Compiler Construction

Lecture 13: Semantic Analysis I (Definition of Attribute Grammars)

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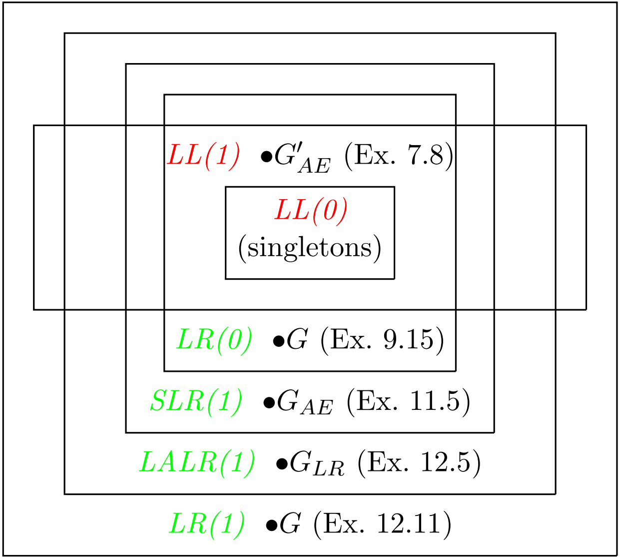
Outline

Overview of Grammar Classes

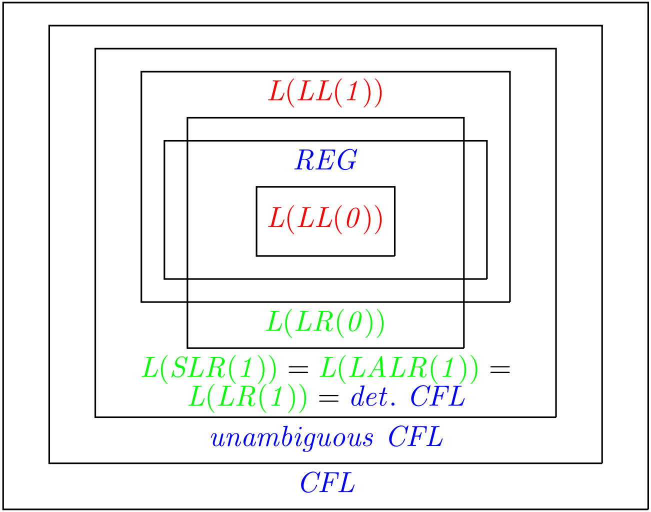


Moreover:

- $LL(k) \subsetneq LL(k+1)$ for every $k \in \mathbb{N}$
- $LR(k) \subsetneq LR(k+1)$ for every $k \in \mathbb{N}$
- $LL(k) \subseteq LR(k)$ for every $k \in \mathbb{N}$

Overview of Language Classes

(cf. O. Mayer: *Syntaxanalyse*, BI-Verlag, 1978, p. 409ff)



Moreover:

- $L(LL(k)) \subsetneq L(LL(k+1)) \subsetneq L(LR(1))$ for every $k \in \mathbb{N}$
- $L(LR(k)) = L(LR(1))$ for every $k \geq 1$

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LL and LR Parsing in Practice

In practice: use of $LL(1)$ or $LALR(1)$

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Detailed comparison (cf. Fischer/LeBlanc: *Crafting a Compiler*, Benjamin/Cummings, 1988):

Simplicity : LL wins

- LL parsing technique easier to understand
- recursive-descent parser easier to debug than LALR action tables

LL and LR Parsing in Practice

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Detailed comparison (cf. Fischer/LeBlanc: *Crafting a Compiler*, Benjamin/Cummings, 1988):

Simplicity : LL wins

Generality : LALR wins

- "almost" $LL(1) \subseteq LALR(1)$ (only pathological counterexamples)
- LL requires elimination of left recursion and left factorization

LL and LR Parsing in Practice

In practice: use of $LL(1)$ or $LALR(1)$

Detailed comparison (cf. Fischer/LeBlanc: *Crafting a Compiler*, Benjamin/Cummings, 1988):

Simplicity : LL wins

Generality : LALR wins

Semantic actions : (see semantic analysis) LL wins

- actions can be placed anywhere in LL parsers without causing conflicts
- in LALR: implicit ε-productions
  $\Longrightarrow$ may generate conflicts

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Semantic actions : (see semantic analysis) LL wins

Error handling : LL wins

- top-down approach provides context information $\implies$ better basis for reporting and/or repairing errors

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Parser size : comparable

- LL: action table
- LALR: action/goto table

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Simplicity	: LL wins
Generality	: LALR wins
Semantic actions	: (see semantic analysis) LL wins
Error handling	: LL wins
Parser size	: comparable
Parsing speed	: comparable

- both linear in length of input program ($LL(1)$: see Lemma 8.7 for ε-free case)
- concrete figures tool dependent

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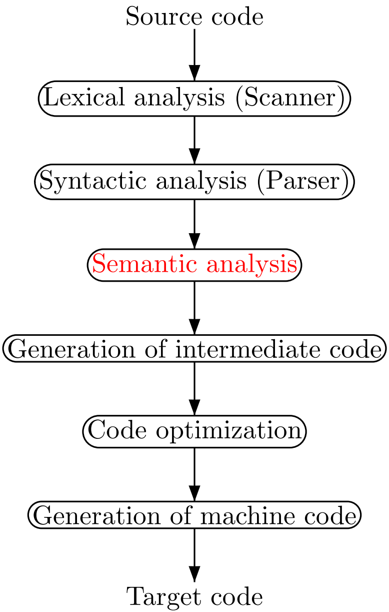
Semantic actions	:	(see semantic analysis) LL wins
Error handling	:	LL wins
Parser size	:	comparable
Parsing speed	:	comparable

Conclusion: choose LL when possible
(depending on available grammars and tools)

Outline

Conceptual Structure of a Compiler



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Beyond Syntax

To generate (efficient) code, the compiler needs to answer many questions:

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These cannot be expressed using context-free grammars!
(e.g., $\{ww \mid w \in \Sigma^*\} \notin CFL_\Sigma$)

Static Semantics

Static semantics refers to properties of program constructs

- which are true for every occurrence of this construct in every program execution (static) and
- can be decided at compile time
- but are context-sensitive and thus not expressible using context-free grammars (semantics).

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Example properties:

Static: type or declaredness of an identifier, number of registers required to evaluate an expression, ...

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Example properties:

Static: type or declaredness of an identifier, number of registers required to evaluate an expression, ...

Dynamic: value of an expression, size of runtime stack, ...

These properties are determined by

Scope rules: defines part of program where a declaration is valid

Visibility rules: defines part of scope where a declaration is visible (overlapping of global and local declarations)

Typing rules: defines type consistency of expressions, statements, ...

Outline

Attribute Grammars I

Goal: compute context-dependent but runtime-independent properties of a given program

Idea: enrich context-free grammar by semantic rules which annotate syntax tree with attribute values

$\Longrightarrow$ Semantic analysis = attribute evaluation

Result: attributed syntax tree

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Result: attributed syntax tree

In greater detail:

- With every nonterminal a set of attributes is associated.
- Two types of attributes are distinguished:

  Synthesized: bottom-up computation (from the leafs to the root)

  Inherited: top-down computation (from the root to the leafs)
- With every production a set of semantic rules is associated.

Attribute Grammars II

Advantage: attribute grammars provide a very flexible and broadly applicable mechanism for transporting information throught the syntax tree ("syntax-directed translation")

- Attribute values: symbol tables, data types, code, error flags, ...
- Application in Compiler Construction:
  - static semantics
  - program analysis for optimization
  - code generation
  - error handling
- Automatic attribute evaluation by compiler generators (cf. `yacc`'s synthesized attributes)
- Originally designed by D. Knuth for defining the semantics of context-free languages (Math. Syst. Theory 2 (1968), pp. 127–145)

Example: Knuth's Binary Numbers I

Example 13.1 (only synthesized attributes)

Binary numbers (with fraction):

$$
\begin{array}{llrcl}
G_B: & \text{Numbers} & N & \rightarrow & L \\
 & & N & \rightarrow & L\,.\,L \\
 & \text{Lists} & L & \rightarrow & B \\
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G_B :					
	Numbers	$N \to L$	$v.0$	$=$	$v.1$
		$N \to L\,.\,L$	$v.0$	$=$	$v.1 + v.3/2^{l.3}$
	Lists	$L \to B$	$v.0$	$=$	$v.1$
			$l.0$	$=$	1
		$L \to LB$	$v.0$	$=$	$2 * v.1 + v.2$
			$l.0$	$=$	$l.1 + 1$
	Bits	$B \to \texttt{0}$	$v.0$	$=$	0
	Bits	$B \to \texttt{1}$	$v.0$	$=$	1

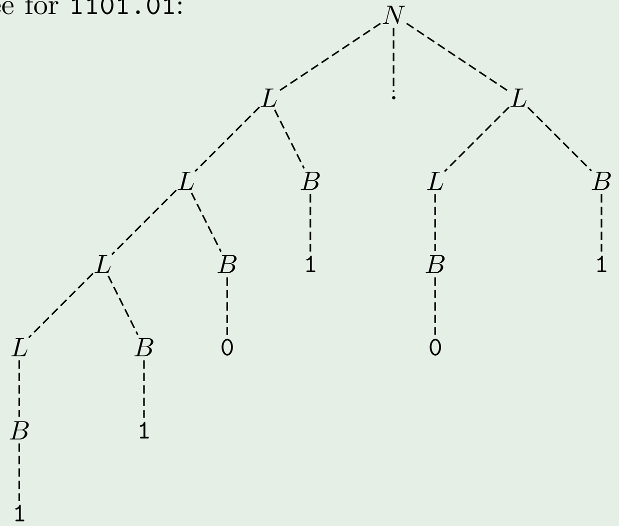
Synthesized attributes of N, L, B: v (value; domain: $V^v := \mathbb{Q}$)
of L: l (length; domain: $V^l := \mathbb{N}$)

Semantic rules: equations with attribute variables
(index = position of symbol; 0 = left-hand side)

Example: Knuth's Binary Numbers II

Example 13.1 (continued)

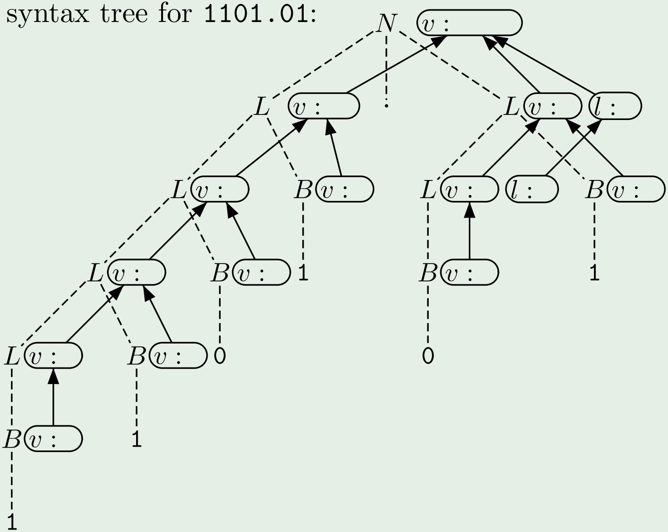
Syntax tree for `1101.01`:



Example: Knuth's Binary Numbers II

Example 13.1 (continued)

Attributed syntax tree for `1101.01`:



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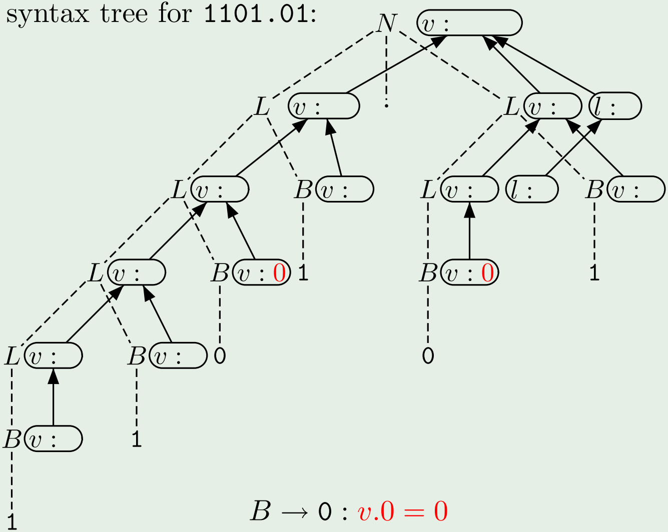

$B \rightarrow 0 : v.0 = 0$

Example 13.1 (continued)

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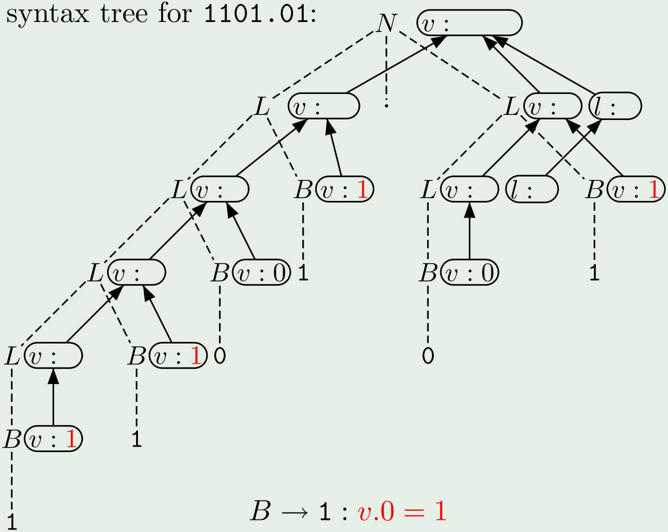

$B \to \texttt{1} : v.0 = 1$

Example 13.1 (continued)

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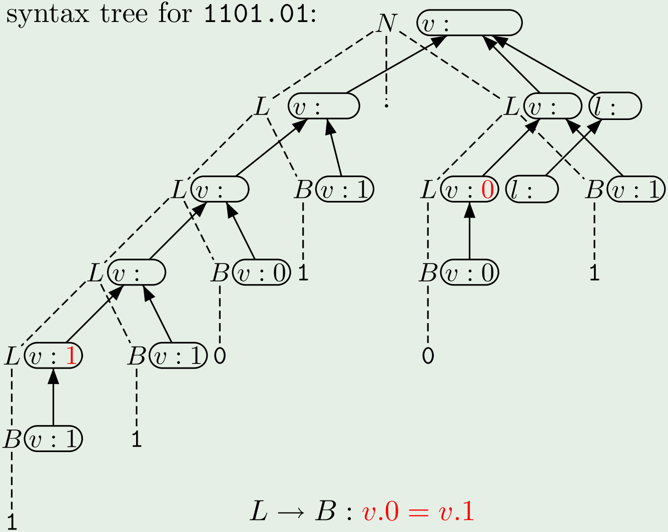

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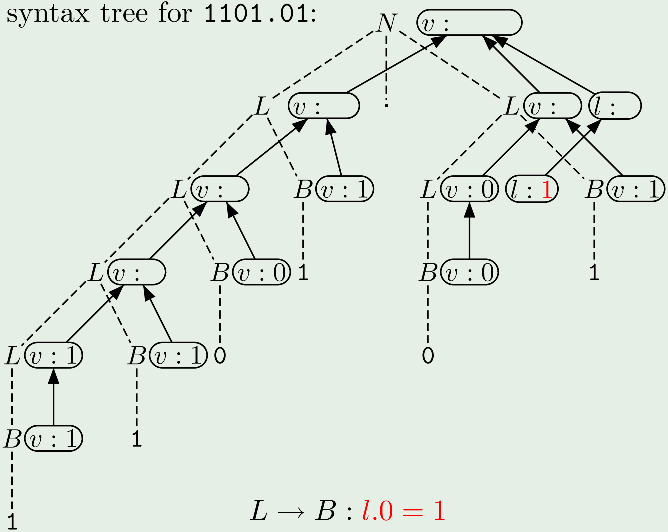

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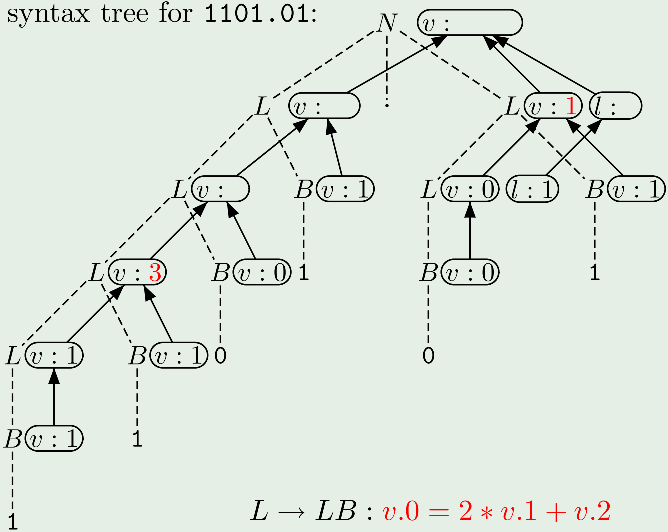

$$L \to LB : v.0 = 2 * v.1 + v.2$$

Example 13.1 (continued)

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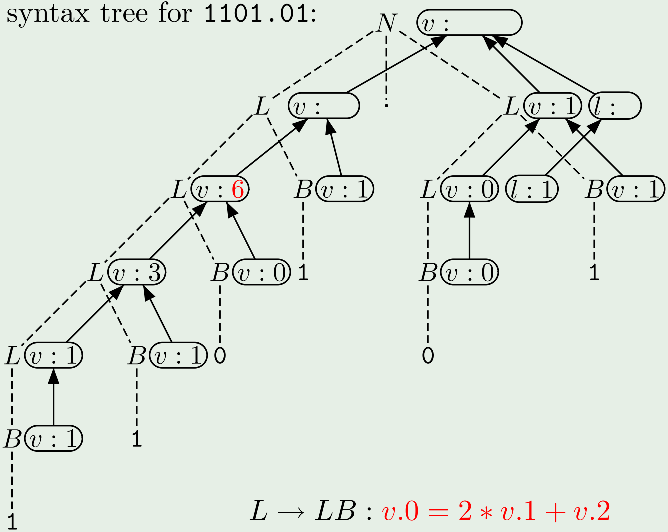

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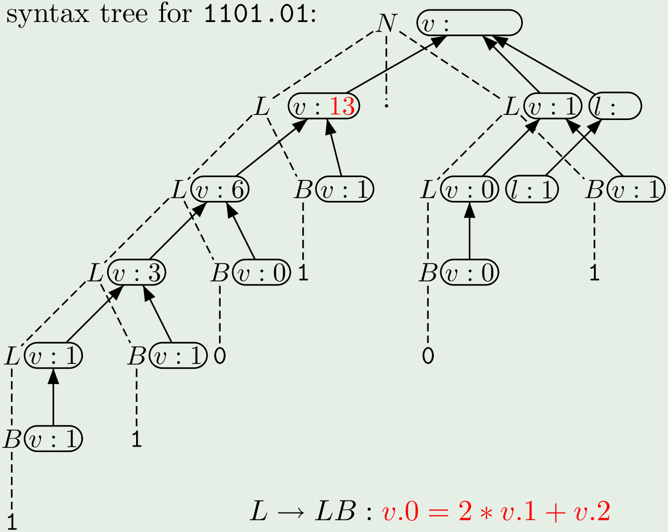

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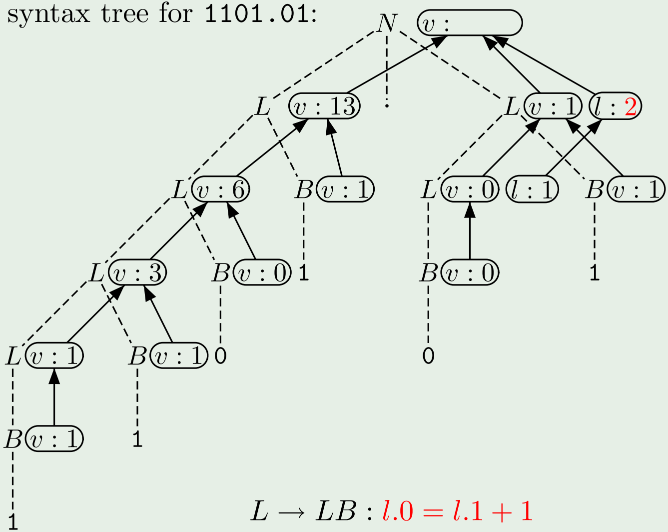

$L \rightarrow LB : l.0 = l.1 + 1$

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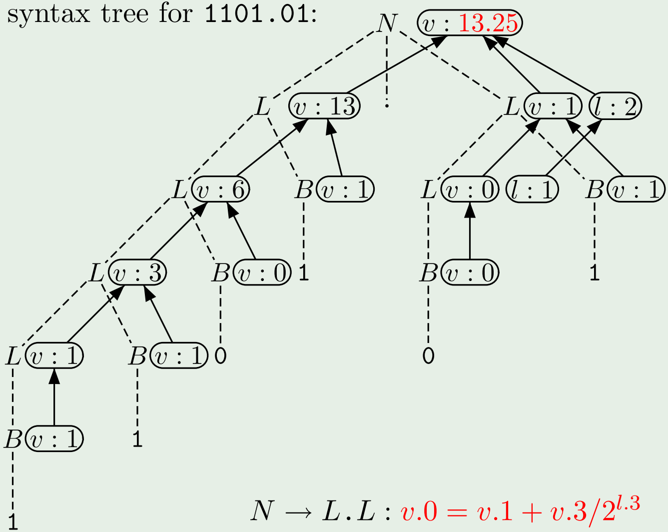

$N \to L\,.\,L : v.0 = v.1 + v.3/2^{l.3}$

Adding Inherited Attributes I

Example 13.2 (synthesized + inherited attributes)

Binary numbers (with fraction):

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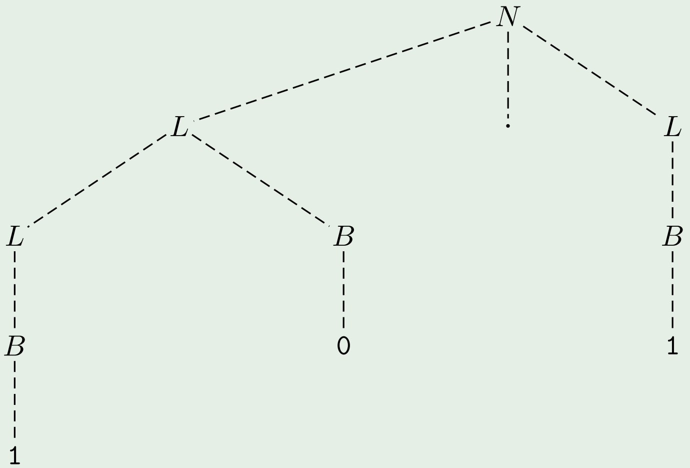
of L: l (length; domain: $V^l := \mathbb{N}$)
Inherited attribute of L, B: p (position; domain: $V^p := \mathbb{Z}$)

Adding Inherited Attributes II

Example 13.2 (continued)

Syntax tree for `10.1`:



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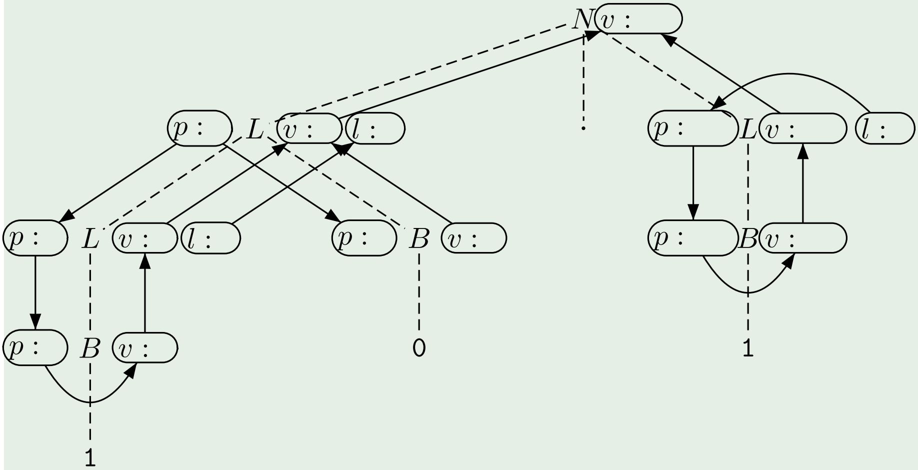

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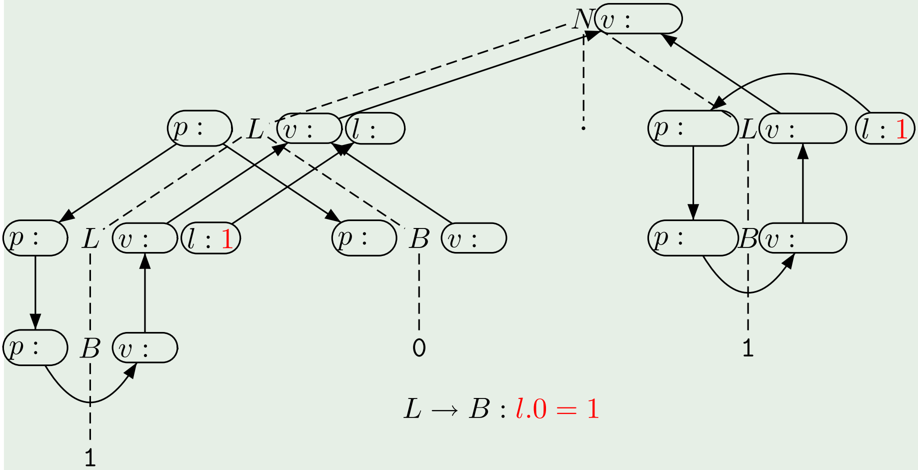

$L \rightarrow B : l.0 = 1$

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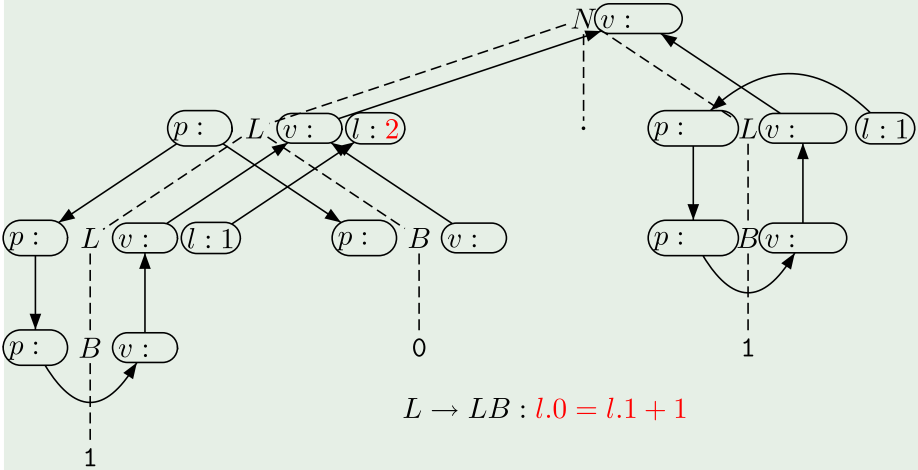

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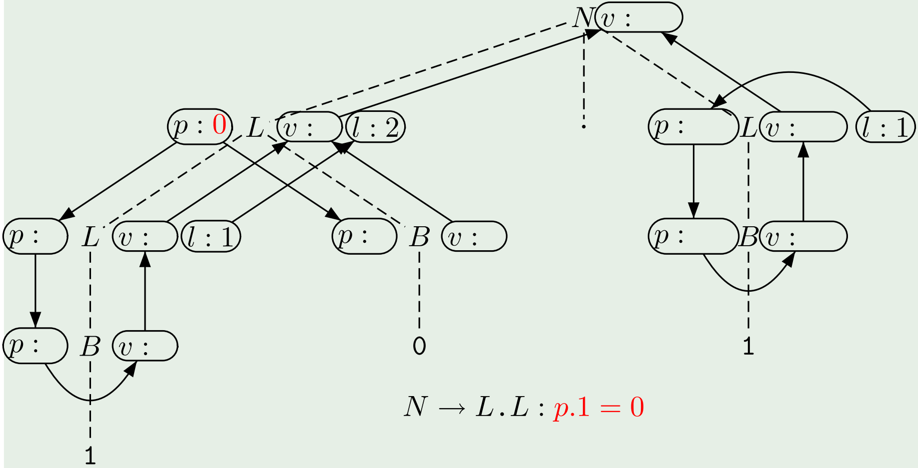

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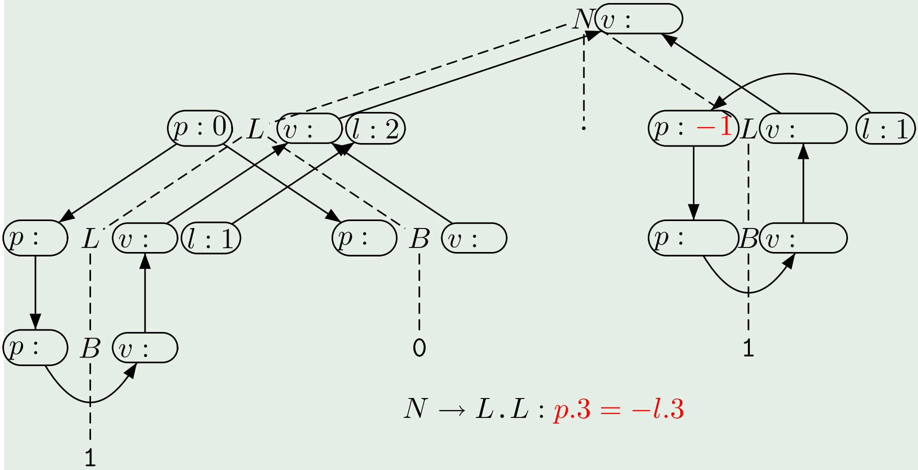

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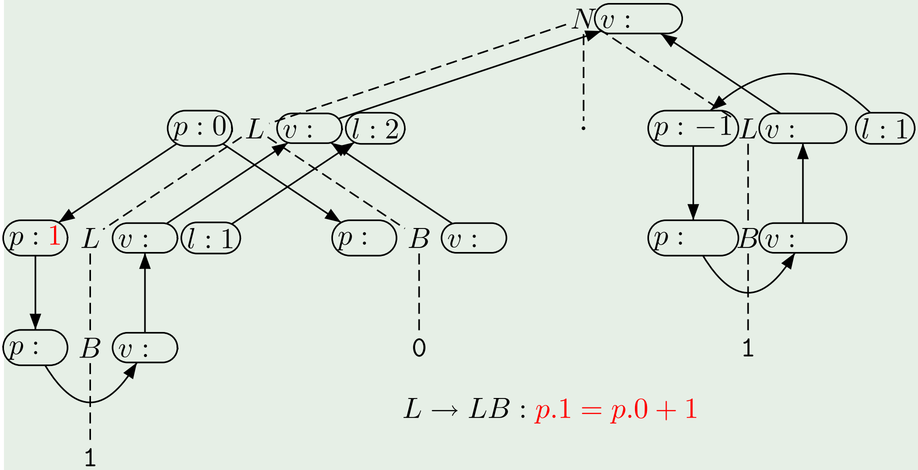

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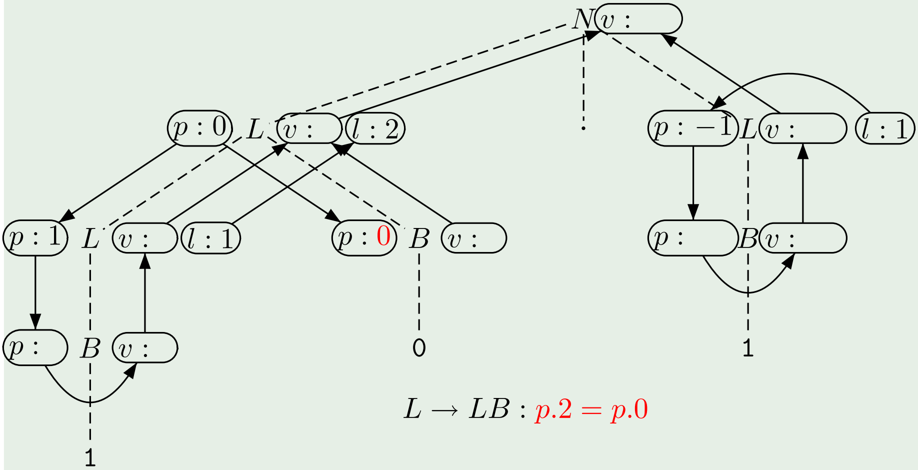

$L \rightarrow LB : p.2 = p.0$

Adding Inherited Attributes II

Example 13.2 (continued)

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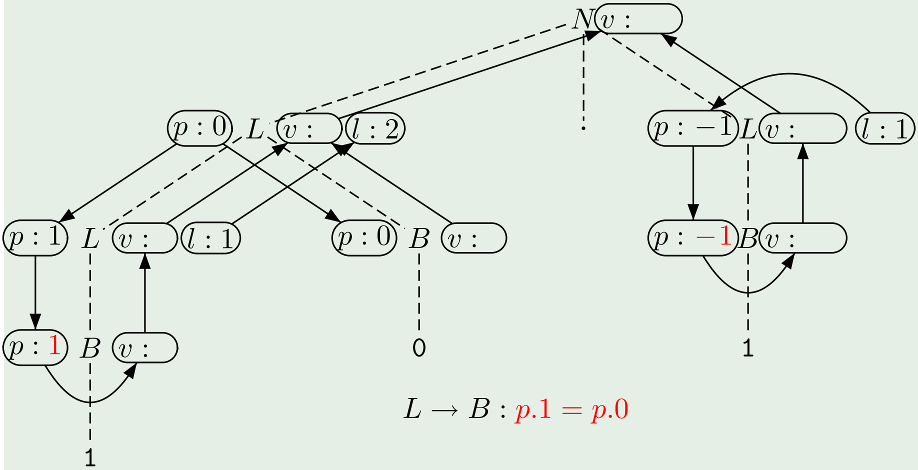

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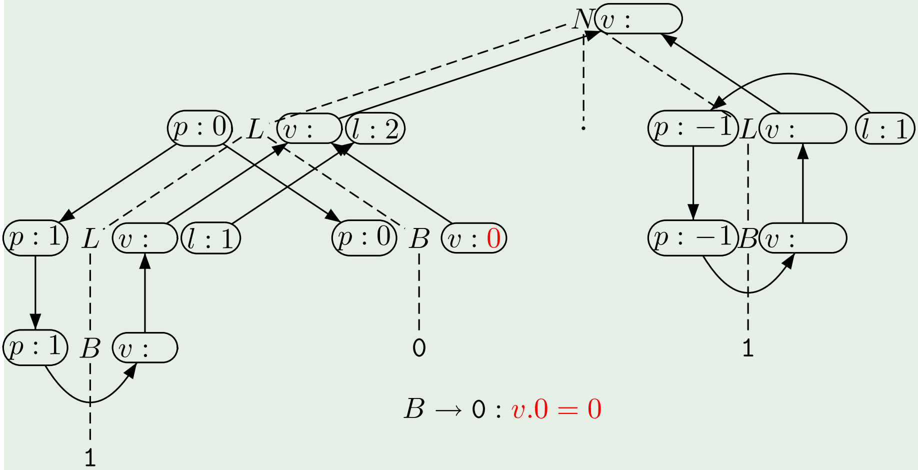

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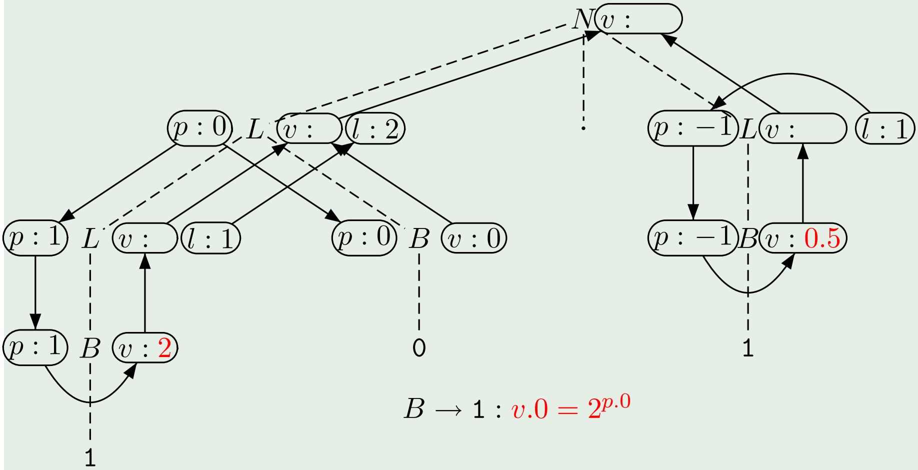

$B \rightarrow \mathtt{1} : v.0 = 2^{p.0}$

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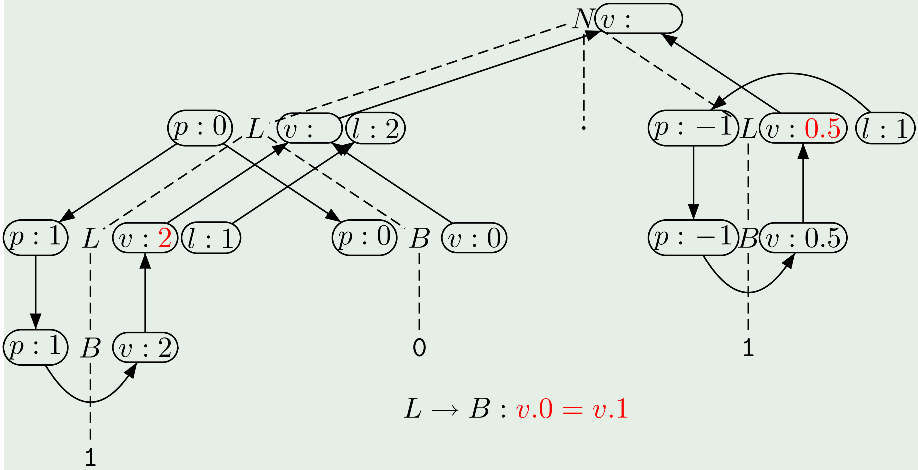

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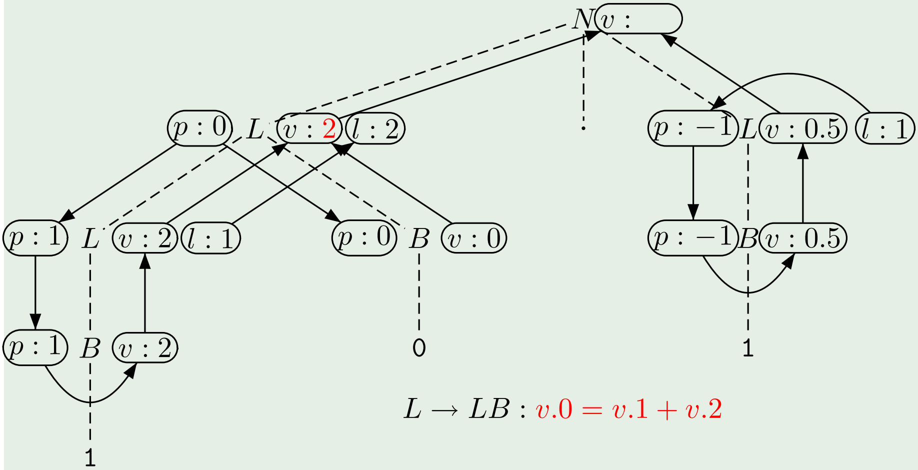

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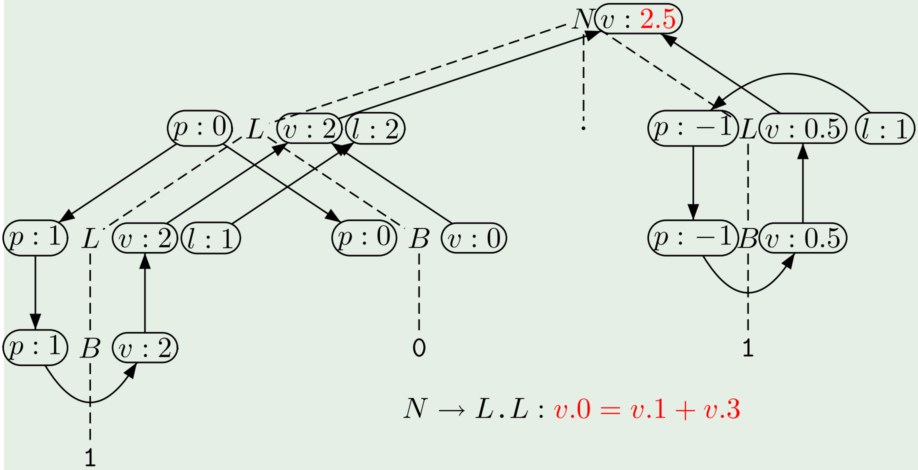

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Outline

Formal Definition of Attribute Grammars I

Definition 13.3 (Attribute grammar)

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Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.

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Definition 13.3 (Attribute grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.
- Let $\text{att} : X \to 2^{Att}$ be an attribute assignment, and let $\text{syn}(Y) := \text{att}(Y) \cap Syn$ and $\text{inh}(Y) := \text{att}(Y) \cap Inh$ for every $Y \in X$.

Formal Definition of Attribute Grammars I

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- Every production $\pi = Y_0 \rightarrow Y_1 \dots Y_r \in P$ determines the set
  $$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \dots, r\}\}$$
  of attribute variables of π with the subsets of inner and outer variables:
  $$\begin{aligned} In_\pi &:= \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\} \\ Out_\pi &:= Var_\pi \setminus In_\pi \end{aligned}$$

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where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \to V^\alpha$.

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Then $\mathfrak{A} := \langle G, E, V\rangle$ is called an attribute grammar: $\mathfrak{A} \in \mathit{AG}$.

Example 13.4 (cf. Example 13.2)

$\mathfrak{A}_B \in AG$ for binary numbers:

- Attributes: $Att = Syn \uplus Inh$ with $Syn = \{v, l\}$ and $Inh = \{p\}$

Formal Definition of Attribute Grammars II

Example 13.4 (cf. Example 13.2)

$\mathfrak{A}_B \in AG$ for binary numbers:

- Attributes: $Att = Syn \uplus Inh$ with $Syn = \{v, l\}$ and $Inh = \{p\}$
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- Attribute assignment:

$Y \in X$	N	L	B	0	1
syn(Y)	$\{v\}$	$\{v, l\}$	$\{v\}$	$\emptyset$	$\emptyset$
inh(Y)	$\emptyset$	$\{p\}$	$\{p\}$	$\emptyset$	$\emptyset$

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- Attribute variables:

$\pi \in P$	$N \to L$	$N \to L\,.\,L$	$L \to B$
In_π	$\{v.0, p.1\}$	$\{v.0, p.1, p.3\}$	$\{v.0, l.0, p.1\}$
Out_π	$\{v.1, l.1\}$	$\{v.1, l.1, v.3, l.3\}$	$\{v.1, p.0\}$

$\pi \in P$	$L \to LB$	$B \to \mathsf{0}$	$B \to \mathsf{1}$
In_π	$\{v.0, l.0, p.1, p.2\}$	$\{v.0\}$	$\{v.0\}$
Out_π	$\{v.1, v.2, l.1, p.0\}$	$\{p.0\}$	$\{p.0\}$

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Out_π	$\{v.1, v.2, l.1, p.0\}$	$\{p.0\}$	$\{p.0\}$

- Semantic rules: see Example 13.2
  (e.g., $E_{N \to L} = \{v.0 = v.1, p.1 = 0\}$)

Attribution of Syntax Trees I

Definition 13.5 (Attribution of syntax trees)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K.

- K determines the set of attribute variables of t:
$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \text{att}(Y)\}.$$

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- The attribute equation system of t is given by
$$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Corollary 13.6

For each $\alpha.k \in Var_t$ except the inherited attribute variables at the root and the synthesized attribute variables at the leafs of t, E_t contains exactly one equation with left-hand side $\alpha.k$.

Attribution of Syntax Trees II

Corollary 13.6

For each $\alpha.k \in Var_t$ *except the inherited attribute variables at the root and the synthesized attribute variables at the leafs of* t, E_t *contains exactly one equation with left-hand side* $\alpha.k$.

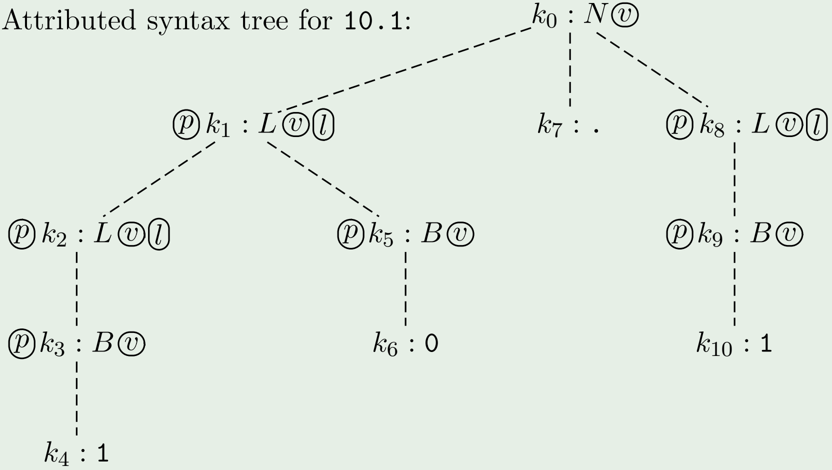
Assumptions:

- The start symbol does not have inherited attributes: $\text{inh}(S) = \emptyset$.
- Synthesized attributes of terminal symbols are provided by the scanner.

Attribution of Syntax Trees III

Example 13.7 (cf. Example 13.2)

Attributed syntax tree for `10.1`:



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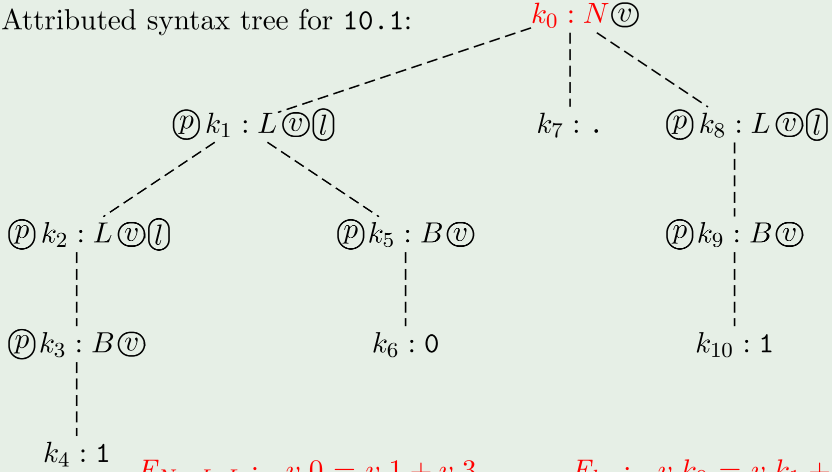

$$E_{N \to L.L}: \quad \begin{aligned} v.0 &= v.1 + v.3 \\ p.1 &= 0 \\ p.3 &= -l.3 \end{aligned} \quad \xrightarrow{\text{subst}} \quad E_{k_0}: \quad \begin{aligned} v.k_0 &= v.k_1 + v.k_8 \\ p.k_1 &= 0 \\ p.k_8 &= -l.k_8 \end{aligned}$$

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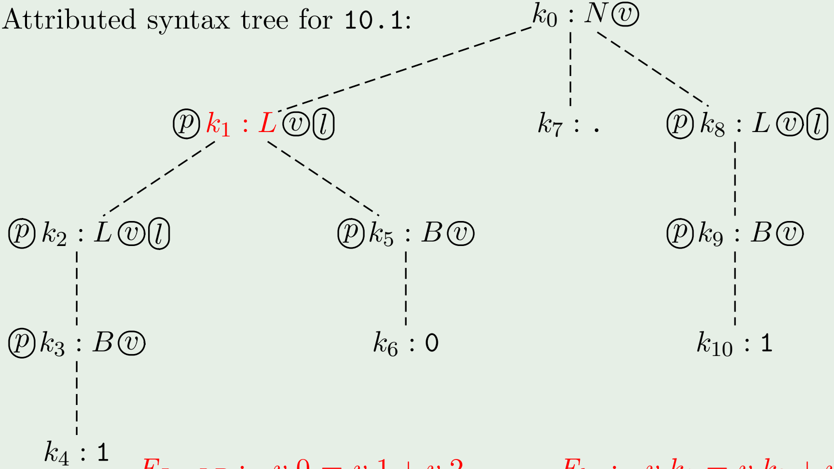

$$E_{L\to LB}: \quad \begin{aligned} v.0 &= v.1 + v.2 \\ l.0 &= l.1 + 1 \\ p.1 &= p.0 + 1 \\ p.2 &= p.0 \end{aligned} \quad \xrightarrow{\text{subst}} \quad E_{k_1}: \quad \begin{aligned} v.k_1 &= v.k_2 + v.k_5 \\ l.k_1 &= l.k_2 + 1 \\ p.k_2 &= p.k_1 + 1 \\ p.k_5 &= p.k_1 \end{aligned}$$