RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK II
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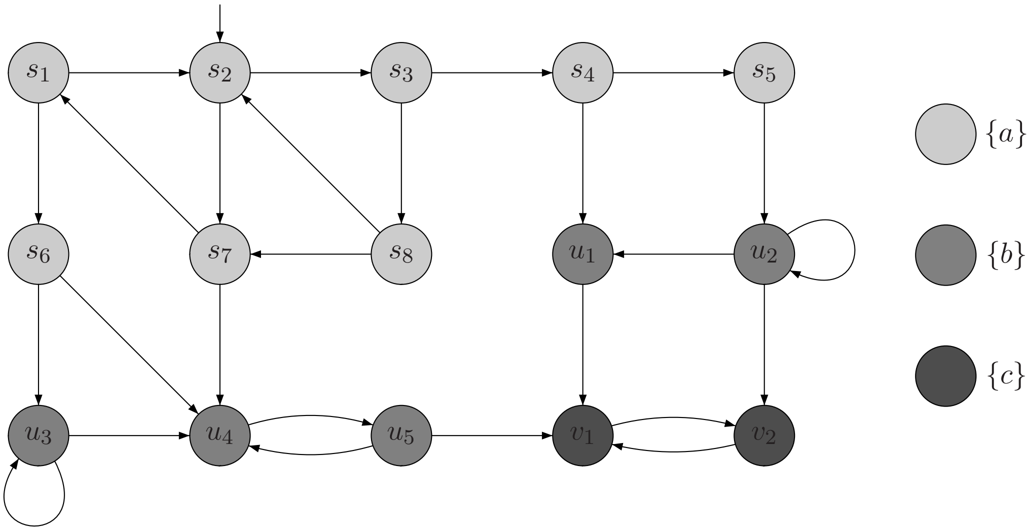
Exercises to the lecture *"Advanced Model Checking"*, winter term 2006

– Assignment 6 –

The solutions are collected on Dec. 1st at the beginning of the exercise class.
Justify your answers!

Exercise 1 (6 points)

Given transition systems TS:



(a) Depict the divergence-sensitive expansion $\overline{TS}$.

(b) Determine the divergence-stutter-bisimulation quotient $(\overline{TS})/\approx$. Apply the algorithm and give for each iteration the partition of the state space.

(c) Depict $TS/{\approx^{div}}$.

(d) Provide CTL$\backslash_{\bigcirc}$ master formulae for the divergence-stutter-bisimulation equivalence class.

Exercise 2 ($5(a) + 7(b) = 12$ points)

Consider the following definition:

Definition 1 *Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP, L_i)$ be transition systems over AP. A* normed simulation *for (TS_1, TS_2) is a triple $(\mathcal{R}, \nu_1, \nu_2)$ consisting of a binary relation $\mathcal{R} \in S_1 \times S_2$ such that:*

$$\forall s_1 \in I_1 . \exists s_2 \in I_2 . (s_1, s_2) \in \mathcal{R}$$

and functions $\nu_1, \nu_2 : S_1 \times S_2 \rightarrow \mathbf{N}$ such that for all $(s_1, s_2) \in \mathcal{R}$:

(I) $L_1(s_1) = L_2(s_2)$

(II) For all $s_1' \in Post(s_1)$*, at least one of the following three conditions holds:*

1) $\exists s_2' \in Post(s_2).(s_1', s_2') \in \mathcal{R}$

2) $(s_1', s_2) \in \mathcal{R}$ *and* $\nu_1(s_1', s_2) < \nu_1(s_1, s_2)$

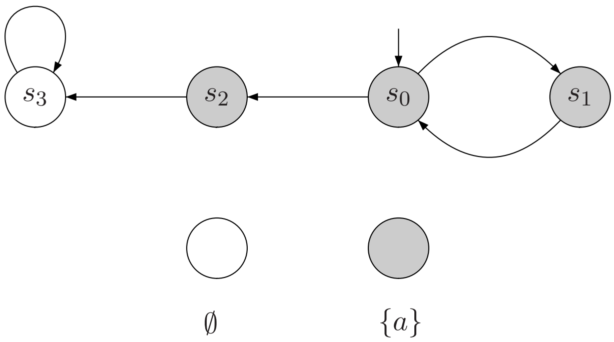
3) $\exists s_2' \in Post(s_2).(s_1, s_2') \in \mathcal{R}$ *and* $\nu_2(s_1, s_2') < \nu_2(s_1, s_2)$

A normed bisimulation *for* (TS_1, TS_2) *is a normed simulation* $(\mathcal{R}, \nu_1, \nu_2)$ *for* (TS_1, TS_2) *such that* $(\mathcal{R}^{-1}, \nu_1^-, \nu_2^-)$ *is a normed simulation for* (TS_2, TS_1)*. Here* ν_i^- *denotes the function* $S_2 \times S_1 \rightarrow \mathbf{N}$ *that results from* ν_i *by swapping the arguments, i.e.* $\nu_i^-(u, v) = \nu_i(v, u)$ *for all* $u \in S_2$ *and* $v \in S_1$*.*

TS_1 *and* TS_2 *are* normed bisimilar*, denoted* $TS_1 \approx^n TS_2$*, if there exists a normed bisimulation for* (TS_1, TS_2)*.*

In this exercise you are asked to do the following:

(a) Given transition systems TS:



Check whether the coarsest equivalence $\mathcal{R}$ which identifies s_0 and s_1 together with $\nu_1(s_0, s_1) = \nu_2(s_1, s_0) = 1$ and $\nu_2(s_1, s_2) = 1$, $\nu_1(s, s) = \nu_2(s, s) = 0$ for all $s \in \{s_0, s_1, s_2, s_3\}$ (and arbitrary values for ν_1 and ν_2 for the remaining cases) is a normed bisimulation.

(b) It is a fact that:

For any s_1*,* s_2 *in transition system* TS*:* $s_1 \approx^n s_2$ *implies* $s_1 \approx^{div} s_2$*.*

In order to prove that, one have to show that for a normed bisimulation $(\mathcal{R}, \nu_1, \nu_2)$, $\mathcal{R}'$ - the symmetric, reflexive, and transitive closure of $\mathcal{R}$ – the coarsest equivalence that contains $\mathcal{R}$ – is a divergence-sensitive stutter bisimulation.

Here you are asked to prove that:

- The condition (2) from the definition of stutter bisimulation holds for $\mathcal{R}'$.