Compiler Construction

Lecture 11: Syntactic Analysis VI ($LR(0)$ Parsing)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH Aachen University

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/cc10/

Winter semester 2010/11

Outline

1. Repetition: Nondeterministic Bottom-Up Parsing
2. $LR(0)$ Grammars
3. $LR(0)$ Parsing

Definition (Nondeterministic bottom-up parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$. The nondeterministic bottom-up parsing automaton of G, $\mathrm{NBA}(G)$, is defined by the following components.

- Input alphabet: Σ
- Pushdown alphabet: X
- Output alphabet: $[p]$
- Configurations: $\Sigma^* \times X^* \times [p]^*$ (top of pushdown to the right)
- Transitions for $w \in \Sigma^*$, $\alpha \in X^*$, and $z \in [p]^*$:
  shifting steps: $(aw, \alpha, z) \vdash (w, \alpha a, z)$ if $a \in \Sigma$
  reduction steps: $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ if $\pi_i = A \to \beta$
- Initial configuration for $w \in \Sigma^*$: $(w, \varepsilon, \varepsilon)$
- Final configurations: $\{\varepsilon\} \times \{S\} \times [p]^*$

Nondeterminisn in NBA(G)

Remark: NBA(G) is generally nondeterministic

- Shift or reduce? Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{ if } \pi_i = A \to a$$

- If reduce: which "handle" β? Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \text{ if } \pi_i = A \to ab \text{ and } \pi_j = B \to b$$

- If reduce β: which left-hand side A? Example:

$$(w, \alpha a, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha B, zj) \end{cases} \text{ if } \pi_i = A \to a \text{ and } \pi_j = B \to a$$

- When to terminate parsing? Example:

$$\underbrace{(\varepsilon, S, z)}_{\text{final}} \vdash (\varepsilon, A, zi) \text{ if } \pi_i = A \to S$$

Outline

1. Repetition: Nondeterministic Bottom-Up Parsing
2. $LR(0)$ Grammars
3. $LR(0)$ Parsing

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

Corollary 11.1 ($LR(0)$ grammar)

$G \in CFG_\Sigma$ has the $LR(0)$ property if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha\beta y \end{cases}$$

it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

Corollary 11.1 ($LR(0)$ grammar)

$G \in CFG_\Sigma$ has the $LR(0)$ property if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha\beta y \end{cases}$$

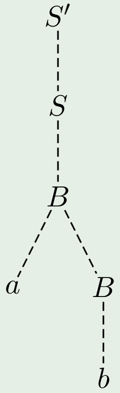
it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

Goal: derive a finite information from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

$LR(0)$ Items and Sets I

Example 11.2

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (1) \\
 & S & \rightarrow B \mid C & (2,3) \\
 & B & \rightarrow aB \mid b & (4,5) \\
 & C & \rightarrow aC \mid c & (6,7)
\end{array}
$$

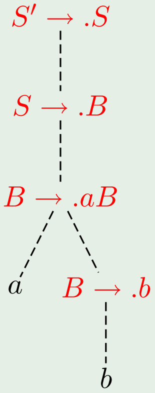


Example 11.2

$$G: \quad \begin{array}{lll} S' & \rightarrow S & (1) \\ S & \rightarrow B \mid C & (2,3) \\ B & \rightarrow aB \mid b & (4,5) \\ C & \rightarrow aC \mid c & (6,7) \end{array}$$

NBA(G):

$(ab, \varepsilon, \varepsilon)$



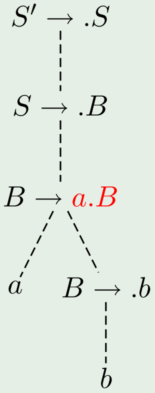
$LR(0)$ Items and Sets I

Example 11.2

$$
\begin{array}{llll}
G: & S' & \to S & (1) \\
 & S & \to B \mid C & (2,3) \\
 & B & \to aB \mid b & (4,5) \\
 & C & \to aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon)
\end{array}
$$



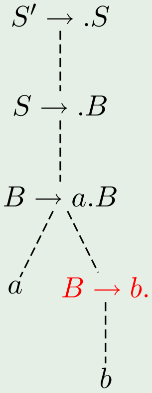
$LR(0)$ Items and Sets I

Example 11.2

$$
\begin{array}{llll}
G: & S' & \to S & (1) \\
 & S & \to B \mid C & (2,3) \\
 & B & \to aB \mid b & (4,5) \\
 & C & \to aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon) \\
\vdash & (\varepsilon, ab, \varepsilon)
\end{array}
$$

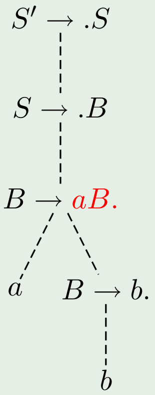


Example 11.2

$$
\begin{array}{llllr}
G: & S' & \to & S & (1) \\
 & S & \to & B \mid C & (2,3) \\
 & B & \to & aB \mid b & (4,5) \\
 & C & \to & aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon) \\
\vdash & (\varepsilon, ab, \varepsilon) \\
\vdash & (\varepsilon, aB, 5)
\end{array}
$$



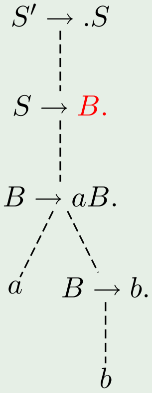
$LR(0)$ Items and Sets I

Example 11.2

$$
\begin{array}{lllll}
G: & S' & \rightarrow & S & (1) \\
 & S & \rightarrow & B \mid C & (2,3) \\
 & B & \rightarrow & aB \mid b & (4,5) \\
 & C & \rightarrow & aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon) \\
\vdash & (\varepsilon, ab, \varepsilon) \\
\vdash & (\varepsilon, aB, 5) \\
\vdash & (\varepsilon, B, 54)
\end{array}
$$

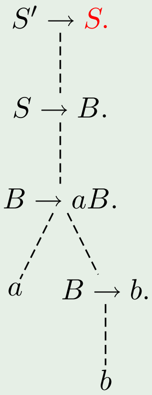


Example 11.2

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (1) \\
 & S & \rightarrow B \mid C & (2,3) \\
 & B & \rightarrow aB \mid b & (4,5) \\
 & C & \rightarrow aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon) \\
\vdash & (\varepsilon, ab, \varepsilon) \\
\vdash & (\varepsilon, aB, 5) \\
\vdash & (\varepsilon, B, 54) \\
\vdash & (\varepsilon, S, 542)
\end{array}
$$



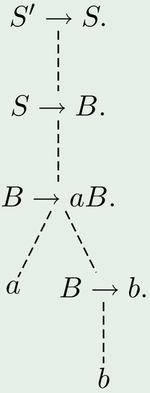
$LR(0)$ Items and Sets I

Example 11.2

$$
\begin{array}{lllll}
G: & S' & \rightarrow & S & (1) \\
 & S & \rightarrow & B \mid C & (2,3) \\
 & B & \rightarrow & aB \mid b & (4,5) \\
 & C & \rightarrow & aC \mid c & (6,7)
\end{array}
$$

NBA(G):

$$
\begin{array}{ll}
 & (ab, \varepsilon, \varepsilon) \\
\vdash & (b, a, \varepsilon) \\
\vdash & (\varepsilon, ab, \varepsilon) \\
\vdash & (\varepsilon, aB, 5) \\
\vdash & (\varepsilon, B, 54) \\
\vdash & (\varepsilon, S, 542) \\
\vdash & (\varepsilon, S', 5421)
\end{array}
$$



Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 11.4

1. *For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.*

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 11.4

1. *For every* $\gamma \in X^*$, $LR(0)(\gamma)$ *is finite.*
2. $LR(0)(G)$ *is finite.*

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 11.4

1. *For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.*
2. *$LR(0)(G)$ is finite.*
3. *The item $[A \to \beta\cdot] \in LR(0)(\gamma)$ indicates the possible reduction $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \to \beta$ and $\gamma = \alpha\beta$.*

$LR(0)$ Items and Sets II

Definition 11.3 ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 11.4

1. *For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.*
2. *$LR(0)(G)$ is finite.*
3. *The item $[A \to \beta\cdot] \in LR(0)(\gamma)$ indicates the possible reduction $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \to \beta$ and $\gamma = \alpha\beta$.*
4. *The item $[A \to \beta_1 \cdot Y\beta_2] \in LR(0)(\gamma)$ indicates an incomplete handle β_1 (to be completed by shift operations or ε-reductions).*

$LR(0)$ Conflicts

Definition 11.5 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta\cdot] \in I.$$

$LR(0)$ Conflicts

Definition 11.5 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta\cdot] \in I.$$
- I has a reduce/reduce conflict if there exist $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha\cdot], [B \to \beta\cdot] \in I.$$

$LR(0)$ Conflicts

Definition 11.5 ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta\cdot] \in I.$$
- I has a reduce/reduce conflict if there exist $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha\cdot], [B \to \beta\cdot] \in I.$$

Lemma 11.6

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Computing $LR(0)$ Sets I

Theorem 11.7 (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. $LR(0)(\varepsilon)$ *is the least set such that*
   - $[S' \to \cdot S] \in LR(0)(\varepsilon)$ *and*
   - *if* $[A \to \cdot B\gamma] \in LR(0)(\varepsilon)$ *and* $B \to \beta \in P$,
     then $[B \to \cdot\beta] \in LR(0)(\varepsilon)$.

Computing $LR(0)$ Sets I

Theorem 11.7 (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. $LR(0)(\varepsilon)$ *is the least set such that*
   - $[S' \to \cdot S] \in LR(0)(\varepsilon)$ *and*
   - *if* $[A \to \cdot B\gamma] \in LR(0)(\varepsilon)$ *and* $B \to \beta \in P$,
     then $[B \to \cdot\beta] \in LR(0)(\varepsilon)$.
2. $LR(0)(\alpha Y)$ *(*$\alpha \in X^*, Y \in X$*) is the least set such that*
   - *if* $[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
     then $[A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ *and*
   - *if* $[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ *and* $B \to \beta \in P$,
     then $[B \to \cdot\beta] \in LR(0)(\alpha Y)$.

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$
\begin{array}{lll}
G: & S' & \to S \\
 & S & \to B \mid C \\
 & B & \to aB \mid b \\
 & C & \to aC \mid c
\end{array}
$$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$[S' \to \cdot S] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon):$ $[S' \to \cdot S]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \to & S \\
 & S & \to & B \mid C \\
 & B & \to & aB \mid b \\
 & C & \to & aC \mid c
\end{array}
$$

$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$
$\Longrightarrow\ [B \to \cdot\beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon): \quad [S' \to \cdot S] \quad [S \to \cdot B]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$
\begin{aligned}
G:\quad & S' \to S \\
& S \to B \mid C \\
& B \to aB \mid b \\
& C \to aC \mid c
\end{aligned}
$$

$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$
$\implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon):$ $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$$
$$\implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon):$ $\quad [S' \to \cdot S] \quad [S \to \cdot B] \quad [S \to \cdot C] \quad [B \to \cdot aB]$
$[B \to \cdot b]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P \implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

Example 11.8 (cf. Example 11.2)

$$
\begin{aligned}
G: \quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{array}{lcl} S' & \rightarrow & S \\ S & \rightarrow & B \mid C \\ B & \rightarrow & aB \mid b \\ C & \rightarrow & aC \mid c \end{array}$$

$$[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$$
$$\Longrightarrow \; [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$\begin{array}{lllll} I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\ I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & & & \\ I_2 := LR(0)(B): & [S \rightarrow B \cdot] & & & \\ I_3 := LR(0)(C): & [S \rightarrow C \cdot] & & & \\ I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & & \end{array}$$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \to \beta \in P \implies [B \to \cdot\beta] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$:	$[S' \to \cdot S]$	$[S \to \cdot B]$	$[S \to \cdot C]$	$[B \to \cdot aB]$
	$[B \to \cdot b]$	$[C \to \cdot aC]$	$[C \to \cdot c]$	
$I_1 := LR(0)(S)$:	$[S' \to S\cdot]$			
$I_2 := LR(0)(B)$:	$[S \to B\cdot]$			
$I_3 := LR(0)(C)$:	$[S \to C\cdot]$			
$I_4 := LR(0)(a)$:	$[B \to a \cdot B]$	$[C \to a \cdot C]$	$[B \to \cdot aB]$	$[B \to \cdot b]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \to \beta \in P$$
$$\Longrightarrow [B \to \cdot\beta] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \to & S \\
 & S & \to & B \mid C \\
 & B & \to & aB \mid b \\
 & C & \to & aC \mid c
\end{array}
$$

$$
[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)
$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$
\begin{aligned}
G:\quad & S' \to S \\
& S \to B \mid C \\
& B \to aB \mid b \\
& C \to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b \cdot]$

$I_6 := LR(0)(c)$: $[C \to c \cdot]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b \cdot]$

$I_6 := LR(0)(c)$: $[C \to c \cdot]$

$I_7 := LR(0)(aB)$: $[B \to aB \cdot]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b \cdot]$

$I_6 := LR(0)(c)$: $[C \to c \cdot]$

$I_7 := LR(0)(aB)$: $[B \to aB \cdot]$

$I_8 := LR(0)(aC)$: $[C \to aC \cdot]$

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \begin{array}{lll} S' & \rightarrow & S \\ S & \rightarrow & B \mid C \\ B & \rightarrow & aB \mid b \\ C & \rightarrow & aC \mid c \end{array}$$

$$\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\
 & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & & & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & & & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & & & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & & & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \rightarrow aC \cdot] & & &
\end{array}$$

($LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$,
$LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

Computing $LR(0)$ Sets II

Example 11.8 (cf. Example 11.2)

$$G: \quad \begin{array}{lll} S' & \rightarrow & S \\ S & \rightarrow & B \mid C \\ B & \rightarrow & aB \mid b \\ C & \rightarrow & aC \mid c \end{array}$$

$$\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\
 & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & & & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & & & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & & & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & & & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \rightarrow aC \cdot] & & &
\end{array}$$

($LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$, $LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

no conflicts $\Longrightarrow$ $G \in LR(0)$

Outline

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields deterministic shift/reduce decision for NBA(G) in a configuration with pushdown γ
$\Longrightarrow$ new pushdown alphabet: $LR(0)(G)$ in place of X

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields deterministic shift/reduce decision for NBA(G) in a configuration with pushdown γ
$\Longrightarrow$ new pushdown alphabet: $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but independent from γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields deterministic shift/reduce decision for NBA(G) in a configuration with pushdown γ
$\Longrightarrow$ new pushdown alphabet: $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but independent from γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

Definition 11.9 ($LR(0)$ goto function)

The function $\text{goto} : LR(0)(G) \times X \to LR(0)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \begin{array}{l}\text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).\end{array}$$

The goto Function II

Example 11.10 (cf. Example 11.8)

$$
\begin{array}{lll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & \\
 & [S \to \cdot B] & [S \to \cdot C] \\
 & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] \\
I_1 := LR(0)(S): & [S' \to S \cdot] & \\
I_2 := LR(0)(B): & [S \to B \cdot] & \\
I_3 := LR(0)(C): & [S \to C \cdot] & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] \\
 & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] \\
I_5 := LR(0)(b): & [B \to b \cdot] & \\
I_6 := LR(0)(c): & [C \to c \cdot] & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & \\
I_9 := \emptyset & &
\end{array}
$$

Example 11.10 (cf. Example 11.8)

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$
$[S \to \cdot B]$ $[S \to \cdot C]$
$[B \to \cdot aB]$ $[B \to \cdot b]$
$[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

$I_3 := LR(0)(C)$: $[S \to C \cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$
$[B \to \cdot aB]$ $[B \to \cdot b]$
$[C \to \cdot aC]$ $[C \to \cdot c]$

$I_5 := LR(0)(b)$: $[B \to b \cdot]$

$I_6 := LR(0)(c)$: $[C \to c \cdot]$

$I_7 := LR(0)(aB)$: $[B \to aB \cdot]$

$I_8 := LR(0)(aC)$: $[C \to aC \cdot]$

$I_9 := \emptyset$

goto	S	B	C	a	b	c
I_0	I_1	I_2	I_3	I_4	I_5	I_6
I_1						
I_2						
I_3						
I_4		I_7	I_8	I_4	I_5	I_6
I_5						
I_6						
I_7						
I_8						
I_9						

(empty $= I_9$)

Example 11.10 (continued)

Representation of goto funtion as finite automaton: