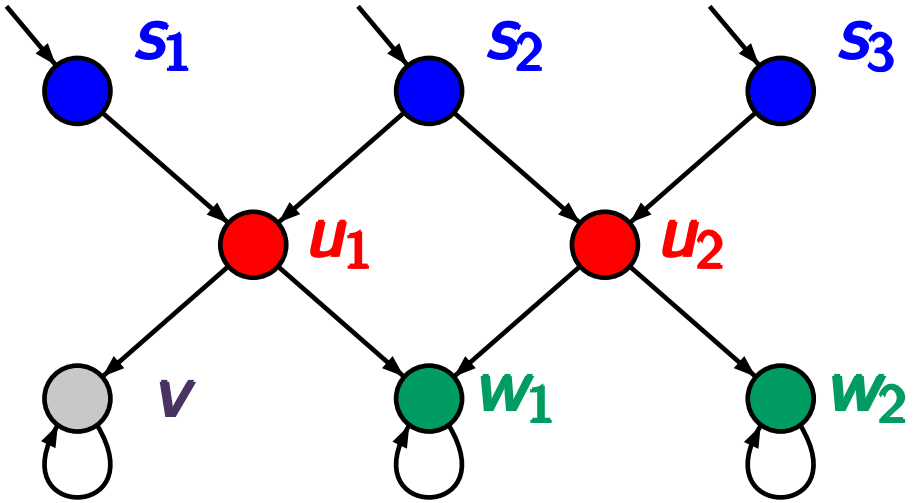

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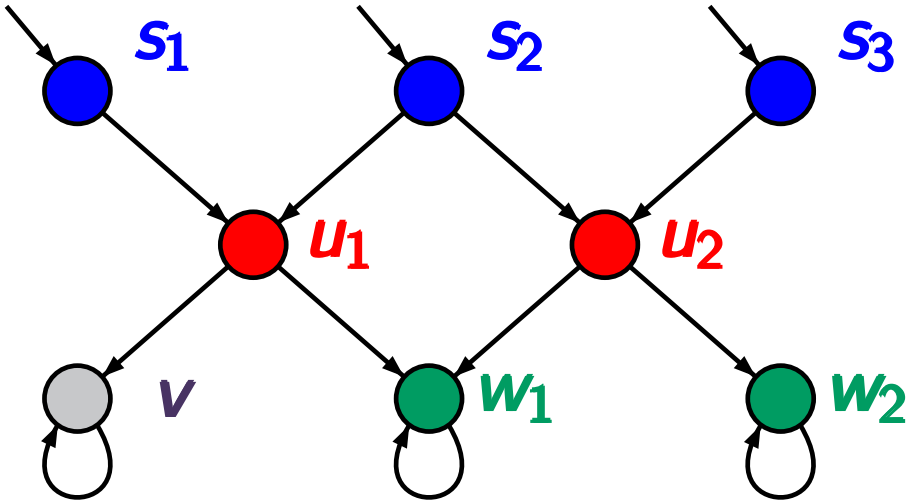

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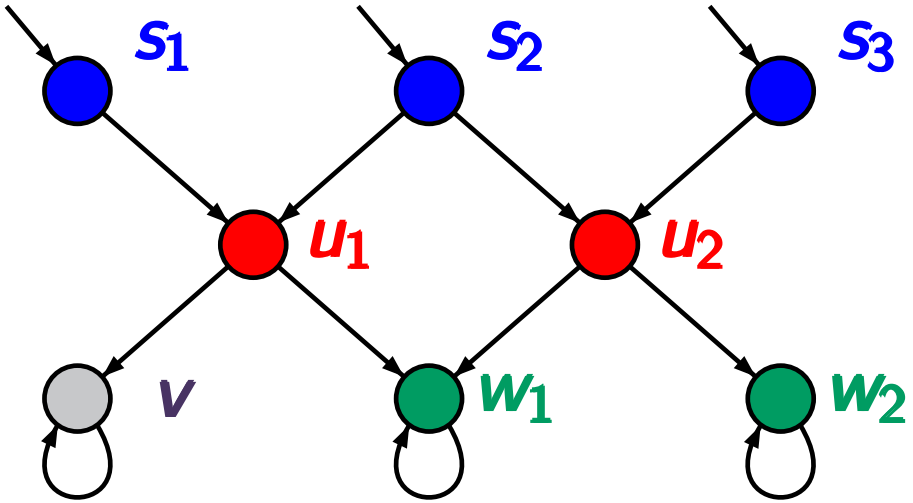

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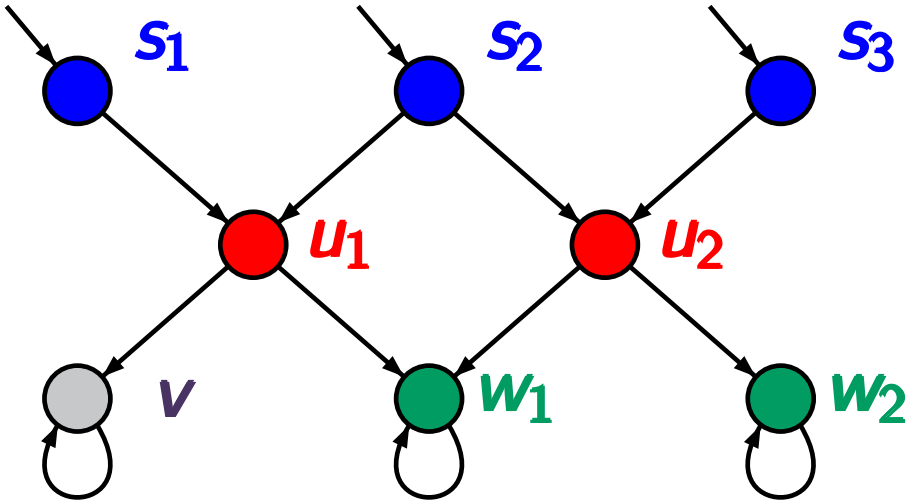

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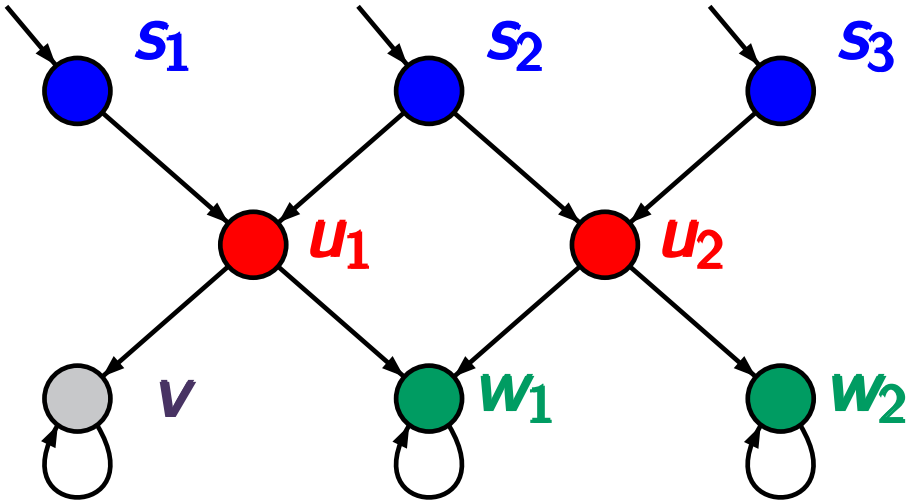

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since $\preceq_T$ is the coarsest simulation, (1) yields:

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$\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \emptyset\}$

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$\ldots$

HHK-algorithm (second version)

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choose such a state s_1' with $Remove(s_1') \neq \emptyset$

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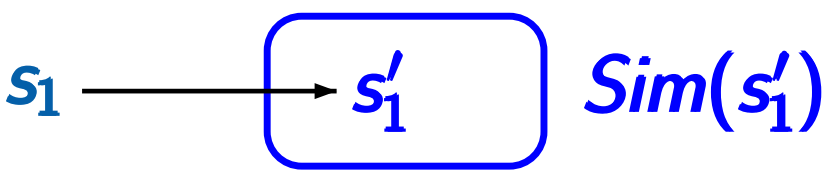

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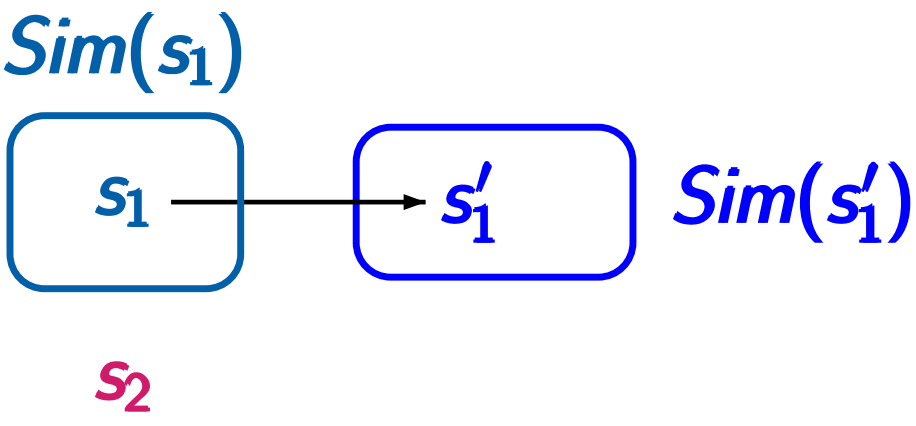

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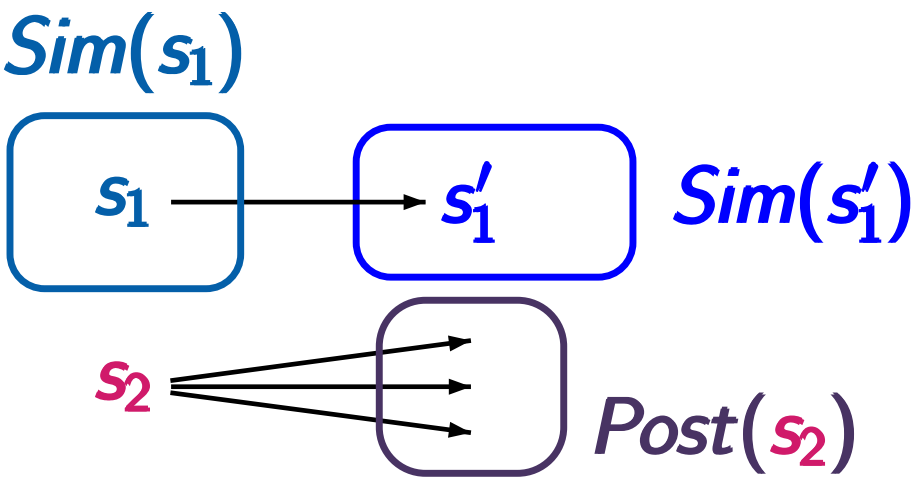

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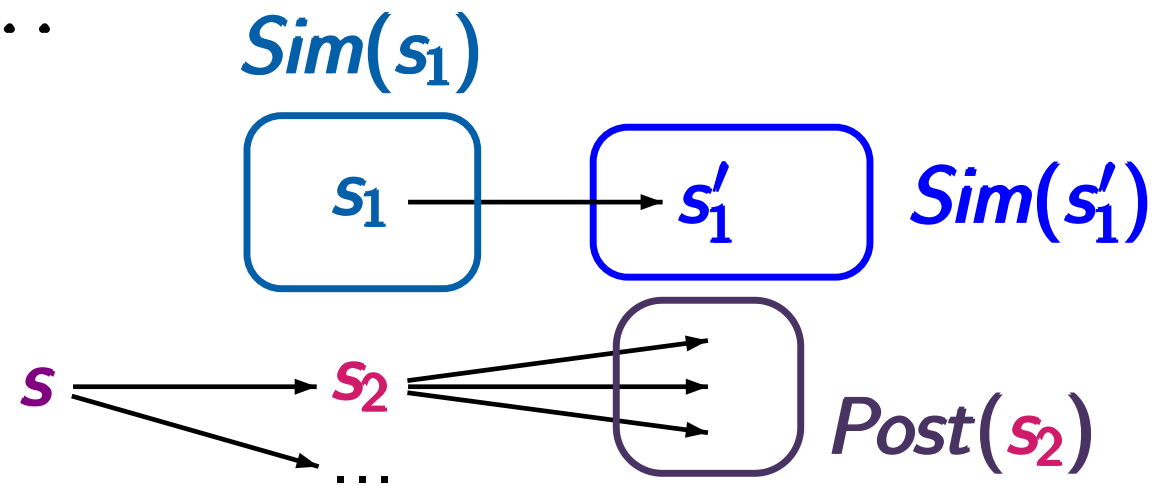

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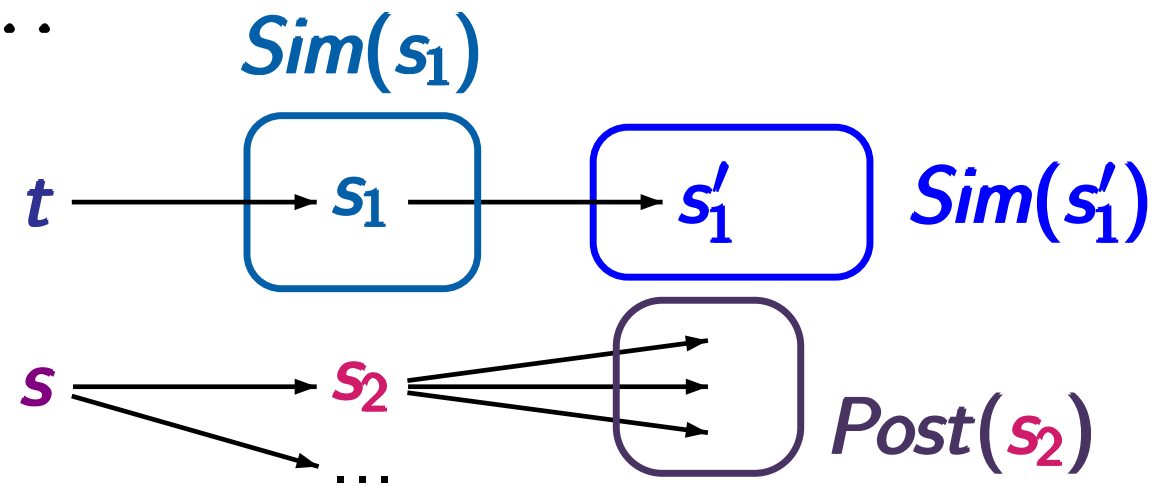

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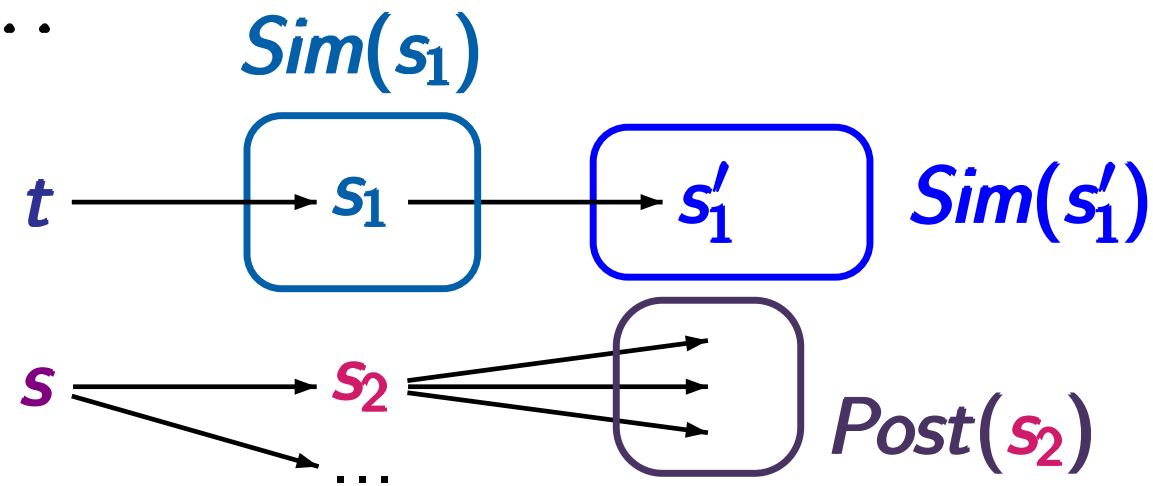

$t \not\preceq_T s$

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for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) *Remove*(s_1') at most once

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```
  ⋮
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    Sim(s1) := Sim(s1) \ {s2};
    Remove(s1) := Remove(s1) ∪
       {s ∈ Pre(s2) : Post(s) ∩ Sim(s1) = ∅}
FI
  ⋮
```

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. $s \longrightarrow s_2$ and $Post(s) \cap Sim(s_1) = \{s_2\}$ immediately before s_2 has been removed from $Sim(s_1)$

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show that the HHK-algorithm can be realized in time:

$$\mathcal{O}(m \cdot |S|)$$

where m = number of edges

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and AP is fixed

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FOR ALL states s_1' DO

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time complexity: $\mathcal{O}(|S| \cdot AP) = \mathcal{O}(|S|)$
(as in the bisimulation algorithms)

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WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO
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OD
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bisimulation $\sim$: $\mathcal{O}(m \cdot \log |S|)$

stutter bisimulation $\approx$ or $\approx^{\mathbf{div}}$: $\mathcal{O}(m \cdot |S|)$

simulation $\preceq$: $\mathcal{O}(m \cdot |S|)$

The End