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**LEHRSTUHL FÜR INFORMATIK 2**
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# Advanced Model Checking
# Summer term 2012

## – Series 3 –

Hand in on May 9'th before the exercise class.

### Exercise 1 (2 + 1 points)

Observational equivalence $\approx_{obs}$ is a slight variant of stutter-bisimulation equivalence where state $s_2$ is allowed to perform a path fragment

$$\underbrace{s_2 u_1 ... u_m}_{stuttersteps} \; \underbrace{v_1 ... v_k s_2'}_{stuttersteps}$$

with arbitrary stutter steps at the beginning and at the end and $s_1' \approx_{obs} s_2'$ to simulate a transition $s_1 \to s_1'$ of an observational equivalent state $s_1$. I.e., it is not required that $s_2$ and states $u_i$ are observationally equivalent, or that $s_2'$ and $v_i$ are observationally equivalent. For the special case where $s_1 \to s_1'$ is a stutter step the path fragment of length 0 (consisting of state $s_2 = s_2'$) can be used to simulate $s_1 \to s_1'$.

The formal definition of observational equivalence is as follows. Let $TS_1$ and $TS_2$ be two transition systems with state-spaces $S_1$ and $S_2$, respectively, and the same set $AP$ of atomic propositions. A binary relation $\mathcal{R} \subseteq S_1 \times S_2$ is called an observational bisimulation for $(TS_1, TS_2)$ iff the following conditions (A) and (B) are satisfied:

(A) Every initial state of $TS_1$ is related to an initial state of $TS_2$, and vice versa. That is,

$$\forall s_1 \in I_1 \; \exists s_2 \in I_2 . (s_1, s_2) \in \mathcal{R} \qquad \text{and} \qquad \forall s_2 \in I_2 \; \exists s_1 \in I_1 . (s_1, s_2) \in \mathcal{R}$$

(B) For all $(s_1, s_2) \in \mathcal{R}$, the following conditions (I), (II) and (III) hold:

(I) $L_1(s_1) = L_2(s_2)$.

(II) If $s_1' \in Post(s_1)$, then there exists a path fragment $u_0 u_1 ... u_n$ such that $n \geq 0$ and $u_0 = s_2$, $(s_1', u_n) \in \mathcal{R}$ and, for some $m \leq n$, $L_2(u_0) = L_2(u_1) = ... = L_2(u_m)$ and $L_2(u_{m+1}) = L_2(u_{m+2}) = ... = L_2(u_n)$.

(III) If $s_2' \in Post(s_2)$, then there exists a path fragment $u_0 u_1 ... u_n$ such that $n \geq 0$ and $u_0 = s_1$, $(u_n, s_2') \in \mathcal{R}$ and, for some $m \leq n$, $L_1(u_0) = L_1(u_1) = ... = L_1(u_m)$ and $L_1(u_{m+1}) = L_1(u_{m+2}) = ... = L_1(u_n)$.

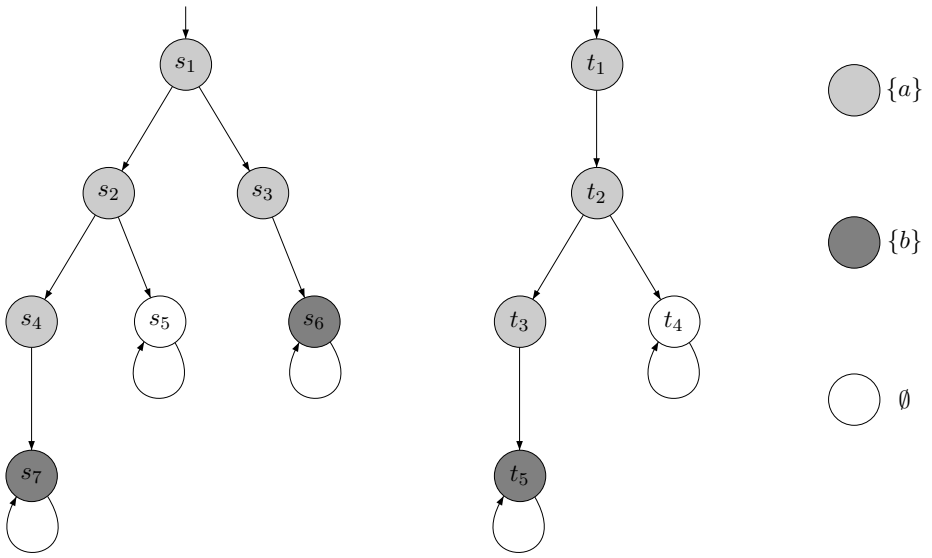
$TS_1$ and $TS_2$ are called observational equivalent, denoted $TS_1 \approx_{obs} TS_2$, iff there exists an observational bisimulation for $(TS_1, TS_2)$.

**Questions:**

The goal of this exercise is to show that $\approx_{obs}$ is strictly coarser than stutter-bisimulation equivalence $\approx$.

(a) Show that $TS_1 \approx TS_2$ implies $TS_1 \approx_{obs} TS_2$.

(b) Consider the two transition systems $TS_1$ and $TS_2$ shown in the following figure. Show that $TS_1 \approx TS_2$ and $TS_1 \approx_{obs} TS_2$.



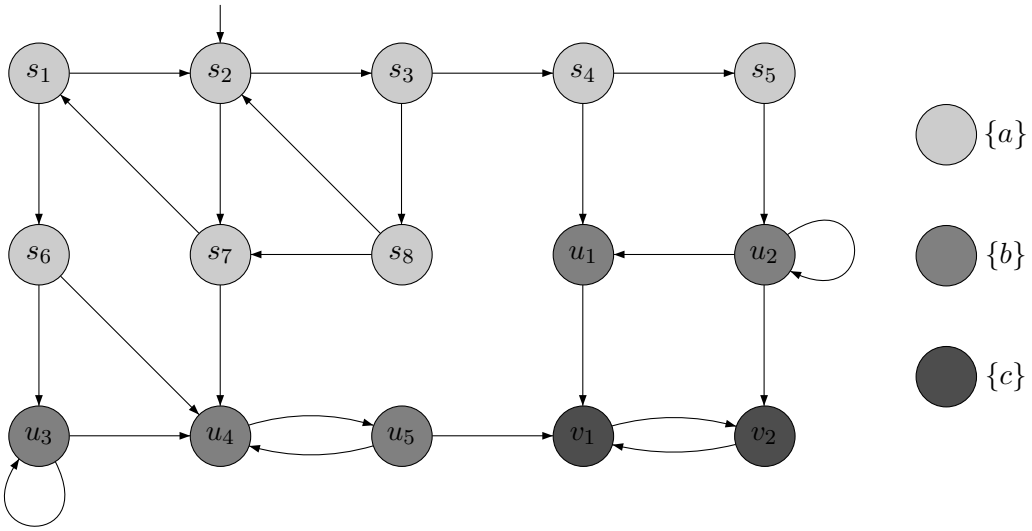

$TS_1$ $TS_2$

## Exercise 2 (3 points)

Let $\varphi$ be an LTL formula such that $Words(\varphi)$ is stutter insensitive.
Show that $\varphi$ is equivalent to some $\text{LTL}_{\setminus\bigcirc}$ formula $\psi$.

## Exercise 3 (1 + 2 + 1 points)

Given transition systems $TS$:




**Questions:**

(a) Depict the divergence-sensitive expansion $\overline{TS}$.

(b) Determine the divergence-stutter-bisimulation quotient $(\overline{TS})/\approx$. Apply the algorithm and give for each iteration the partition of the state space.

(c) Depict $TS/\approx^{div}$.