Compiler Construction

Lecture 9: Syntactic Analysis V ($LR(k)$ Grammars)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH Aachen University

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/cc08/

Summer semester 2008

Outline

Example

Grammar for
arithmetic expressions:

$$G_{AE}: \quad \begin{array}{lll} E \to E\texttt{+}T \mid T & & (1,2) \\ T \to T\texttt{*}F \mid F & & (3,4) \\ F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7) \end{array}$$

Example

E

Grammar for arithmetic expressions:

$$
\begin{aligned}
G_{AE}:\quad E &\rightarrow E\texttt{+}T \mid T & (1,2)\\
T &\rightarrow T\texttt{*}F \mid F & (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Leftmost analysis of `(a)*b`:

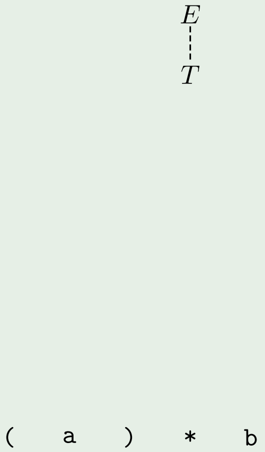
(a) * b

Example

Grammar for
arithmetic expressions:

$$G_{AE}: \begin{array}{lll} E \rightarrow E\texttt{+}T \mid T & (1,2) \\ T \rightarrow T\texttt{*}F \mid F & (3,4) \\ F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

Leftmost analysis of `(a)*b`:
2



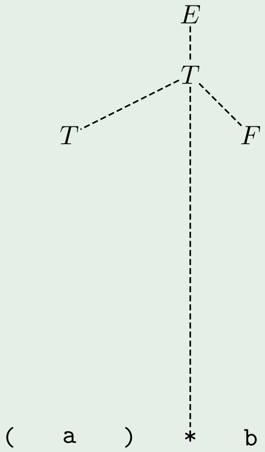
Example

Grammar for arithmetic expressions:

$$G_{AE}: \quad \begin{array}{lll} E \rightarrow E\texttt{+}T \mid T & (1,2) \\ T \rightarrow T\texttt{*}F \mid F & (3,4) \\ F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

Leftmost analysis of `(a)*b`:

2 3

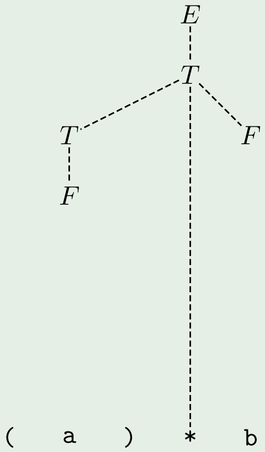


Example

Grammar for
arithmetic expressions:

$$
\begin{aligned}
G_{AE}: \quad E &\rightarrow E\texttt{+}T \mid T && (1,2)\\
T &\rightarrow T\texttt{*}F \mid F && (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Leftmost analysis of `(a)*b`:
2 3 4



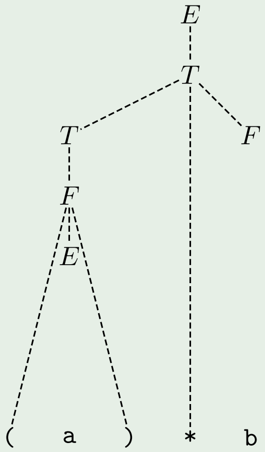
Example

Grammar for arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \to E\texttt{+}T \mid T & (1,2) \\
 & T \to T\texttt{*}F \mid F & (3,4) \\
 & F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

Leftmost analysis of `(a)*b`:

2 3 4 5



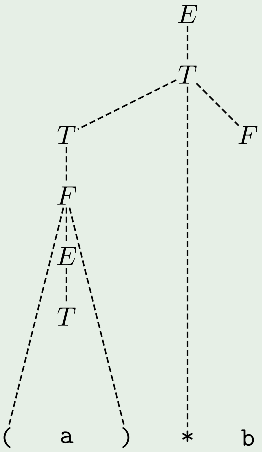
Top-Down Parsing

Example

Grammar for
arithmetic expressions:

$$
\begin{aligned}
G_{AE}: \quad E &\rightarrow E\texttt{+}T \mid T & (1,2)\\
T &\rightarrow T\texttt{*}F \mid F & (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Leftmost analysis of `(a)*b`:
2 3 4 5 2



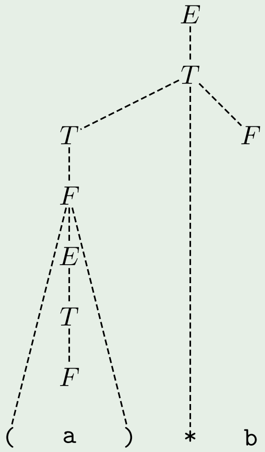
Top-Down Parsing

Example

Grammar for
arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Leftmost analysis of `(a)*b`:
2 3 4 5 2 4

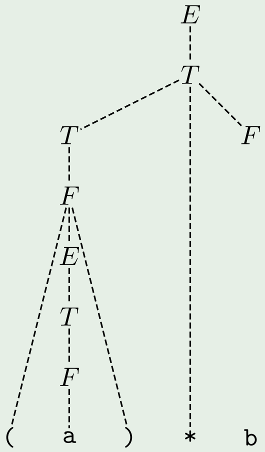


Example

Grammar for
arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Leftmost analysis of `(a)*b`:
2 3 4 5 2 4 6

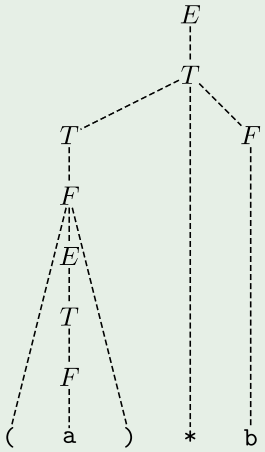


Example

Grammar for
arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

Leftmost analysis of `(a)*b`:
2 3 4 5 2 4 6 7



Outline

Example 9.1

Grammar for
arithmetic expressions:

$$
\begin{aligned}
G_{AE}: \quad & E \rightarrow E\texttt{+}T \mid T && (1,2)\\
& T \rightarrow T\texttt{*}F \mid F && (3,4)\\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Example 9.1

Grammar for arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

Reversed rightmost analysis of `(a)*b`:

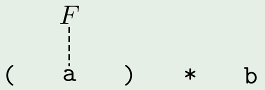
`(` `a` `)` `*` `b`

Example 9.1

Grammar for
arithmetic expressions:

$$G_{AE}: \quad \begin{array}{lll} E \rightarrow E\texttt{+}T \mid T & & (1,2) \\ T \rightarrow T\texttt{*}F \mid F & & (3,4) \\ F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7) \end{array}$$

Reversed rightmost analysis
of `(a)*b`:
6



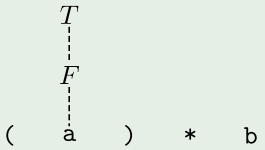
Example 9.1

Grammar for arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Reversed rightmost analysis of `(a)*b`:

6 4



Bottom-Up Parsing I

Example 9.1

Grammar for
arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

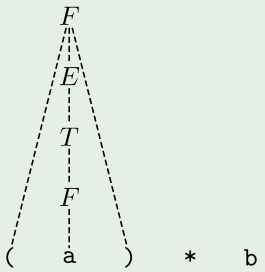
Reversed rightmost analysis
of `(a)*b`:
6 4 2

Example 9.1

Grammar for
arithmetic expressions:

$$
\begin{aligned}
G_{AE}: \quad & E \to E\texttt{+}T \mid T && (1,2)\\
& T \to T\texttt{*}F \mid F && (3,4)\\
& F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Reversed rightmost analysis
of `(a)*b`:
6 4 2 5

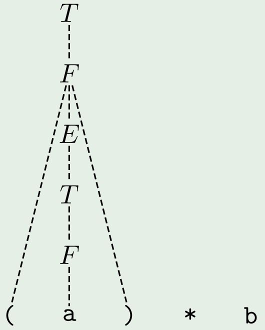


Example 9.1

Grammar for
arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
 & T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
 & F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Reversed rightmost analysis
of `(a)*b`:
6 4 2 5 4



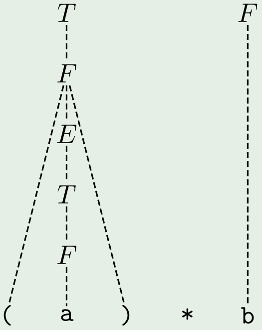
Bottom-Up Parsing I

Example 9.1

Grammar for
arithmetic expressions:

$$G_{AE}: \quad \begin{array}{lcll} E & \rightarrow & E\texttt{+}T \mid T & (1,2) \\ T & \rightarrow & T\texttt{*}F \mid F & (3,4) \\ F & \rightarrow & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

Reversed rightmost analysis
of `(a)*b`:
6 4 2 5 4 7



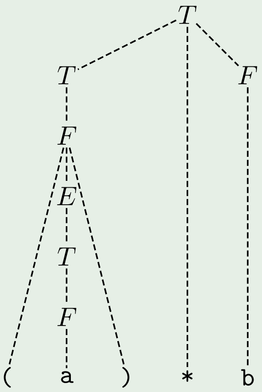
Example 9.1

Grammar for arithmetic expressions:

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Reversed rightmost analysis of `(a)*b`:

6 4 2 5 4 7 3



Bottom-Up Parsing I

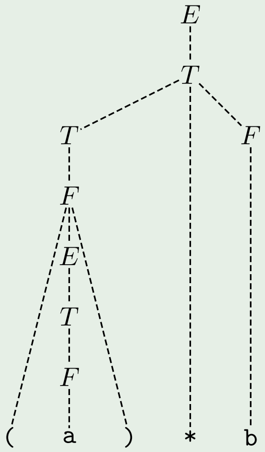
Example 9.1

Grammar for arithmetic expressions:

$$
\begin{aligned}
G_{AE}: \quad E &\rightarrow E\texttt{+}T \mid T & (1,2)\\
T &\rightarrow T\texttt{*}F \mid F & (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Reversed rightmost analysis of `(a)*b`:

6 4 2 5 4 7 3 2



Bottom-Up Parsing II

Approach:

1. Given $G \in CFG_\Sigma$, construct a nondeterministic bottom-up parsing automaton (NBA) which accepts $L(G)$ and which additionally computes corresponding (reversed) rightmost analyses
   - input alphabet: Σ
   - pushdown alphabet: X
   - output alphabet: $[p]$ (where $p := |P|$)
   - state set: omitted
   - transitions:

     shift: shifting input symbols onto the pushdown

     reduce: replacing the right-hand side of a production by its left-hand side (= inverse expansion steps)

Bottom-Up Parsing II

Approach:

1. Given $G \in CFG_\Sigma$, construct a nondeterministic bottom-up parsing automaton (NBA) which accepts $L(G)$ and which additionally computes corresponding (reversed) rightmost analyses
   - input alphabet: Σ
   - pushdown alphabet: X
   - output alphabet: $[p]$ (where $p := |P|$)
   - state set: omitted
   - transitions:

     shift: shifting input symbols onto the pushdown

     reduce: replacing the right-hand side of a production by its left-hand side (= inverse expansion steps)

2. Remove nondeterminism by allowing lookahead on the input: $G \in LR(k)$ iff $L(G)$ recognizable by deterministic bottom-up parsing automaton with lookahead of k symbols

Outline

1. Repetition: Top-Down Parsing
2. Bottom-Up Parsing
3. Nondeterministic Bottom-Up Parsing
4. $LR(k)$ Grammars
5. $LR(0)$ Grammars

Nondeterministic Bottom-Up Automaton I

Definition 9.2 (Nondeterministic bottom-up parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$. The nondeterministic bottom-up parsing automaton of G, NBA(G), is defined by the following components.

- Input alphabet: Σ
- Pushdown alphabet: X
- Output alphabet: $[p]$
- Configurations: $\Sigma^* \times X^* \times [p]^*$ (top of pushdown to the right)
- Transitions for $w \in \Sigma^*$, $\alpha \in X^*$, and $z \in [p]^*$:
  shifting steps: $(aw, \alpha, z) \vdash (w, \alpha a, z)$ if $a \in \Sigma$
  reduction steps: $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ if $\pi(i) = A \to \beta$
- Initial configuration for $w \in \Sigma^*$: $(w, \varepsilon, \varepsilon)$
- Final configurations: $\{\varepsilon\} \times \{S\} \times [p]^*$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for
arithmetic expressions
(cf. Example 5.11):

$$
\begin{aligned}
G_{AE}: E &\to E\texttt{+}T \mid T && (1,2)\\
T &\to T\texttt{*}F \mid F && (3,4)\\
F &\to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE} : E &\rightarrow E\texttt{+}T \mid T && (1,2)\\
T &\rightarrow T\texttt{*}F \mid F && (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$(\texttt{(a)*b}, \varepsilon \quad , \varepsilon \qquad)$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE}: E &\to E\texttt{+}T \mid T & (1,2)\\
T &\to T\texttt{*}F \mid F & (3,4)\\
F &\to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{aligned}
& (\texttt{(a)*b}, \varepsilon, \varepsilon)\\
\vdash\ & (\texttt{a)*b}, \texttt{(}, \varepsilon)
\end{aligned}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$G_{AE} : \begin{array}{lll} E \rightarrow E\texttt{+}T \mid T & & (1,2) \\ T \rightarrow T\texttt{*}F \mid F & & (3,4) \\ F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7) \end{array}$$

Bottom-up parsing of `(a)*b`:

$$\begin{array}{ll} & (\texttt{(a)*b}, \varepsilon, \varepsilon) \\ \vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\ \vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \end{array}$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$G_{AE}: \begin{array}{lcll} E & \rightarrow & E\texttt{+}T \mid T & (1,2) \\ T & \rightarrow & T\texttt{*}F \mid F & (3,4) \\ F & \rightarrow & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

Bottom-up parsing of `(a)*b`:

$$\begin{array}{ll} & (\texttt{(a)*b}, \varepsilon, \varepsilon) \\ \vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\ \vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \\ \vdash & (\texttt{)*b}, \texttt{(}F, 6) \end{array}$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$G_{AE} : \begin{array}{rcll} {\color{red}E} & \to & E\texttt{+}T \mid {\color{red}T} & (1,2) \\ T & \to & T\texttt{*}F \mid F & (3,4) \\ F & \to & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

Bottom-up parsing of `(a)*b`:

$$\begin{array}{ll} & (\texttt{(a)*b}, \varepsilon, \varepsilon) \\ \vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\ \vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \\ \vdash & (\texttt{)*b}, \texttt{(}F, 6) \\ \vdash & (\texttt{)*b}, \texttt{(}{\color{red}T}, 64) \end{array}$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{array}{rlll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{array}{rl}
 & (\texttt{(a)*b}, \varepsilon, \varepsilon) \\
\vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(}F, 6) \\
\vdash & (\texttt{)*b}, \texttt{(}T, 64) \\
\vdash & (\texttt{)*b}, \texttt{(}E, 642)
\end{array}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE}: E &\rightarrow E\texttt{+}T \mid T & (1,2)\\
T &\rightarrow T\texttt{*}F \mid F & (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{aligned}
& (\texttt{(a)*b}, \varepsilon, \varepsilon)\\
\vdash\ & (\texttt{a)*b}, \texttt{(}, \varepsilon)\\
\vdash\ & (\texttt{)*b}, \texttt{(a}, \varepsilon)\\
\vdash\ & (\texttt{)*b}, \texttt{(}F, 6)\\
\vdash\ & (\texttt{)*b}, \texttt{(}T, 64)\\
\vdash\ & (\texttt{)*b}, \texttt{(}E, 642)\\
\vdash\ & (\texttt{*b}, \texttt{(}E\texttt{)}, 642)
\end{aligned}
$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE}: E &\rightarrow E\texttt{+}T \mid T && (1,2)\\
{\color{red}T} &\rightarrow T\texttt{*}F \mid {\color{red}F} && (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{aligned}
&(\texttt{(a)*b}, \varepsilon, \varepsilon)\\
\vdash &(\texttt{a)*b}, \texttt{(}, \varepsilon)\\
\vdash &(\texttt{)*b}, \texttt{(a}, \varepsilon)\\
\vdash &(\texttt{)*b}, \texttt{(}F, 6)\\
\vdash &(\texttt{)*b}, \texttt{(}T, 64)\\
\vdash &(\texttt{)*b}, \texttt{(}E, 642)\\
\vdash &(\texttt{*b}, \texttt{(}E\texttt{)}, 642)\\
\vdash &(\texttt{*b}, {\color{red}F}, 6425)
\end{aligned}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{array}{lllll}
& (\texttt{(a)*b}, & \varepsilon & , \varepsilon &) \\
\vdash & (\texttt{a)*b}, & \texttt{(} & , \varepsilon &) \\
\vdash & (\texttt{)*b}, & \texttt{(a} & , \varepsilon &) \\
\vdash & (\texttt{)*b}, & \texttt{(}F & , 6 &) \\
\vdash & (\texttt{)*b}, & \texttt{(}T & , 64 &) \\
\vdash & (\texttt{)*b}, & \texttt{(}E & , 642 &) \\
\vdash & (\texttt{*b}, & \texttt{(}E\texttt{)} & , 642 &) \\
\vdash & (\texttt{*b}, & F & , 6425 &) \\
\vdash & (\texttt{*b}, & T & , 64254 &)
\end{array}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$G_{AE}: \begin{array}{lll} E \to E\texttt{+}T \mid T & & (1,2) \\ T \to T\texttt{*}F \mid F & & (3,4) \\ F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7) \end{array}$$

Bottom-up parsing of `(a)*b`:

$$\begin{array}{rlll}
 & (\texttt{(a)*b}, & \varepsilon & , \varepsilon) \\
\vdash & (\texttt{a)*b}, & \texttt{(} & , \varepsilon) \\
\vdash & (\texttt{)*b}, & \texttt{(a} & , \varepsilon) \\
\vdash & (\texttt{)*b}, & \texttt{(}F & , 6) \\
\vdash & (\texttt{)*b}, & \texttt{(}T & , 64) \\
\vdash & (\texttt{)*b}, & \texttt{(}E & , 642) \\
\vdash & (\texttt{*b}, & \texttt{(}E\texttt{)} & , 642) \\
\vdash & (\texttt{*b}, & F & , 6425) \\
\vdash & (\texttt{*b}, & T & , 64254) \\
\vdash & (\texttt{b}, & T\texttt{*} & , 64254)
\end{array}$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7)
\end{array}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{array}{ll}
& (\texttt{(a)*b}, \varepsilon, \varepsilon) \\
\vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(}F, 6) \\
\vdash & (\texttt{)*b}, \texttt{(}T, 64) \\
\vdash & (\texttt{)*b}, \texttt{(}E, 642) \\
\vdash & (\texttt{*b}, \texttt{(}E\texttt{)}, 642) \\
\vdash & (\texttt{*b}, F, 6425) \\
\vdash & (\texttt{*b}, T, 64254) \\
\vdash & (\texttt{b}, T\texttt{*}, 64254) \\
\vdash & (\varepsilon, T\texttt{*b}, 64254)
\end{array}
$$

Nondeterministic Bottom-Up Automaton II

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{array}{llll}
G_{AE}: & E \rightarrow E\texttt{+}T \mid T & (1,2) \\
& T \rightarrow T\texttt{*}F \mid F & (3,4) \\
& F \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

Bottom-up parsing of (a)*b:

$$
\begin{array}{ll}
& (\texttt{(a)*b}, \varepsilon, \varepsilon) \\
\vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon) \\
\vdash & (\texttt{)*b}, \texttt{(}F, 6) \\
\vdash & (\texttt{)*b}, \texttt{(}T, 64) \\
\vdash & (\texttt{)*b}, \texttt{(}E, 642) \\
\vdash & (\texttt{*b}, \texttt{(}E\texttt{)}, 642) \\
\vdash & (\texttt{*b}, F, 6425) \\
\vdash & (\texttt{*b}, T, 64254) \\
\vdash & (\texttt{b}, T\texttt{*}, 64254) \\
\vdash & (\varepsilon, T\texttt{*b}, 64254) \\
\vdash & (\varepsilon, T\texttt{*}F, 642547)
\end{array}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE} : E &\to E\texttt{+}T \mid T && (1,2)\\
T &\to T\texttt{*}F \mid F && (3,4)\\
F &\to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} && (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{aligned}
& (\texttt{(a)*b}, \varepsilon, \varepsilon)\\
\vdash\ & (\texttt{a)*b}, \texttt{(}, \varepsilon)\\
\vdash\ & (\texttt{)*b}, \texttt{(a}, \varepsilon)\\
\vdash\ & (\texttt{)*b}, \texttt{(}F, 6)\\
\vdash\ & (\texttt{)*b}, \texttt{(}T, 64)\\
\vdash\ & (\texttt{)*b}, \texttt{(}E, 642)\\
\vdash\ & (\texttt{*b}, \texttt{(}E\texttt{)}, 642)\\
\vdash\ & (\texttt{*b}, F, 6425)\\
\vdash\ & (\texttt{*b}, T, 64254)\\
\vdash\ & (\texttt{b}, T\texttt{*}, 64254)\\
\vdash\ & (\varepsilon, T\texttt{*b}, 64254)\\
\vdash\ & (\varepsilon, T\texttt{*}F, 642547)\\
\vdash\ & (\varepsilon, T, 6425473)
\end{aligned}
$$

Example 9.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{aligned}
G_{AE} : E &\rightarrow E\texttt{+}T \mid T & (1,2)\\
T &\rightarrow T\texttt{*}F \mid F & (3,4)\\
F &\rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{aligned}
$$

Bottom-up parsing of `(a)*b`:

$$
\begin{array}{ll}
 & (\texttt{(a)*b}, \varepsilon, \varepsilon)\\
\vdash & (\texttt{a)*b}, \texttt{(}, \varepsilon)\\
\vdash & (\texttt{)*b}, \texttt{(a}, \varepsilon)\\
\vdash & (\texttt{)*b}, \texttt{(}F, 6)\\
\vdash & (\texttt{)*b}, \texttt{(}T, 64)\\
\vdash & (\texttt{)*b}, \texttt{(}E, 642)\\
\vdash & (\texttt{*b}, \texttt{(}E\texttt{)}, 642)\\
\vdash & (\texttt{*b}, F, 6425)\\
\vdash & (\texttt{*b}, T, 64254)\\
\vdash & (\texttt{b}, T\texttt{*}, 64254)\\
\vdash & (\varepsilon, T\texttt{*b}, 64254)\\
\vdash & (\varepsilon, T\texttt{*}F, 642547)\\
\vdash & (\varepsilon, T, 6425473)\\
\vdash & (\varepsilon, E, 64254732)
\end{array}
$$

Correctness of NBA(G)

Theorem 9.4 (Correctness of NBA(G))

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ *and* NBA(G) *as before. Then, for every* $w \in \Sigma^*$ *and* $z \in [p]^*$,

$$(w, \varepsilon, \varepsilon) \vdash^* (\varepsilon, S, z) \quad \textit{iff} \quad \overleftarrow{z} \textit{ is a rightmost analysis of } w$$

Correctness of NBA(G)

Theorem 9.4 (Correctness of NBA(G))

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ *and* NBA(G) *as before. Then, for every* $w \in \Sigma^*$ *and* $z \in [p]^*$,

$$(w, \varepsilon, \varepsilon) \vdash^* (\varepsilon, S, z) \quad \textit{iff} \quad \overleftarrow{z} \textit{ is a rightmost analysis of } w$$

Proof.

similar to the top-down case (Theorem 5.15) □

Nondeterminisn in NTA(G)

Remark: NTA(G) is generally nondeterministic

- Shift or reduce? Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{ if } \pi(i) = A \to a$$

Nondeterminisn in NTA(G)

Remark: NTA(G) is generally nondeterministic

- Shift or reduce? Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{ if } \pi(i) = A \to a$$

- If reduce: which “handle” β? Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \text{ if } \pi(i) = A \to ab \text{ and } \pi(j) = B \to b$$

Nondeterminisn in NTA(G)

Remark: NTA(G) is generally nondeterministic

- Shift or reduce? Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{if } \pi(i) = A \to a$$

- If reduce: which "handle" β? Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \text{if } \pi(i) = A \to ab \text{ and } \pi(j) = B \to b$$

- If reduce β: which left-hand side A? Example:

$$(w, \alpha a, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha B, zj) \end{cases} \text{if } \pi(i) = A \to a \text{ and } \pi(j) = B \to a$$

Nondeterminisn in NTA(G)

Remark: NTA(G) is generally nondeterministic

- Shift or reduce? Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{if } \pi(i) = A \to a$$

- If reduce: which “handle” β? Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \text{if } \pi(i) = A \to ab \text{ and } \pi(j) = B \to b$$

- If reduce β: which left-hand side A? Example:

$$(w, \alpha a, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha B, zj) \end{cases} \text{if } \pi(i) = A \to a \text{ and } \pi(j) = B \to a$$

- When to terminate parsing? Example:

$$\underbrace{(\varepsilon, S, z)}_{\text{final}} \vdash (\varepsilon, A, zi) \text{ if } \pi(i) = A \to S$$

Resolving Termination Nondeterminism I

General assumption in the following: every grammar is start separated

Definition 9.5 (Start separation)

A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ is called start separated if S only occurs in productions of the form $S \to A$ where $A \neq S$.

Resolving Termination Nondeterminism I

General assumption in the following: every grammar is start separated

Definition 9.5 (Start separation)

A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ is called start separated if S only occurs in productions of the form $S \to A$ where $A \neq S$.

Remarks:

- Start separation always possible by adding $S' \to S$ with new start symbol S'
- From now on consider only reduced grammars of this form ($\pi(0) = S' \to S$)

Resolving Termination Nondeterminism II

Start separation removes last form of nondeterminism (when to terminate parsing):

Corollary 9.6

If $G \in CFG_\Sigma$ *is start separated, then no successor of a final configuration* (ε, S', z) *in* $\mathrm{NBA}(G)$ *is again a final configuration. (Thus parsing should be stopped in the first final configuration.)*

Resolving Termination Nondeterminism II

Start separation removes last form of nondeterminism (when to terminate parsing):

Corollary 9.6

If $G \in CFG_\Sigma$ is start separated, then no successor of a final configuration (ε, S', z) in $\mathrm{NBA}(G)$ is again a final configuration. (Thus parsing should be stopped in the first final configuration.)

Corollary 9.7

- *To (ε, S', z), only reductions by ε-productions can be applied:*

$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \textit{if } \pi(i) = A \to \varepsilon$$

Resolving Termination Nondeterminism II

Start separation removes last form of nondeterminism (when to terminate parsing):

Corollary 9.6

If $G \in CFG_\Sigma$ is start separated, then no successor of a final configuration (ε, S', z) in $\mathrm{NBA}(G)$ is again a final configuration. (Thus parsing should be stopped in the first final configuration.)

Corollary 9.7

- *To (ε, S', z), only reductions by ε-productions can be applied:*
$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \text{if } \pi(i) = A \rightarrow \varepsilon$$
- *Thereafter, only reductions by productions of the form $A_0 \rightarrow A_1 \ldots A_n$ $(n \geq 0)$ can be applied*

Resolving Termination Nondeterminism II

Start separation removes last form of nondeterminism (when to terminate parsing):

Corollary 9.6

If $G \in CFG_\Sigma$ is start separated, then no successor of a final configuration (ε, S', z) in $\mathrm{NBA}(G)$ is again a final configuration. (Thus parsing should be stopped in the first final configuration.)

Corollary 9.7

- *To (ε, S', z), only reductions by ε-productions can be applied:*
$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \textit{if } \pi(i) = A \to \varepsilon$$
- *Thereafter, only reductions by productions of the form $A_0 \to A_1 \dots A_n$ $(n \geq 0)$ can be applied*
- *Every resulting configuration is of the (non-final) form*
$$(\varepsilon, S'B_1 \dots B_k, z) \quad \textit{where } k \geq 1$$

Outline

$LR(k)$ Grammars

Goal: resolve remaining nondeterminism of NBA(G) by supporting lookahead of $k \in \mathbb{N}$ symbols on the input

$\Longrightarrow$ $LR(k)$: reading of input from left to right with k-lookahead, computing a rightmost analysis

$LR(k)$ Grammars

Goal: resolve remaining nondeterminism of NBA(G) by supporting lookahead of $k \in \mathbb{N}$ symbols on the input

$\Longrightarrow$ $LR(k)$: reading of input from left to right with k-lookahead, computing a rightmost analysis

Definition 9.8 ($LR(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated and $k \in \mathbb{N}$. Then G has the $LR(k)$ property (notation: $G \in LR(k)$) if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \ \alpha A w \ \Rightarrow_r \ \alpha\beta w \\ \Rightarrow_r^* \ \alpha' A' w' \ \Rightarrow_r \ \alpha\beta v \end{cases}$$

such that $\text{first}_k(w) = \text{first}_k(v)$, it follows that $\alpha' = \alpha$, $A' = A$, and $w' = v$.

$LR(k)$ Grammars

Goal: resolve remaining nondeterminism of NBA(G) by supporting lookahead of $k \in \mathbb{N}$ symbols on the input

$\Longrightarrow$ $LR(k)$: reading of input from left to right with k-lookahead, computing a rightmost analysis

Definition 9.8 ($LR(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated and $k \in \mathbb{N}$. Then G has the $LR(k)$ property (notation: $G \in LR(k)$) if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \ \alpha A w \ \Rightarrow_r \ \alpha\beta w \\ \Rightarrow_r^* \ \alpha' A' w' \ \Rightarrow_r \ \alpha\beta v \end{cases}$$

such that $\mathrm{first}_k(w) = \mathrm{first}_k(v)$, it follows that $\alpha' = \alpha$, $A' = A$, and $w' = v$.

Remarks:

- If $G \in LR(k)$, then the reduction of $\alpha\beta w$ to $\alpha A w$ is already determined by $\alpha\beta\, \mathrm{first}_k(w)$.
- Therefore NBA(G) in configuration $(w, \alpha\beta, z)$ can decide whether to shift or to reduce and, in the second case, how to reduce.

Outline

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

Corollary 9.9 ($LR(0)$ grammar)

$G \in CFG_\Sigma$ has the $LR(0)$ property if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \ \alpha A w \ \Rightarrow_r \ \alpha\beta w \\ \Rightarrow_r^* \ \alpha' A' w' \ \Rightarrow_r \ \alpha\beta v \end{cases}$$

it follows that $\alpha' = \alpha$, $A' = A$, and $w' = v$.

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

Corollary 9.9 ($LR(0)$ grammar)

$G \in CFG_\Sigma$ has the $LR(0)$ property if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* & \alpha A w & \Rightarrow_r \alpha\beta w \\ \Rightarrow_r^* & \alpha' A' w' & \Rightarrow_r \alpha\beta v \end{cases}$$

it follows that $\alpha' = \alpha$, $A' = A$, and $w' = v$.

Goal: derive a finite information from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \rightarrow \beta_1\beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 9.11

1. *For every* $\gamma \in X^*$, $LR(0)(\gamma)$ *is finite.*

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 9.11

1. *For every* $\gamma \in X^*$, $LR(0)(\gamma)$ *is finite.*
2. $LR(0)(G)$ *is finite.*

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 9.11

1. *For every* $\gamma \in X^*$, $LR(0)(\gamma)$ *is finite.*
2. $LR(0)(G)$ *is finite.*
3. *The item* $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$ *indicates the possible reduction* $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ *where* $\pi(i) = A \rightarrow \beta$ *and* $\gamma = \alpha\beta$.

$LR(0)$ Items and Sets

Definition 9.10 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 9.11

1. *For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.*
2. *$LR(0)(G)$ is finite.*
3. *The item $[A \to \beta \cdot] \in LR(0)(\gamma)$ indicates the possible reduction $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ where $\pi(i) = A \to \beta$ and $\gamma = \alpha\beta$.*
4. *The item $[A \to \beta_1 \cdot Y\beta_2] \in LR(0)(\gamma)$ indicates a possible shift step (with incomplete handle β_1).*

$LR(0)$ Conflicts

Definition 9.12 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a \alpha_2], [B \to \beta \cdot] \in I.$$

$LR(0)$ Conflicts

Definition 9.12 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that

$$[A \rightarrow \alpha_1 \cdot a\alpha_2], [B \rightarrow \beta\cdot] \in I.$$

- I has a reduce/reduce conflict if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha\cdot], [B \rightarrow \beta\cdot] \in I.$$

$LR(0)$ Conflicts

Definition 9.12 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that
$$[A \rightarrow \alpha_1 \cdot a\alpha_2], [B \rightarrow \beta \cdot] \in I.$$
- I has a reduce/reduce conflict if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

Lemma 9.13

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Computing $LR(0)$ Sets I

Theorem 9.14 (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. *$LR(0)(\varepsilon)$ is the least set such that*
   - *$[S' \to \cdot S] \in LR(0)(\varepsilon)$ and*
   - *if $[A \to \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \to \beta \in P$, then $[B \to \cdot\beta] \in LR(0)(\varepsilon)$.*

Computing $LR(0)$ Sets I

Theorem 9.14 (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \rightarrow S$ and reduced.

1. *$LR(0)(\varepsilon)$ is the least set such that*
   - *$[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and*
   - *if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$, then $[B \rightarrow \cdot\beta] \in LR(0)(\varepsilon)$.*
2. *$LR(0)(\alpha Y)$ ($\alpha \in X^*, Y \in X$) is the least set such that*
   - *if $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$, then $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ and*
   - *if $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ and $B \rightarrow \beta \in P$, then $[B \rightarrow \cdot\beta] \in LR(0)(\alpha Y)$.*

Computing $LR(0)$ Sets II

Example 9.15

$$
G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}
$$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$[S' \to \cdot S] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon):$ $[S' \to \cdot S]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lcl} S' & \rightarrow & S \\ S & \rightarrow & B \mid C \\ B & \rightarrow & aB \mid b \\ C & \rightarrow & aC \mid c \end{array}$$

$$[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$$
$$\Longrightarrow [B \rightarrow \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon): \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$$
$$\implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon): \quad [S' \to \cdot S] \quad [S \to \cdot B] \quad [S \to \cdot C]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P \implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P \implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon)$:

$[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$
$[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon):$ $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$
$[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S):$ $[S' \to S\cdot]$

Example 9.15

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

Computing $LR(0)$ Sets II

Example 9.15

$$
\begin{array}{llll}
G: & S' & \to & S \\
 & S & \to & B \mid C \\
 & B & \to & aB \mid b \\
 & C & \to & aC \mid c
\end{array}
$$

$$
[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)
$$
$$
\Longrightarrow\ [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)
$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example 9.15

$$
\begin{aligned}
G:\quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$$
$$\Longrightarrow\ [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lll} S' & \rightarrow & S \\ S & \rightarrow & B \mid C \\ B & \rightarrow & aB \mid b \\ C & \rightarrow & aC \mid c \end{array}$$

$$[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$$
$$\Longrightarrow [B \rightarrow \cdot\beta] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S)$: $[S' \rightarrow S\cdot]$

$I_2 := LR(0)(B)$: $[S \rightarrow B\cdot]$

$I_3 := LR(0)(C)$: $[S \rightarrow C\cdot]$

$I_4 := LR(0)(a)$: $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$

Computing $LR(0)$ Sets II

Example 9.15

$$\begin{aligned} G: \quad S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \to \beta \in P$$
$$\implies [B \to \cdot\beta] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b\cdot]$

Computing $LR(0)$ Sets II

Example 9.15

$$\begin{array}{llll} G: & S' & \to & S \\ & S & \to & B \mid C \\ & B & \to & aB \mid b \\ & C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$:	$[S' \to \cdot S]$	$[S \to \cdot B]$	$[S \to \cdot C]$	$[B \to \cdot aB]$
	$[B \to \cdot b]$	$[C \to \cdot aC]$	$[C \to \cdot c]$	
$I_1 := LR(0)(S)$:	$[S' \to S\cdot]$			
$I_2 := LR(0)(B)$:	$[S \to B\cdot]$			
$I_3 := LR(0)(C)$:	$[S \to C\cdot]$			
$I_4 := LR(0)(a)$:	$[B \to a \cdot B]$	$[C \to a \cdot C]$	$[B \to \cdot aB]$	$[B \to \cdot b]$
	$[C \to \cdot aC]$	$[C \to \cdot c]$		
$I_5 := LR(0)(b)$:	$[B \to b\cdot]$			
$I_6 := LR(0)(c)$:	$[C \to c\cdot]$			

Computing $LR(0)$ Sets II

Example 9.15

$$
\begin{aligned}
G:\quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b\cdot]$

$I_6 := LR(0)(c)$: $[C \to c\cdot]$

$I_7 := LR(0)(aB)$: $[B \to aB\cdot]$

Computing $LR(0)$ Sets II

Example 9.15

$$\begin{aligned} G: \quad S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b \cdot]$

$I_6 := LR(0)(c)$: $[C \to c \cdot]$

$I_7 := LR(0)(aB)$: $[B \to aB \cdot]$

$I_8 := LR(0)(aC)$: $[C \to aC \cdot]$

Computing $LR(0)$ Sets II

Example 9.15

$$G: \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & & &
\end{array}$$

($LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$,
$LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

Computing $LR(0)$ Sets II

Example 9.15

$$\begin{aligned} G: \quad S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$\begin{array}{llllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & & &
\end{array}$$

($LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$,
$LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

no conflicts $\implies G \in LR(0)$