

Exercises to the lecture “Advanced Model Checking”, winter term 2006

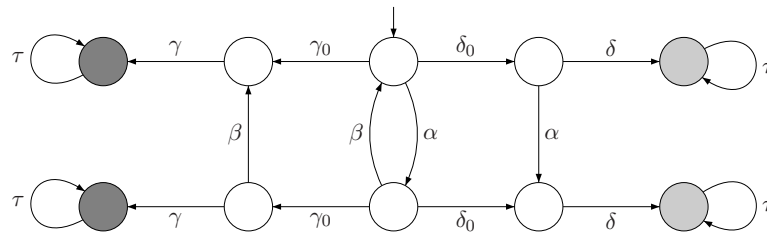
– Assignment 8 –

The solutions are collected on Dec. 15th at the beginning of the exercise class.
Justify your answers!

Exercise 1

(4 points)

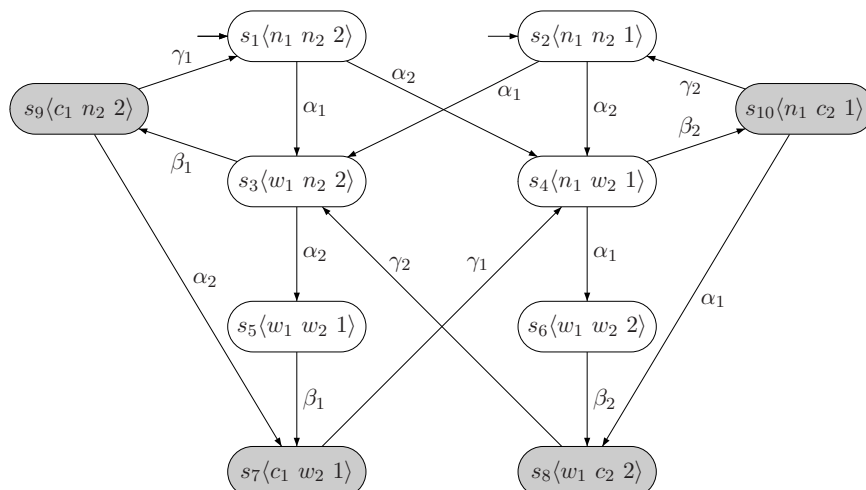
Consider the transition system TS shown in the following figure. Show that (A1)-(A4) does not allow for any state reduction, although there is a smaller subsystem \widehat{TS} that is stutter-trace equivalent to TS .



Exercise 2

(4 points)

Consider the transition system TS_{Pet} for the Peterson mutual exclusion algorithm.
(For more details of the algorithm, cf. page 59-63 of the lecture notes.)



Questions:

- (a) Which actions are independent?
- (b) Apply the partial order reduction approach to TS_{Pet} with “small” ample sets according to Algorithm 30 (page 520) for checking the invariant “always $\neg(crit_1 \wedge crit_2)$ ”, where $AP = \{crit_1, crit_2\}$. Note that c_i in the figure is an abbreviation for $crit_i$.

Exercise 3

(3 points)

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be an action-deterministic transition system and let \mathcal{I}_{st} be the set of all pairs $(\alpha, \beta) \in Act \times Act$ of independent actions α and β where α or β (or both) is a stutter action. Let *stutter permutation equivalence* \cong_{perm} be the finest equivalence on Act^* such that

$$\bar{\gamma}\alpha\beta\bar{\delta} \cong_{perm} \bar{\gamma}\beta\alpha\bar{\delta}$$

if $\bar{\gamma}, \bar{\delta} \in Act^*$ and $(\alpha, \beta) \in \mathcal{I}_{st}$.

The extension of \cong_{perm} to an equivalence for infinite action sequences is defined as follows. If $\tilde{\alpha} = \alpha_1\alpha_2\alpha_3\dots$ and $\tilde{\beta} = \beta_1\beta_2\beta_3\dots$ are actions sequences in Act^ω , then $\tilde{\alpha} \sqsubseteq_{perm} \tilde{\beta}$ if for all finite prefixes $\alpha_1\dots\alpha_n$ of $\tilde{\alpha}$ there exists a finite prefix $\beta_1\dots\beta_m$ of $\tilde{\beta}$ with $m \geq n$ and a finite word $\bar{\gamma} \in Act^*$ such that

$$\alpha_1\dots\alpha_n\bar{\gamma} \cong_{perm} \beta_1\dots\beta_m$$

We then define the binary relation \cong_{perm}^ω on Act^ω by

$$\tilde{\alpha} \cong_{perm}^\omega \tilde{\beta} \quad \text{iff} \quad \tilde{\alpha} \sqsubseteq_{perm} \tilde{\beta} \quad \text{and} \quad \tilde{\beta} \sqsubseteq_{perm} \tilde{\alpha}$$

Questions:

- (a) Show that \cong_{perm}^ω is an equivalence.

Exercise 4

(5 points)

Consider the following definition:

Definition 1 Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP, L_i)$ be transition systems over AP . A normed simulation for (TS_1, TS_2) is a triple $(\mathcal{R}, \nu_1, \nu_2)$ consisting of a binary relation $\mathcal{R} \in S_1 \times S_2$ such that:

$$\forall s_1 \in I_1. \exists s_2 \in I_2. (s_1, s_2) \in \mathcal{R}$$

and functions $\nu_1, \nu_2 : S_1 \times S_2 \rightarrow \mathbf{N}$ such that for all $(s_1, s_2) \in \mathcal{R}$:

$$(I) \quad L_1(s_1) = L_2(s_2)$$

(II) For all $s'_1 \in Post(s_1)$, at least one of the following three conditions holds:

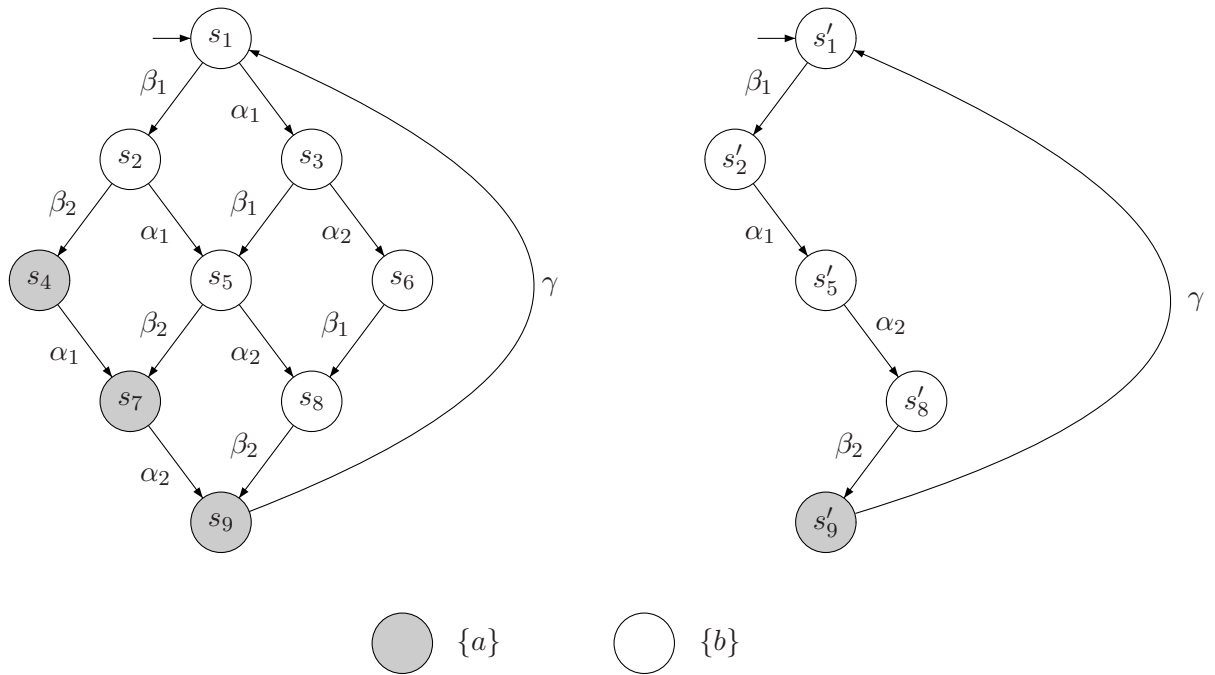
- 1) $\exists s'_2 \in Post(s_2). (s'_1, s'_2) \in \mathcal{R}$
- 2) $(s'_1, s_2) \in \mathcal{R}$ and $\nu_1(s'_1, s_2) < \nu_1(s_1, s_2)$
- 3) $\exists s'_2 \in Post(s_2). (s_1, s'_2) \in \mathcal{R}$ and $\nu_2(s_1, s'_2) < \nu_2(s_1, s_2)$

A normed bisimulation for (TS_1, TS_2) is a normed simulation $(\mathcal{R}, \nu_1, \nu_2)$ for (TS_1, TS_2) such that $(\mathcal{R}^{-1}, \nu_1^-, \nu_2^-)$ is a normed simulation for (TS_2, TS_1) . Here ν_i^- denotes the function $S_2 \times S_1 \rightarrow \mathbf{N}$ that results from ν_i by swapping the arguments, i.e. $\nu_i^-(u, v) = \nu_i(v, u)$ for all $u \in S_2$ and $v \in S_1$.

TS_1 and TS_2 are normed bisimilar, denoted $TS_1 \approx^n TS_2$, if there exists a normed bisimulation for (TS_1, TS_2) .

Questions:

For two transition systems TS (left) and \widehat{TS} (right) show that:



- The ample sets $ample(\cdot)$ which reduce TS to \widehat{TS} satisfy conditions (A1)-(A5).
- Provide a normed bisimulation for (TS, \widehat{TS}) .