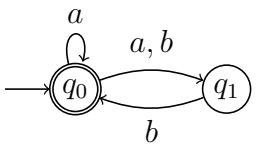
Exercise 1 (Regular Languages). (12 points)

(i) Give a regular expression that describes the language 3

$$L := \{w \in \{a, b\}^* \mid w \text{ contains } ab \text{ and ends with } b\}.$$

(ii) Give a nondeterministic finite automaton, possibly with ϵ-transitions (ϵ-NFA), that recognizes the same language L. (You can either construct it directly or by translation from the previous regular expression.) 3

(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. 3



(iv) Is $\mathfrak{A}'$ minimal? Please justify your answer in the following way: 3

"yes": give a distinguishing word for each pair of states;

"no": give two equivalent states and explain why they are equivalent.

Exercise 2 (Context-Free Languages). (13 points)

(i) Give a context-free grammar G which generates the language 5

$$L := \{a^k b^l \mid k, l \geq 1, k \neq l\}.$$

(ii) Give a derivation of the word $aaabb \in L$ from the start symbol of G. 2

(iii) Let G' be the following context-free grammar: 6

$$\begin{aligned} S &\to AB \mid AC \\ A &\to a \\ B &\to b \\ C &\to SB \end{aligned}$$

and let $w := aaabbb$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 6)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5 a_6$.

$i \backslash j$	1	2	3	4	5	6
1						
2	X					
3	X	X				
4	X	X	X			
5	X	X	X	X		
6	X	X	X	X	X	