Compiler Construction

Lecture 12: Syntactic Analysis VII ($SLR(1)$ Parsing)

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Outline

1. Repetition: $LR(0)$ Parsing
2. Examples of $LR(0)$ Conflicts
3. The $LR(0)$ Parsing Automaton
4. $SLR(1)$ Parsing

$LR(0)$ Items and Sets

Definition ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for γ, called the $LR(0)$ set (or: $LR(0)$ information) of γ.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary

1. *For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.*
2. *$LR(0)(G)$ is finite.*
3. *The item $[A \to \beta\cdot] \in LR(0)(\gamma)$ indicates the possible reduction $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \to \beta$ and $\gamma = \alpha\beta$.*
4. *The item $[A \to \beta_1 \cdot Y\beta_2] \in LR(0)(\gamma)$ indicates an incomplete handle β_1 (to be completed by shift operations or ε-reductions).*

$LR(0)$ Conflicts

Definition ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta\cdot] \in I.$$
- I has a reduce/reduce conflict if there exist $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha\cdot], [B \to \beta\cdot] \in I.$$

Lemma

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Computing $LR(0)$ Sets I

Theorem (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. $LR(0)(\varepsilon)$ *is the least set such that*
   - $[S' \to \cdot S] \in LR(0)(\varepsilon)$ *and*
   - *if* $[A \to \cdot B\gamma] \in LR(0)(\varepsilon)$ *and* $B \to \beta \in P$, *then* $[B \to \cdot\beta] \in LR(0)(\varepsilon)$.
2. $LR(0)(\alpha Y)$ $(\alpha \in X^*, Y \in X)$ *is the least set such that*
   - *if* $[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$, *then* $[A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ *and*
   - *if* $[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ *and* $B \to \beta \in P$, *then* $[B \to \cdot\beta] \in LR(0)(\alpha Y)$.

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}
$$

Example (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[S' \to \cdot S] \in LR(0)(\varepsilon)$$

$$I_0 := LR(0)(\varepsilon): \qquad [S' \to \cdot S]$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$$
$$\Longrightarrow \ [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon): \qquad [S' \to \cdot S] \qquad [S \to \cdot B]$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$\begin{aligned} G:\quad S' &\to S \\ S &\to B \mid C \\ B &\to aB \mid b \\ C &\to aC \mid c \end{aligned}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$$
$$\Longrightarrow\ [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$I_0 := LR(0)(\varepsilon):\qquad [S' \to \cdot S] \qquad [S \to \cdot B] \qquad [S \to \cdot C]$

Example (cf. Example 11.2)

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P \implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$$I_0 := LR(0)(\varepsilon): \quad [S' \to \cdot S] \quad [S \to \cdot B] \quad [S \to \cdot C] \quad [B \to \cdot aB] \quad [B \to \cdot b]$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$G: \quad \begin{array}{rl} S' & \to S \\ S & \to B \mid C \\ B & \to aB \mid b \\ C & \to aC \mid c \end{array}$$

$$[A \to \cdot B\gamma] \in LR(0)(\varepsilon), B \to \beta \in P$$
$$\implies [B \to \cdot\beta] \in LR(0)(\varepsilon)$$

$$I_0 := LR(0)(\varepsilon): \quad \begin{array}{llll} [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\ [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \end{array}$$

Example (cf. Example 11.2)

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$\begin{array}{lllll} I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\ & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\ I_1 := LR(0)(S): & [S' \to S \cdot] & & & \end{array}$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{aligned}
G:\quad & S' \to S\\
& S \to B \mid C\\
& B \to aB \mid b\\
& C \to aC \mid c
\end{aligned}
$$

$$
\begin{aligned}
& [A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)\\
\Longrightarrow\ & [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)
\end{aligned}
$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$ $[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(B)$: $[S \to B \cdot]$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{aligned}
G: \quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{aligned}
G:\quad & S' \to S \\
& S \to B \mid C \\
& B \to aB \mid b \\
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\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]$ $[S \to \cdot B]$ $[S \to \cdot C]$ $[B \to \cdot aB]$
$[B \to \cdot b]$ $[C \to \cdot aC]$ $[C \to \cdot c]$

$I_1 := LR(0)(S)$: $[S' \to S\cdot]$

$I_2 := LR(0)(B)$: $[S \to B\cdot]$

$I_3 := LR(0)(C)$: $[S \to C\cdot]$

$I_4 := LR(0)(a)$: $[B \to a \cdot B]$ $[C \to a \cdot C]$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$G: \quad \begin{aligned} S' &\rightarrow S \\ S &\rightarrow B \mid C \\ B &\rightarrow aB \mid b \\ C &\rightarrow aC \mid c \end{aligned}$$

$$[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$$
$$\Longrightarrow \; [B \rightarrow \cdot\beta] \in LR(0)(\alpha Y)$$

$I_0 := LR(0)(\varepsilon)$: $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S)$: $[S' \rightarrow S\cdot]$

$I_2 := LR(0)(B)$: $[S \rightarrow B\cdot]$

$I_3 := LR(0)(C)$: $[S \rightarrow C\cdot]$

$I_4 := LR(0)(a)$: $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \rightarrow & S \\
 & S & \rightarrow & B \mid C \\
 & B & \rightarrow & aB \mid b \\
 & C & \rightarrow & aC \mid c
\end{array}
$$

$$[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$$
$$\Longrightarrow\ [B \rightarrow \cdot\beta] \in LR(0)(\alpha Y)$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\
 & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\
I_1 := LR(0)(S): & [S' \rightarrow S\cdot] & & & \\
I_2 := LR(0)(B): & [S \rightarrow B\cdot] & & & \\
I_3 := LR(0)(C): & [S \rightarrow C\cdot] & & & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \rightarrow & S \\
 & S & \rightarrow & B \mid C \\
 & B & \rightarrow & aB \mid b \\
 & C & \rightarrow & aC \mid c
\end{array}
$$

$$
[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)
$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\
 & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & & & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & & & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & & & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{aligned}
G:\quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$
\begin{array}{llllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \rightarrow & S \\
 & S & \rightarrow & B \mid C \\
 & B & \rightarrow & aB \mid b \\
 & C & \rightarrow & aC \mid c
\end{array}
$$

$$[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$$
$$\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\
 & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & & & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & & & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & & & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & & & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{aligned}
G:\quad S' &\to S \\
S &\to B \mid C \\
B &\to aB \mid b \\
C &\to aC \mid c
\end{aligned}
$$

$$[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & & &
\end{array}
$$

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$
\begin{array}{llll}
G: & S' & \to & S \\
 & S & \to & B \mid C \\
 & B & \to & aB \mid b \\
 & C & \to & aC \mid c
\end{array}
$$

$$
\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & & &
\end{array}
$$

$(LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$,
$LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

Computing $LR(0)$ Sets II

Example (cf. Example 11.2)

$$G: \begin{array}{lll} S' & \to & S \\ S & \to & B \mid C \\ B & \to & aB \mid b \\ C & \to & aC \mid c \end{array}$$

$$\begin{array}{lllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot B] & [S \to \cdot C] & [B \to \cdot aB] \\
 & [B \to \cdot b] & [C \to \cdot aC] & [C \to \cdot c] & \\
I_1 := LR(0)(S): & [S' \to S \cdot] & & & \\
I_2 := LR(0)(B): & [S \to B \cdot] & & & \\
I_3 := LR(0)(C): & [S \to C \cdot] & & & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] & & \\
I_5 := LR(0)(b): & [B \to b \cdot] & & & \\
I_6 := LR(0)(c): & [C \to c \cdot] & & & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & & & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & & &
\end{array}$$

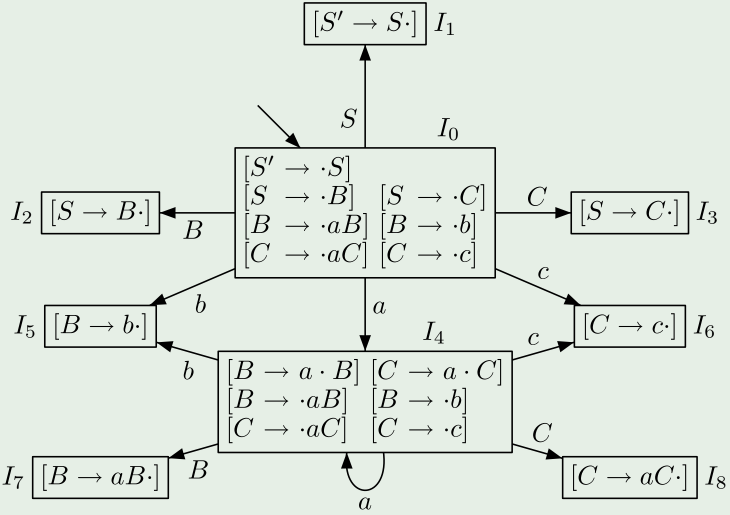
($LR(0)(aa) = LR(0)(a) = I_4$, $LR(0)(ab) = LR(0)(b) = I_5$, $LR(0)(ac) = LR(0)(c) = I_6$, $I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

no conflicts $\implies$ $G \in LR(0)$

The goto Function

Example (continued)

Representation of goto funtion as finite automaton:



Outline

Reduce/Reduce Conflicts

Example 12.1

$$G: \quad \begin{aligned} S' &\to S \\ S &\to Aa \mid Bb \\ A &\to a \\ B &\to a \end{aligned}$$

Example 12.1

$$
\begin{aligned}
G: \quad & S' \to S \\
& S \to Aa \mid Bb \\
& A \to a \\
& B \to a
\end{aligned}
$$

$$
\begin{array}{lllll}
LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot Aa] & [S \to \cdot Bb] & [A \to \cdot a] \quad [B \to \cdot a] \\
LR(0)(S): & [S' \to S\cdot] & & & \\
LR(0)(A): & [S \to A \cdot a] & & & \\
LR(0)(B): & [S \to B \cdot a] & & & \\
LR(0)(a): & \color{red}{[A \to a\cdot]} & \color{red}{[B \to a\cdot]} & & \\
LR(0)(Aa): & [S \to Aa\cdot] & & & \\
LR(0)(Ba): & [S \to Ba\cdot] & & &
\end{array}
$$

Reduce/Reduce Conflicts

Example 12.1

$$\begin{aligned} G: \quad S' &\to S \\ S &\to Aa \mid Bb \\ A &\to a \\ B &\to a \end{aligned}$$

$$\begin{array}{llllll}
LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot Aa] & [S \to \cdot Bb] & [A \to \cdot a] & [B \to \cdot a] \\
LR(0)(S): & [S' \to S \cdot] \\
LR(0)(A): & [S \to A \cdot a] \\
LR(0)(B): & [S \to B \cdot a] \\
LR(0)(a): & [A \to a \cdot] & [B \to a \cdot] \\
LR(0)(Aa): & [S \to Aa \cdot] \\
LR(0)(Ba): & [S \to Ba \cdot]
\end{array}$$

Note: G is unambiguous

Shift/Reduce Conflicts

Example 12.2

$$
G: \quad \begin{aligned} S' &\to S \\ S &\to aS \mid a \end{aligned}
$$

Shift/Reduce Conflicts

Example 12.2

$$G: \quad \begin{aligned} S' &\to S \\ S &\to aS \mid a \end{aligned}$$

$$\begin{array}{lllll}
LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot aS] & [S \to \cdot a] & \\
LR(0)(S): & [S' \to S \cdot] & & & \\
LR(0)(a): & [S \to a \cdot S] & [S \to \cdot aS] & \color{red}{[S \to \cdot a]} & \color{red}{[S \to a \cdot]} \\
LR(0)(aS): & [S \to aS \cdot] & & &
\end{array}$$

Example 12.2

$$G: \quad \begin{array}{lcl} S' & \to & S \\ S & \to & aS \mid a \end{array}$$

$$\begin{array}{lllll}
LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot aS] & [S \to \cdot a] & \\
LR(0)(S): & [S' \to S \cdot] & & & \\
LR(0)(a): & [S \to a \cdot S] & [S \to \cdot aS] & [S \to \cdot a] & [S \to a \cdot] \\
LR(0)(aS): & [S \to aS \cdot] & & &
\end{array}$$

Note: G is unambiguous

Outline

The $LR(0)$ Action Function

The automaton will be defined using another table, the action function, which determines the shift/reduce decision.
(Reminder: $\pi_0 = S' \to S$)

The $LR(0)$ Action Function

The automaton will be defined using another table, the action function, which determines the shift/reduce decision.
(Reminder: $\pi_0 = S' \to S$)

Definition 12.3 ($LR(0)$ action function)

The $LR(0)$ action function

$$\mathrm{act} : LR(0)(G) \to \{\mathsf{red}\, i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I) := \begin{cases} \mathsf{red}\, i & \text{if } \pi_i = A \to \alpha \text{ and } [A \to \alpha \cdot] \in I \ (i \neq 0) \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot a\alpha_2] \in I \\ \mathsf{accept} & \text{if } [S' \to S \cdot] \in I \\ \mathsf{error} & \text{if } I = \emptyset \end{cases}$$

The $LR(0)$ Action Function

The automaton will be defined using another table, the action function, which determines the shift/reduce decision.
(Reminder: $\pi_0 = S' \to S$)

Definition 12.3 ($LR(0)$ action function)

The $LR(0)$ action function

$$\text{act} : LR(0)(G) \to \{\mathsf{red}\ i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

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Corollary 12.4

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act *is well defined.*

The $LR(0)$ Action Function

The automaton will be defined using another table, the action function, which determines the shift/reduce decision.
(Reminder: $\pi_0 = S' \to S$)

Definition 12.3 ($LR(0)$ action function)

The $LR(0)$ action function

$$\mathrm{act} : LR(0)(G) \to \{\mathsf{red}\, i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I) := \begin{cases} \mathsf{red}\, i & \text{if } \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot] \in I \ (i \neq 0) \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot a\alpha_2] \in I \\ \mathsf{accept} & \text{if } [S' \to S\cdot] \in I \\ \mathsf{error} & \text{if } I = \emptyset \end{cases}$$

Corollary 12.4

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act *is well defined.*

Together, act and goto form the $LR(0)$ parsing table of G.

The $LR(0)$ Parsing Table

Example 12.5 (cf. Example 11.10)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$$
\begin{array}{lll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & \\
 & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] \\
 & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] \\
 & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & \\
I_8 := LR(0)(aC): & [C \rightarrow aC \cdot] & \\
I_9 := \emptyset & &
\end{array}
$$

Example 12.5 (cf. Example 11.10)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
& S & \rightarrow B \mid C & (1,2) \\
& B & \rightarrow aB \mid b & (3,4) \\
& C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$$
\begin{array}{lll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & \\
& [S \rightarrow \cdot B] & [S \rightarrow \cdot C] \\
& [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
& [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] \\
& [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
& [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & \\
I_8 := LR(0)(aC): & [C \rightarrow aC \cdot] & \\
I_9 := \emptyset & &
\end{array}
$$

The $LR(0)$ Parsing Automaton I

Definition 12.6 ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of G is defined by the following components.

- Input alphabet Σ
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [p] \cup \{0, \mathsf{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration (w, I_0, ε) where $I_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I) = \mathsf{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce: $(w, \alpha I I_1 \ldots I_n, z) \vdash (w, \alpha I I', zi)$ if $\mathrm{act}(I_n) = \mathsf{red}\, i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathrm{act}(I) = \mathsf{accept}$

error: $(w, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\mathsf{error})$ if $\mathrm{act}(I) = \mathsf{error}$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{lllll}
G: & S' & \to & S & (0) \\
 & S & \to & B \mid C & (1,2) \\
 & B & \to & aB \mid b & (3,4) \\
 & C & \to & aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$(aac,\ I_0\qquad,\ \varepsilon\qquad)$$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac,\ I_0 \quad , \varepsilon \quad) \\
\vdash & (\ ac,\ I_0 I_4 \quad , \varepsilon \quad)
\end{array}
$$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$G: \begin{array}{lll} S' & \to S & (0) \\ S & \to B \mid C & (1,2) \\ B & \to aB \mid b & (3,4) \\ C & \to aC \mid c & (5,6) \end{array}$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$\begin{array}{ll} & (aac, I_0, \varepsilon) \\ \vdash & (ac, I_0 I_4, \varepsilon) \\ \vdash & (c, I_0 I_4 I_4, \varepsilon) \end{array}$$

Example 12.7 (cf. Example 12.5)

$$
\begin{aligned}
G: \quad S' &\to S && (0)\\
S &\to B \mid C && (1,2)\\
B &\to aB \mid b && (3,4)\\
\color{red}{C} &\to aC \mid \color{red}{c} && (5,6)
\end{aligned}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	$\color{red}{I_8}$	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{aligned}
&(aac,\ I_0,\ \varepsilon)\\
\vdash\ &(ac,\ I_0I_4,\ \varepsilon)\\
\vdash\ &(c,\ I_0I_4I_4,\ \varepsilon)\\
\vdash\ &(\varepsilon,\ I_0I_4I_4\color{red}{I_6},\ \varepsilon)
\end{aligned}
$$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \to S & (0) \\
 & S & \to B \mid C & (1,2) \\
 & B & \to aB \mid b & (3,4) \\
 & C & \to aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0I_4,\ \varepsilon) \\
\vdash & (c,\ I_0I_4I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_8,\ 6)
\end{array}
$$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{lllll}
G: & S' & \to & S & (0) \\
 & S & \to & B \mid C & (1,2) \\
 & B & \to & aB \mid b & (3,4) \\
 & C & \to & aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0 I_4,\ \varepsilon) \\
\vdash & (c,\ I_0 I_4 I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_8,\ 6) \\
(*) & \\
\vdash & (\varepsilon,\ I_0 I_4 I_8,\ 65)
\end{array}
$$

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0I_4,\ \varepsilon) \\
\vdash & (c,\ I_0I_4I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_8,\ 6) \\
(*) & \\
\vdash & (\varepsilon,\ I_0I_4I_8,\ 65) \\
\vdash & (\varepsilon,\ I_0I_3,\ 655)
\end{array}
$$

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \to S & (0) \\
 & S & \to B \mid C & (1,2) \\
 & B & \to aB \mid b & (3,4) \\
 & C & \to aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac, I_0, \varepsilon) \\
\vdash & (ac, I_0I_4, \varepsilon) \\
\vdash & (c, I_0I_4I_4, \varepsilon) \\
\vdash & (\varepsilon, I_0I_4I_4I_6, \varepsilon) \\
\vdash & (\varepsilon, I_0I_4I_4I_8, 6) \\
(*) & \\
\vdash & (\varepsilon, I_0I_4I_8, 65) \\
\vdash & (\varepsilon, I_0I_3, 655) \\
\vdash & (\varepsilon, I_0I_1, 6552)
\end{array}
$$

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0I_4,\ \varepsilon) \\
\vdash & (c,\ I_0I_4I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0I_4I_4I_8,\ 6) \\
(*) & \\
\vdash & (\varepsilon,\ I_0I_4I_8,\ 65) \\
\vdash & (\varepsilon,\ I_0I_3,\ 655) \\
\vdash & (\varepsilon,\ I_0I_1,\ 6552) \\
\vdash & (\varepsilon,\ \varepsilon,\ 65520)
\end{array}
$$

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac, I_0, \varepsilon) \\
\vdash & (ac, I_0 I_4, \varepsilon) \\
\vdash & (c, I_0 I_4 I_4, \varepsilon) \\
\vdash & (\varepsilon, I_0 I_4 I_4 I_6, \varepsilon) \\
\vdash & (\varepsilon, I_0 I_4 I_4 I_8, 6) \\
(*) & \\
\vdash & (\varepsilon, I_0 I_4 I_8, 65) \\
\vdash & (\varepsilon, I_0 I_3, 655) \\
\vdash & (\varepsilon, I_0 I_1, 6552) \\
\vdash & (\varepsilon, \varepsilon, 65520)
\end{array}
$$

Check by rightmost derivation (on the board)

The $LR(0)$ Parsing Automaton II

Example 12.7 (cf. Example 12.5)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
I_0	shift	I_1	I_2	I_3	I_4	I_5	I_6
I_1	accept						
I_2	red 1						
I_3	red 2						
I_4	shift		I_7	I_8	I_4	I_5	I_6
I_5	red 4						
I_6	red 6						
I_7	red 3						
I_8	red 5						
I_9	error						

(empty $= I_9$)

$LR(0)$ parsing of aac:

$$
\begin{array}{ll}
 & (aac, I_0, \varepsilon) \\
\vdash & (ac, I_0 I_4, \varepsilon) \\
\vdash & (c, I_0 I_4 I_4, \varepsilon) \\
\vdash & (\varepsilon, I_0 I_4 I_4 I_6, \varepsilon) \\
\vdash & (\varepsilon, I_0 I_4 I_4 I_8, 6) \\
(*) & \\
\vdash & (\varepsilon, I_0 I_4 I_8, 65) \\
\vdash & (\varepsilon, I_0 I_3, 655) \\
\vdash & (\varepsilon, I_0 I_1, 6552) \\
\vdash & (\varepsilon, \varepsilon, 65520)
\end{array}
$$

Check by rightmost derivation (on the board)

Remark: in the corresponding computation of NBA(G), $(*)$ is nondeterministic

Theorem 12.8 (Correctness of $LR(0)$ Parsing Automaton)

If $G \in LR(0)$, then the $LR(0)$ parsing automaton of G is deterministic, and for every $w \in \Sigma^*$ and $z \in \{0, \ldots, p\}^*$:

$$(w, I_0, \varepsilon) \vdash^* (\varepsilon, \varepsilon, z) \quad \textit{iff} \quad \overleftarrow{z} \textit{ is a rightmost analysis of } w$$

Proof.

omitted □

Outline

Removing Conflicts in $LR(0)$ Parsing

In practice: often $G \notin LR(0)$

Example 12.9

$$
\begin{array}{lll}
G_{AE}: & E' \to E & E \to E\texttt{+}T \mid T \\
& T \to T\texttt{*}F \mid F & F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b}
\end{array}
$$

In practice: often $G \notin LR(0)$

Example 12.9

$$
\begin{array}{llll}
G_{AE}: & E' \to E & & E \to E\texttt{+}T \mid T \\
 & T \to T\texttt{*}F \mid F & & F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b}
\end{array}
$$

$LR(0)(G_{AE})$ with conflicts:

$$
\begin{array}{llllll}
I_0: & [E' \to \cdot E] & [E \to \cdot E\texttt{+}T] & I_1: & [E' \to E\cdot] & [E \to E\cdot\texttt{+}T] \\
 & [E \to \cdot T] & [T \to \cdot T\texttt{*}F] & I_2: & [E \to T\cdot] & [T \to T\cdot\texttt{*}F] \\
 & [T \to \cdot F] & [F \to \cdot\texttt{(}E\texttt{)}] & I_3: & [T \to F\cdot] & \\
 & [F \to \cdot\texttt{a}] & [F \to \cdot\texttt{b}] & & & \\
I_4: & [F \to \texttt{(}\cdot E\texttt{)}] & [E \to \cdot E\texttt{+}T] & I_5: & [F \to \texttt{a}\cdot] & \\
 & [E \to \cdot T] & [T \to \cdot T\texttt{*}F] & I_6: & [F \to \texttt{b}\cdot] & \\
 & [T \to \cdot F] & [F \to \cdot\texttt{(}E\texttt{)}] & I_7: & [E \to E\texttt{+}\cdot T] & [T \to \cdot T\texttt{*}F] \\
 & [F \to \cdot\texttt{a}] & [F \to \cdot\texttt{b}] & & [T \to \cdot F] & [F \to \cdot\texttt{(}E\texttt{)}] \\
 & & & & [F \to \cdot\texttt{a}] & [F \to \cdot\texttt{b}] \\
I_8: & [T \to T\texttt{*}\cdot F] & [F \to \cdot\texttt{(}E\texttt{)}] & I_9: & [F \to \texttt{(}E\cdot\texttt{)}] & [E \to E\cdot\texttt{+}T] \\
 & [F \to \cdot\texttt{a}] & [F \to \cdot\texttt{b}] & I_{10}: & [E \to E\texttt{+}T\cdot] & [T \to T\cdot\texttt{*}F] \\
I_{11}: & [T \to T\texttt{*}F\cdot] & & I_{12}: & [F \to \texttt{(}E\texttt{)}\cdot] &
\end{array}
$$

Adding Lookahead I

Goal: resolving conflicts by considering first input symbol

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Observations:

- $[A \to \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
  $\implies S' \Rightarrow_r^* \alpha A w \Rightarrow_r \qquad \alpha\beta_1 a\beta_2 w$

  pushdown   next input symbol

  Thus: shift only on lookahead a

Adding Lookahead I

Goal: resolving conflicts by considering first input symbol

Observations:

- $[A \to \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
  $\implies S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1 a\beta_2 w$

  pushdown ($\alpha\beta_1$) next input symbol (a)

  Thus: shift only on lookahead a
- $[A \to \beta\cdot] \in LR(0)(\alpha\beta)$
  $\implies S' \Rightarrow_r^* \alpha A x w \Rightarrow_r \alpha\beta x w$

  pushdown ($\alpha\beta$) input (xw)

  $\implies x \in \mathrm{fo}(A) \subseteq \Sigma_\varepsilon$ ($x = \varepsilon$ only if $w = \varepsilon$)

  Thus: reduce with $A \to \beta$ only if lookahead $x \in \mathrm{fo}(A)$

Adding Lookahead II

Example 12.10 (cf. Example 12.9)

$$
\begin{array}{llll}
G_{AE}: & E' & \to E & (0) \\
& E & \to E\texttt{+}T \mid T & (1,2) \\
& T & \to T\texttt{*}F \mid F & (3,4) \\
& F & \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$

Adding Lookahead II

Example 12.10 (cf. Example 12.9)

$$
\begin{array}{llll}
G_{AE}: & E' & \rightarrow E & (0) \\
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\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E \cdot \texttt{+}T]\}$:
  - accept on lookahead ε
  - shift on lookahead $\texttt{+}$

Example 12.10 (cf. Example 12.9)

$$
\begin{array}{lllll}
G_{AE}: & E' & \rightarrow & E & (0) \\
 & E & \rightarrow & E\texttt{+}T \mid T & (1,2) \\
 & T & \rightarrow & T\texttt{*}F \mid F & (3,4) \\
 & F & \rightarrow & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E \cdot \texttt{+}T]\}$:
  - accept on lookahead ε
  - shift on lookahead `+`
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T \cdot \texttt{*}F]\}$:
  - red 2 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead `*`

Adding Lookahead II

Example 12.10 (cf. Example 12.9)

$$G_{AE}: \quad \begin{array}{lcll} E' & \to & E & (0) \\ E & \to & E\texttt{+}T \mid T & (1,2) \\ T & \to & T\texttt{*}F \mid F & (3,4) \\ F & \to & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$

- $I_1 = \{[E' \to E\cdot], [E \to E \cdot \texttt{+}T]\}$:
  - accept on lookahead ε
  - shift on lookahead $\texttt{+}$
- $I_2 = \{[E \to T\cdot], [T \to T \cdot \texttt{*}F]\}$:
  - red 2 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead $\texttt{*}$
- $I_{10} = \{[E \to E\texttt{+}T\cdot], [T \to T \cdot \texttt{*}F]\}$:
  - red 1 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead $\texttt{*}$

Adding Lookahead II

Example 12.10 (cf. Example 12.9)

$$
\begin{array}{llll}
G_{AE}: & E' & \rightarrow E & (0)\\
& E & \rightarrow E\texttt{+}T \mid T & (1,2)\\
& T & \rightarrow T\texttt{*}F \mid F & (3,4)\\
& F & \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E \cdot \texttt{+}T]\}$:
  - accept on lookahead ε
  - shift on lookahead +
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T \cdot \texttt{*}F]\}$:
  - red 2 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead *
- $I_{10} = \{[E \rightarrow E\texttt{+}T\cdot], [T \rightarrow T \cdot \texttt{*}F]\}$:
  - red 1 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead *

$\Longrightarrow$ $SLR(1)$ parsing (Simple $LR(1)$)

The $SLR(1)$ Action Function

Definition 12.11 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\mathrm{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\, i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \mathsf{red}\, i & \text{if } \pi_i = A \to \alpha, [A \to \alpha \cdot] \in I \ (i \neq 0), \\ & \text{and } x \in \mathrm{fo}(A) \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2] \in I \text{ and } x \in \Sigma \\ \mathsf{accept} & \text{if } [S' \to S\cdot] \in I \text{ and } x = \varepsilon \\ \mathsf{error} & \text{otherwise} \end{cases}$$

The $SLR(1)$ Action Function

Definition 12.11 ($SLR(1)$ action function)

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$$\mathrm{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\, i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

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Definition 12.12 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

The $SLR(1)$ Action Function

Definition 12.11 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\, i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \mathsf{red}\, i & \text{if } \pi_i = A \to \alpha, [A \to \alpha \cdot] \in I \ (i \neq 0), \\ & \text{and } x \in \text{fo}(A) \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2] \in I \text{ and } x \in \Sigma \\ \mathsf{accept} & \text{if } [S' \to S\cdot] \in I \text{ and } x = \varepsilon \\ \mathsf{error} & \text{otherwise} \end{cases}$$

Definition 12.12 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

Together, act and the $LR(0)$ goto function (cf. Definition 11.9) form the $SLR(1)$ parsing table of G.

Example 12.13 (cf. Example 12.9)

I_0 : $[E' \to \cdot E]$ $[E \to \cdot E\texttt{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_1 : $[E' \to E\cdot]$ $[E \to E \cdot \texttt{+}T]$

I_2 : $[E \to T\cdot]$ $[T \to T \cdot \texttt{*}F]$

I_3 : $[T \to F\cdot]$

I_4 : $[F \to \texttt{(} \cdot E\texttt{)}]$ $[E \to \cdot E\texttt{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_5 : $[F \to \texttt{a}\cdot]$

I_6 : $[F \to \texttt{b}\cdot]$

I_7 : $[E \to E\texttt{+} \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_8 : $[T \to T\texttt{*} \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_9 : $[F \to \texttt{(}E \cdot \texttt{)}]$ $[E \to E \cdot \texttt{+}T]$

I_{10} : $[E \to E\texttt{+}T\cdot]$ $[T \to T \cdot \texttt{*}F]$

I_{11} : $[T \to T\texttt{*}F\cdot]$

I_{12} : $[F \to \texttt{(}E\texttt{)}\cdot]$

$A \in N$	fo(A)
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$
T	$\{\texttt{+}, \texttt{*}, \texttt{)}, \varepsilon\}$
F	$\{\texttt{+}, \texttt{*}, \texttt{)}, \varepsilon\}$

The $SLR(1)$ Parsing Table

Example 12.13 (cf. Example 12.9)

I_0 : $[E' \to \cdot E]$ $[E \to \cdot E\texttt{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_1 : $[E' \to E\cdot]$ $[E \to E \cdot \texttt{+}T]$

I_2 : $[E \to T\cdot]$ $[T \to T \cdot \texttt{*}F]$

I_3 : $[T \to F\cdot]$

I_4 : $[F \to \texttt{(} \cdot E\texttt{)}]$ $[E \to \cdot E\texttt{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_5 : $[F \to \texttt{a}\cdot]$

I_6 : $[F \to \texttt{b}\cdot]$

I_7 : $[E \to E\texttt{+} \cdot T]$ $[T \to \cdot T\texttt{*}F]$ $[T \to \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_8 : $[T \to T\texttt{*} \cdot F]$ $[F \to \cdot\texttt{(}E\texttt{)}]$ $[F \to \cdot\texttt{a}]$ $[F \to \cdot\texttt{b}]$

I_9 : $[F \to \texttt{(}E \cdot \texttt{)}]$ $[E \to E \cdot \texttt{+}T]$

I_{10} : $[E \to E\texttt{+}T\cdot]$ $[T \to T \cdot \texttt{*}F]$

I_{11} : $[T \to T\texttt{*}F\cdot]$

I_{12} : $[F \to \texttt{(}E\texttt{)}\cdot]$

$A \in N$	$\mathrm{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{\texttt{+}, \texttt{)}, \varepsilon\}$
T	$\{\texttt{+}, \texttt{*}, \texttt{)}, \varepsilon\}$
F	$\{\texttt{+}, \texttt{*}, \texttt{)}, \varepsilon\}$

$LR(0)(G_{AE})$	act							goto								
	+	*	(	)	a	b	ε	E	T	F	+	*	(	)	a	b
I_0			shift		shift	shift		I_1	I_2	I_3			I_4		I_5	I_6
I_1	shift						accept				I_7					
I_2	red 2	shift		red 2			red 2					I_8				
I_3	red 4	red 4		red 4			red 4									
I_4			shift		shift	shift		I_9	I_2	I_3			I_4		I_5	I_6
I_5	red 6	red 6		red 6			red 6									
I_6	red 7	red 7		red 7			red 7									
I_7			shift		shift	shift			I_{10}	I_3			I_4		I_5	I_6
I_8			shift		shift	shift				I_{11}			I_4		I_5	I_6
I_9	shift			shift							I_7			I_{12}		
I_{10}	red 1	shift		red 1			red 1					I_8				
I_{11}	red 3	red 3		red 3			red 3									
I_{12}	red 5	red 5		red 5			red 5									

Definition 12.14 ($SLR(1)$ parsing automaton)

The $SLR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 12.6), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathsf{act}(I, a) = \mathsf{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce$_a$: $(aw, \alpha I I_1 \ldots I_n, z) \vdash (aw, \alpha I I', zi)$ if $\mathsf{act}(I_n, a) = \mathsf{red}\, i$, $\pi_i = A \rightarrow Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

reduce$_\varepsilon$: $(\varepsilon, \alpha I I_1 \ldots I_n, z) \vdash (\varepsilon, \alpha I I', zi)$ if $\mathsf{act}(I_n, \varepsilon) = \mathsf{red}\, i$, $\pi_i = A \rightarrow Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathsf{act}(I, \varepsilon) = \mathsf{accept}$

error$_a$: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\mathsf{error})$ if $\mathsf{act}(I, a) = \mathsf{error}$

error$_\varepsilon$: $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\mathsf{error})$ if $\mathsf{act}(I, \varepsilon) = \mathsf{error}$