Name: ______________________

Student number: ______________________

Exercise 1 (Regular Languages). (13 points)

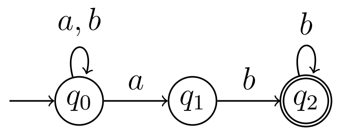
(i) Give a regular expression that describes the language [3]

$$L := \{w \in \{a,b\}^* \mid \text{each occurrence of } b \text{ is followed by at least two } a\text{s}\}.$$

(ii) Give a nondeterministic finite automaton $\mathfrak{A}_L$, possibly with ϵ-transitions (ϵ-NFA), that recognizes the same language L. (You can either construct it directly or by translation from the previous regular expression.) [3]

(iii) Show that $\mathfrak{A}_L$ accepts the word $abaa \in L$. [1]

(iv) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. [3]



(v) Is $\mathfrak{A}'$ minimal? Please justify your answer in the following way: [3]

"yes": give a distinguishing word for each pair of states;

"no": give two equivalent states and explain why they are equivalent.

Exercise 2 (Context-Free Languages). (12 points)

(i) Give a context-free grammar G_1 which generates the language 5

$$L := \{a^k b^l c^{k+l} \mid k, l \geq 1\}.$$

(ii) Give a derivation of the word $aabccc \in L$ from the start symbol of G_1. 2

(iii) Let G_2 be the following context-free grammar: 5

$$\begin{array}{lcl} S & \to & AB \mid BC \\ A & \to & BA \mid a \\ B & \to & CC \mid b \\ C & \to & AB \mid a \end{array}$$

and let $w := aaaaa$. Employing the CYK-Algorithm, show that $w \in L(G_2)$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a^{j-i+1}$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	