

Compiler Construction

Lecture 13: Syntactic Analysis VIII ($LR(1)$ Parsing)

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1 Repetition: $SLR(1)$ Parsing

2 $LR(1)$ Parsing

3 $LALR(1)$ Parsing

Adding Lookahead

Goal: resolving conflicts by considering first input symbol

Observations:

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$

$$\implies S' \Rightarrow_r^* \alpha A w \Rightarrow_r$$

$\alpha\beta_1 a\beta_2 w$
 $\nearrow$ $\nwarrow$
pushdown next input symbol

Thus: shift only on lookahead a

- $[A \rightarrow \beta \cdot] \in LR(0)(\alpha\beta)$

$$\implies S' \Rightarrow_r^* \alpha A x w \Rightarrow_r$$

$\alpha\beta x w$
 $\nearrow$ $\nwarrow$
pushdown input

$$\implies x \in \text{fo}(A) \subseteq \Sigma_\epsilon \text{ (} x = \epsilon \text{ only if } w = \epsilon \text{)}$$

Thus: reduce with $A \rightarrow \beta$ only if lookahead $x \in \text{fo}(A)$

The $SLR(1)$ Action Function

Definition ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I \ (i \neq 0), \\ & \text{and } x \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

Together, act and the $LR(0)$ goto function (cf. Definition 11.9) form the $SLR(1)$ parsing table of G .

The $SLR(1)$ Parsing Table

Example (cf. Example 12.9)

$I_0 :$	$\begin{bmatrix} E' \rightarrow \cdot E \\ E \rightarrow \cdot T \\ T \rightarrow \cdot F \\ F \rightarrow \cdot a \end{bmatrix}$	$\begin{bmatrix} E \rightarrow \cdot E+T \\ T \rightarrow \cdot T*F \\ F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix}$	$I_1 :$	$\begin{bmatrix} E' \rightarrow E \cdot \end{bmatrix}$	$\begin{bmatrix} E \rightarrow E \cdot +T \\ T \rightarrow T \cdot *F \end{bmatrix}$
$I_4 :$	$\begin{bmatrix} F \rightarrow (\cdot E) \\ E \rightarrow \cdot T \\ T \rightarrow \cdot F \\ F \rightarrow \cdot a \end{bmatrix}$	$\begin{bmatrix} E \rightarrow \cdot E+T \\ T \rightarrow \cdot T*F \\ F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix}$	$I_5 :$	$\begin{bmatrix} F \rightarrow a \cdot \end{bmatrix}$	
			$I_6 :$	$\begin{bmatrix} F \rightarrow b \cdot \end{bmatrix}$	
			$I_7 :$	$\begin{bmatrix} E \rightarrow E+ \cdot T \\ T \rightarrow \cdot F \\ F \rightarrow \cdot a \end{bmatrix}$	$\begin{bmatrix} T \rightarrow T \cdot *F \\ F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix}$
$I_8 :$	$\begin{bmatrix} T \rightarrow T* \cdot F \\ F \rightarrow \cdot a \end{bmatrix}$	$\begin{bmatrix} F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix}$	$I_9 :$	$\begin{bmatrix} F \rightarrow (E \cdot) \end{bmatrix}$	$\begin{bmatrix} E \rightarrow E \cdot +T \\ T \rightarrow T \cdot *F \end{bmatrix}$
$I_{11} :$	$\begin{bmatrix} T \rightarrow T*F \cdot \end{bmatrix}$		$I_{10} :$	$\begin{bmatrix} E \rightarrow E+T \cdot \end{bmatrix}$	
			$I_{12} :$	$\begin{bmatrix} F \rightarrow (E) \cdot \end{bmatrix}$	

$A \in N$	$fo(A)$
E'	$\{\varepsilon\}$
E	$\{+, \cdot, \varepsilon\}$
T	$\{+, *, \cdot, \varepsilon\}$
F	$\{+, *, \cdot, \varepsilon\}$

The $SLR(1)$ Parsing Table

Example (cf. Example 12.9)

I_0 :	$[E' \rightarrow \cdot E]$ $[E \rightarrow \cdot T]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$	$[E \rightarrow \cdot E+T]$ $[T \rightarrow \cdot T*F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$	I_1 :	$[E' \rightarrow E \cdot]$ I_2 :	$[E \rightarrow T \cdot]$ I_3 :	$[T \rightarrow F \cdot]$	$[E \rightarrow E \cdot +T]$ $[T \rightarrow T \cdot *F]$
I_4 :	$[F \rightarrow (\cdot E)]$ $[E \rightarrow \cdot T]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$	$[E \rightarrow \cdot E+T]$ $[T \rightarrow \cdot T*F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$	I_5 :	$[F \rightarrow a \cdot]$ I_6 :	$[F \rightarrow b \cdot]$ I_7 :	$[E \rightarrow E+ \cdot T]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$	$[T \rightarrow \cdot T*F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$
I_8 :	$[T \rightarrow T* \cdot F]$ $[F \rightarrow \cdot a]$ I_{11} :	$[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$ $[T \rightarrow T*F \cdot]$	I_9 :	$[F \rightarrow (E \cdot)]$ I_{10} :	$[E \rightarrow E+T \cdot]$ I_{12} :	$[F \rightarrow (E) \cdot]$	$[E \rightarrow E \cdot +T]$ $[T \rightarrow T \cdot *F]$

$A \in N$	$fo(A)$
E'	$\{\varepsilon\}$
E	$\{+, \cdot, \varepsilon\}$
T	$\{+, *, \cdot, \varepsilon\}$
F	$\{+, *, \cdot, \varepsilon\}$

$LR(0)(G_{AE})$	act							goto								
	+	*	(	)	a	b	ϵ	E	T	F	+	*	(	)	a	b
I_0				shift		shift	shift		I_1	I_2	I_3			I_4		I_5 I_6
I_1	shift						accept									
I_2	red 2	shift			red 2		red 2									
I_3	red 4	red 4			red 4		red 4									
I_4			shift			shift	shift	I_9	I_2	I_3				I_4		I_5 I_6
I_5	red 6	red 6			red 6		red 6									
I_6	red 7	red 7			red 7		red 7									
I_7			shift			shift	shift									
I_8			shift			shift	shift		I_{10}	I_3				I_4		I_5 I_6
I_9	shift				shift					I_{11}				I_4		I_5 I_6
I_{10}	red 1	shift			red 1		red 1									
I_{11}	red 3	red 3			red 3		red 3									
I_{12}	red 5	red 5			red 5		red 5									

The $SLR(1)$ Parsing Automaton

Definition ($SLR(1)$ parsing automaton)

The **$SLR(1)$ parsing automaton** is defined as in the $LR(0)$ case (see Definition 12.6), except for the **transition relation**:

shift: $(aw, \alpha I, z) \vdash (w, \alpha II', z)$ if $\text{act}(I, a) = \text{shift}$ and $\text{goto}(I, a) = I'$

reduce_a: $(aw, \alpha II_1 \dots I_n, z) \vdash (aw, \alpha II', zi)$ if $\text{act}(I_n, a) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

reduce _{ε} : $(\varepsilon, \alpha II_1 \dots I_n, z) \vdash (\varepsilon, \alpha II', zi)$ if $\text{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(I, \varepsilon) = \text{accept}$

error_a: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, a) = \text{error}$

error _{ε} : $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, \varepsilon) = \text{error}$

- 1 Repetition: $SLR(1)$ Parsing
- 2 $LR(1)$ Parsing
- 3 $LALR(1)$ Parsing

SLR(1) Conflicts

Problem: not all conflicts can be resolved using fo sets

Example 13.1

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \textcolor{blue}{a} \quad R \rightarrow L$

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$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \textcolor{blue}{a} \quad R \rightarrow L$

$LR(0)(G_{LR})$:

$I_0 := LR(0)(\varepsilon) :$	$[S' \rightarrow \cdot S]$ $[L \rightarrow \cdot *R]$	$[S \rightarrow \cdot L=R]$ $[L \rightarrow \cdot \textcolor{blue}{a}]$	$[S \rightarrow \cdot R]$ $[R \rightarrow \cdot L]$
$I_1 := LR(0)(S) :$	$[S' \rightarrow S \cdot]$		
$I_2 := LR(0)(L) :$	$[S \rightarrow L \cdot =R]$	$[R \rightarrow L \cdot]$	
$I_3 := LR(0)(R) :$	$[S \rightarrow R \cdot]$		
$I_4 := LR(0)(*) :$	$[L \rightarrow * \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot \textcolor{blue}{a}]$
$I_5 := LR(0)(\textcolor{blue}{a}) :$	$[L \rightarrow \textcolor{blue}{a} \cdot]$		
$I_6 := LR(0)(L=) :$	$[S \rightarrow L = \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot \textcolor{blue}{a}]$
$I_7 := LR(0)(*R) :$	$[L \rightarrow *R \cdot]$		
$I_8 := LR(0)(*L) :$	$[R \rightarrow L \cdot]$		
$I_9 := LR(0)(L=R) :$	$[S \rightarrow L=R \cdot]$		

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$LR(0)(G_{LR})$:

$I_0 := LR(0)(\varepsilon) :$	$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot L=R]$	$[S \rightarrow \cdot R]$
	$[L \rightarrow \cdot *R]$	$[L \rightarrow \cdot \textcolor{blue}{a}]$	$[R \rightarrow \cdot L]$
$I_1 := LR(0)(S) :$	$[S' \rightarrow S \cdot]$		
$I_2 := LR(0)(L) :$	$[S \rightarrow L \cdot =R]$	$[R \rightarrow L \cdot]$	
$I_3 := LR(0)(R) :$	$[S \rightarrow R \cdot]$		
$I_4 := LR(0)(*) :$	$[L \rightarrow * \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot \textcolor{blue}{a}]$
$I_5 := LR(0)(\textcolor{blue}{a}) :$	$[L \rightarrow \textcolor{blue}{a} \cdot]$		
$I_6 := LR(0)(L=) :$	$[S \rightarrow L= \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot \textcolor{blue}{a}]$
$I_7 := LR(0)(*R) :$	$[L \rightarrow *R \cdot]$		
$I_8 := LR(0)(*L) :$	$[R \rightarrow L \cdot]$		
$I_9 := LR(0)(L=R) :$	$[S \rightarrow L=R \cdot]$		

But: conflict in I_2 **not** $SLR(1)$ -solvable since $= \in \text{fo}(R)$

Observation: not every element of $\text{fo}(A)$ can follow every occurrence of A

$\implies$ refinement of $LR(0)$ items by adding possible lookahead symbols

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Definition 13.2 ($LR(1)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha \beta_1 \beta_2 a w$, then $[A \rightarrow \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha \beta_1$.

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$, then $[A \rightarrow \beta_1 \cdot \beta_2, \varepsilon]$ is called an **$LR(1)$ item** for $\alpha \beta_1$.

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$, then $[A \rightarrow \beta_1 \cdot \beta_2, \varepsilon]$ is called an **LR(1) item** for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ , called the **LR(1) set** (or: **LR(1) information**) of γ .

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- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ , called the **$LR(1)$ set** (or: **$LR(1)$ information**) of γ .
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$.

Corollary 13.3

- 1 For every $\gamma \in X^*$, $LR(1)(\gamma)$ is finite.

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- ❶ For every $\gamma \in X^*$, $LR(1)(\gamma)$ is finite.
- ❷ $LR(1)(G)$ is finite.
- ❸ For every $\gamma \in X^*$, $LR(1)(\gamma)$ “contains” $LR(0)(\gamma)$, i.e.,

$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

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- ❸ For every $\gamma \in X^*$, $LR(1)(\gamma)$ “contains” $LR(0)(\gamma)$, i.e.,

$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

- ❹ $[A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(G) \implies x \in \text{fo}(A)$

Definition 13.4 ($LR(1)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $I \in LR(1)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ and $x \in \Sigma_{\varepsilon}$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2, x], [B \rightarrow \beta \cdot, a] \in I.$$

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- I has a **reduce/reduce conflict** if there exist $x \in \Sigma_{\varepsilon}$ and $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot, x], [B \rightarrow \beta \cdot, x] \in I.$$

Definition 13.4 ($LR(1)$ conflicts)

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- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ and $x \in \Sigma_\epsilon$ such that

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- I has a **reduce/reduce conflict** if there exist $x \in \Sigma_\epsilon$ and $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot, x], [B \rightarrow \beta \cdot, x] \in I.$$

Lemma 13.5

$G \in LR(1)$ iff no $I \in LR(1)(G)$ contains conflicting items.

Computing $LR(1)$ Sets I

The computation of $LR(0)$ sets (cf. Theorem 11.7) can be extended to cover right contexts:

Theorem 13.6 (Computing $LR(1)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

- 1 $LR(1)(\varepsilon)$ is the least set such that
 - $[S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$ and
 - if $[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon)$, $B \rightarrow \beta \in P$, and $y \in \text{fi}(\gamma x)$, then $[B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$.

The computation of $LR(0)$ sets (cf. Theorem 11.7) can be extended to cover right contexts:

Theorem 13.6 (Computing $LR(1)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \rightarrow S$ and reduced.

- ① $LR(1)(\varepsilon)$ is the least set such that
 - $[S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$ and
 - if $[A \rightarrow \cdot B\gamma, \mathbf{x}] \in LR(1)(\varepsilon)$, $B \rightarrow \beta \in P$, and $\mathbf{y} \in \mathbf{fi}(\gamma\mathbf{x})$, then $[B \rightarrow \cdot \beta, \mathbf{y}] \in LR(1)(\varepsilon)$.
- ② $LR(1)(\alpha Y)$ ($\alpha \in X^*, Y \in X$) is the least set such that
 - if $[A \rightarrow \gamma_1 \cdot Y \gamma_2, \mathbf{x}] \in LR(1)(\alpha)$, then $[A \rightarrow \gamma_1 Y \cdot \gamma_2, \mathbf{x}] \in LR(1)(\alpha Y)$ and
 - if $[A \rightarrow \gamma_1 \cdot B \gamma_2, \mathbf{x}] \in LR(1)(\alpha Y)$, $B \rightarrow \beta \in P$, and $\mathbf{y} \in \mathbf{fi}(\gamma_2 \mathbf{x})$, then $[B \rightarrow \cdot \beta, \mathbf{y}] \in LR(1)(\alpha Y)$.

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \textcolor{blue}{a} \quad R \rightarrow L$

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$G_{LR} : S' \rightarrow S \mid S \rightarrow L=R \mid R \mid L \rightarrow *R \mid \mathbf{a} \mid R \rightarrow L$

$LR(1)(G_{LR}) : [S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : [S' \rightarrow \cdot S, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : \quad [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \quad [S \rightarrow \cdot R, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot a, =]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \cdot B \gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR})$: $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, \varepsilon]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$ $[L \rightarrow \cdot \mathbf{a}, =]$	$[S \rightarrow \cdot L=R, \varepsilon]$ $[R \rightarrow \cdot L, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$ $[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$ $[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$ $[L \rightarrow *R \cdot, =]$	$[L \rightarrow * \cdot R, \varepsilon]$ $[L \rightarrow * \mathbf{a}, =]$	$[R \rightarrow \cdot L, =]$ $[L \rightarrow \cdot *R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$ $[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[\mathbf{R} \rightarrow L = \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \mid S \rightarrow L=R \mid R \mid L \rightarrow *R \mid \mathbf{a} \mid R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \mid S \rightarrow L=R \mid R \mid L \rightarrow *R \mid \mathbf{a} \mid R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L=R \cdot, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_{12} := LR(1)(L=\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_{12} := LR(1)(L=\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \mid S \rightarrow L=R \mid R \mid L \rightarrow *R \mid a \mid R \rightarrow L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$
$I'_{12} := LR(1)(L=a) :$	$[L \rightarrow a \cdot, \varepsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \varepsilon]$			
$I'_{14} := \emptyset$				

Computing $LR(1)$ Sets II

Example 13.7 (cf. Example 13.1)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\varepsilon) :$	$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot \mathbf{a}, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \varepsilon]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \varepsilon]$
	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow \cdot \mathbf{a}, =]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_5 := LR(1)(\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, =]$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \varepsilon]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot \mathbf{a}, \varepsilon]$
$I'_{12} := LR(1)(L=\mathbf{a}) :$	$[L \rightarrow \mathbf{a} \cdot, \varepsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \varepsilon]$			
$I'_{14} := \emptyset$				

In I'_2 : shift on $=$ /reduce on $\varepsilon \implies G_{LR} \in LR(1)$

The $LR(1)$ Action Function

Definition 13.8 ($LR(1)$ action function)

The $LR(1)$ action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $LR(1)$ Action Function

Definition 13.8 ($LR(1)$ action function)

The $LR(1)$ action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Corollary 13.9

For every $G \in CFG_\Sigma$, $G \in LR(1)$ iff its $LR(1)$ action function is well defined.

The $LR(1)$ goto Function

The goto function is defined in analogy to the $LR(0)$ case (Definition 11.9).

Definition 13.10 ($LR(1)$ goto function)

The function $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \begin{array}{l} \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y). \end{array}$$

The $LR(1)$ goto Function

The goto function is defined in analogy to the $LR(0)$ case (Definition 11.9).

Definition 13.10 ($LR(1)$ goto function)

The function $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \begin{array}{l} \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y). \end{array}$$

Again, act and goto form the $LR(1)$ parsing table of G .

The $LR(1)$ Parsing Table

Example 13.11 (cf. Example 13.7)

$LR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5			I'_8	I'_7
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

The $LR(1)$ Parsing Automaton I

Definition 13.12 ($LR(1)$ parsing automaton)

The $LR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 12.6), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha II', z)$ if $\text{act}(I, a) = \text{shift}$ and $\text{goto}(I, a) = I'$

reduce_a: $(aw, \alpha II_1 \dots I_n, z) \vdash (aw, \alpha II', zi)$ if $\text{act}(I_n, a) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

reduce _{ε} : $(\varepsilon, \alpha II_1 \dots I_n, z) \vdash (\varepsilon, \alpha II', zi)$ if $\text{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(I, \varepsilon) = \text{accept}$

error_a: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, a) = \text{error}$

error _{ε} : $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, \varepsilon) = \text{error}$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5			I'_8	I'_7
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

($a=*a$, I'_0 , ϵ)

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5			I'_8	I'_7
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}			I'_{10}	I'_9
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}			I'_{10}	I'_{13}
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{13}, 445)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ \mathcal{N}$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{13}, 445)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{10}, 4453)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R$ | R (1,2) $L \rightarrow *R$ | a (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{13}, 445)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{10}, 4453)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_9, 44535)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ _N$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{13}, 445)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{10}, 4453)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_9, 44535)$
 $\vdash (\epsilon, I'_0 I'_1, 445351)$

The $LR(1)$ Parsing Automaton II

Example 13.13 (cf. Example 13.7)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

$LR(1)(G_{LR})$	act/goto $ \Sigma_\epsilon$				goto $ _N$		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4					
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3					
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

$LR(1)$ parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{13}, 445)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{10}, 4453)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_9, 44535)$
 $\vdash (\epsilon, I'_0 I'_1, 445351)$
 $\vdash (\epsilon, \epsilon, 4453510)$

- 1 Repetition: $SLR(1)$ Parsing
- 2 $LR(1)$ Parsing
- 3 $LALR(1)$ Parsing

- **Motivation:** resolving conflicts using *LR*(1) too expensive

- **Motivation:** resolving conflicts using $LR(1)$ too expensive
- Example 13.1/13.7: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$

- **Motivation:** resolving conflicts using $LR(1)$ **too expensive**
- Example 13.1/13.7: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$
- A. Johnstone, E. Scott: *Generalised Reduction Modified LR Parsing for Domain Specific Language Prototyping*, HICSS '02, IEEE, 2002, <http://doi.ieeecomputersociety.org/10.1109/HICSS.2002.994495>:

Grammar	$ LR(0)(G) $	$ LR(1)(G) $
Ansi-C	381	1788
Pascal	368	1395

- **Motivation:** resolving conflicts using $LR(1)$ **too expensive**
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- **Observation:** potential **redundancy by containment** of $LR(0)$ sets in $LR(1)$ sets (cf. Corollary 13.3)

- **Motivation:** resolving conflicts using $LR(1)$ **too expensive**
- Example 13.1/13.7: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$
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Grammar	$ LR(0)(G) $	$ LR(1)(G) $
Ansi-C	381	1788
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- **Observation:** potential **redundancy by containment** of $LR(0)$ sets in $LR(1)$ sets (cf. Corollary 13.3)

Definition 13.14 ($LR(0)$ equivalence)

Let $\text{lr}_0 : LR(1)(G) \rightarrow LR(0)(G)$ be defined by

$$\text{lr}_0(I) := \{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in I\}.$$

Two sets $I_1, I_2 \in LR(1)(G)$ are called **$LR(0)$ -equivalent** (notation: $I_1 \sim_0 I_2$) if $\text{lr}_0(I_1) = \text{lr}_0(I_2)$.

Example 13.15 (cf. Example 13.1/13.7)

$$G_{LR} : \quad \begin{array}{l} S' \rightarrow S \quad S \rightarrow L=R \mid R \\ L \rightarrow *R \mid a \quad R \rightarrow L \end{array}$$

$$LR(0)(G_{LR}) :$$

$$I_0(\varepsilon) : \quad \begin{array}{ll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot L=R] \\ [S \rightarrow \cdot R] & [L \rightarrow \cdot *R] \\ [L \rightarrow \cdot a] & [R \rightarrow \cdot L] \end{array}$$

$$I_1(S) : [S' \rightarrow S \cdot]$$

$$I_2(L) : [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$$

$$I_3(R) : [S \rightarrow R \cdot]$$

$$I_4(*) : \quad \begin{array}{ll} [L \rightarrow * \cdot R] & [R \rightarrow \cdot L] \\ [L \rightarrow *R \cdot] & [L \rightarrow \cdot a] \end{array}$$

$$I_5(a) : [L \rightarrow a \cdot]$$

$$I_6(L=) : \quad \begin{array}{ll} [S \rightarrow L= \cdot R] & [R \rightarrow \cdot L] \\ [L \rightarrow *R \cdot] & [L \rightarrow \cdot a] \end{array}$$

$$I_7(*R) : [L \rightarrow *R \cdot]$$

$$I_8(*L) : [R \rightarrow L \cdot]$$

$$I_9(L=R) : [S \rightarrow L=R \cdot]$$

$$LR(1)(G_{LR}) :$$

$$I'_0(\varepsilon) : \quad \begin{array}{ll} [S' \rightarrow \cdot S, \varepsilon] & [S \rightarrow \cdot L=R, \varepsilon] \\ [S \rightarrow \cdot R, \varepsilon] & [L \rightarrow \cdot *R, =] \\ [L \rightarrow \cdot a, =] & [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] & [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_1(S) : [S' \rightarrow S \cdot, \varepsilon]$$

$$I'_2(L) : [S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I'_3(R) : [S \rightarrow R \cdot, \varepsilon]$$

$$I'_4(*) : \quad \begin{array}{ll} [L \rightarrow * \cdot R, =] & [L \rightarrow * \cdot R, \varepsilon] \\ [R \rightarrow \cdot L, =] & [R \rightarrow \cdot L, \varepsilon] \end{array}$$

$$\quad \begin{array}{ll} [L \rightarrow *R \cdot, =] & [L \rightarrow \cdot a, =] \\ [L \rightarrow *R \cdot, \varepsilon] & [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_5(a) : [L \rightarrow a \cdot, =] \quad [L \rightarrow a \cdot, \varepsilon]$$

$$I'_6(L=) : \quad \begin{array}{ll} [S \rightarrow L= \cdot R, \varepsilon] & [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow *R \cdot, \varepsilon] & [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_7(*R) : [L \rightarrow *R \cdot, =] \quad [L \rightarrow *R \cdot, \varepsilon]$$

$$I'_8(*L) : [R \rightarrow L \cdot, =] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I'_9(L=R) : [S \rightarrow L=R \cdot, \varepsilon]$$

$$I'_{10}(L=L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I'_{11}(L=*) : \quad \begin{array}{ll} [L \rightarrow * \cdot R, \varepsilon] & [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow *R \cdot, \varepsilon] & [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_{12}(L=a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I'_{13}(L=*R) : [L \rightarrow *R \cdot, \varepsilon]$$

Example 13.15 (cf. Example 13.1/13.7)

$$G_{LR} : \quad \begin{array}{l} S' \rightarrow S \quad S \rightarrow L=R \mid R \\ L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L \end{array}$$

$$LR(0)(G_{LR}) :$$

$$I_0(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \\ [S \rightarrow \cdot R] \quad [L \rightarrow \cdot *R] \\ [L \rightarrow \cdot \mathbf{a}] \quad [R \rightarrow \cdot L] \end{array}$$

$$I_1(S) : [S' \rightarrow S \cdot]$$

$$I_2(L) : [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$$

$$I_3(R) : [S \rightarrow R \cdot]$$

$$I_4(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot \mathbf{a}] \end{array}$$

$$I_5(\mathbf{a}) : [L \rightarrow \mathbf{a} \cdot]$$

$$I_6(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot \mathbf{a}] \end{array}$$

$$I_7(*R) : [L \rightarrow *R \cdot]$$

$$I_8(*L) : [R \rightarrow L \cdot]$$

$$I_9(L=R) : [S \rightarrow L=R \cdot]$$

$$\Rightarrow I_4' \sim_0 I_{11}'$$

$$LR(1)(G_{LR}) :$$

$$I_0'(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \\ [S \rightarrow \cdot R, \varepsilon] \quad [L \rightarrow \cdot *R, =] \\ [L \rightarrow \cdot \mathbf{a}, =] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, \varepsilon] \end{array}$$

$$I_1'(S) : [S' \rightarrow S \cdot, \varepsilon]$$

$$I_2'(L) : [S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I_3'(R) : [S \rightarrow R \cdot, \varepsilon]$$

$$I_4'(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, =] \quad [L \rightarrow * \cdot R, \varepsilon] \\ [R \rightarrow \cdot L, =] \quad [R \rightarrow \cdot L, \varepsilon] \end{array}$$

$$\quad \begin{array}{l} [L \rightarrow \cdot *R, =] \quad [L \rightarrow \cdot \mathbf{a}, =] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, \varepsilon] \end{array}$$

$$I_5'(\mathbf{a}) : [L \rightarrow \mathbf{a} \cdot, \varepsilon]$$

$$I_6'(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, \varepsilon] \end{array}$$

$$I_7'(*R) : [L \rightarrow *R \cdot, \varepsilon]$$

$$I_8'(*L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I_9'(L=R) : [S \rightarrow L=R \cdot, \varepsilon]$$

$$I_{10}'(L=L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I_{11}'(L=*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, \varepsilon] \end{array}$$

$$I_{12}'(L=\mathbf{a}) : [L \rightarrow \mathbf{a} \cdot, \varepsilon]$$

$$I_{13}'(L=*R) : [L \rightarrow *R \cdot, \varepsilon]$$

Example 13.15 (cf. Example 13.1/13.7)

$$G_{LR} : \quad \begin{array}{l} S' \rightarrow S \quad S \rightarrow L=R \mid R \\ L \rightarrow *R \mid a \quad R \rightarrow L \end{array}$$

$$LR(0)(G_{LR}) :$$

$$I_0(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \\ [S \rightarrow \cdot R] \quad [L \rightarrow \cdot *R] \\ [L \rightarrow \cdot a] \quad [R \rightarrow \cdot L] \end{array}$$

$$I_1(S) : [S' \rightarrow S \cdot]$$

$$I_2(L) : [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$$

$$I_3(R) : [S \rightarrow R \cdot]$$

$$I_4(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_5(a) : [L \rightarrow a \cdot]$$

$$I_6(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_7(*R) : [L \rightarrow *R \cdot]$$

$$I_8(*L) : [R \rightarrow L \cdot]$$

$$I_9(L=R) : [S \rightarrow L=R \cdot]$$

$$\Rightarrow \quad \begin{array}{l} I_4' \sim_0 I_{11}' \\ I_5' \sim_0 I_{12}' \end{array}$$

$$LR(1)(G_{LR}) :$$

$$I_0'(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \\ [S \rightarrow \cdot R, \varepsilon] \quad [L \rightarrow \cdot *R, =] \\ [L \rightarrow \cdot a, =] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_1'(S) : [S' \rightarrow S \cdot, \varepsilon]$$

$$I_2'(L) : [S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I_3'(R) : [S \rightarrow R \cdot, \varepsilon]$$

$$I_4'(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, =] \quad [L \rightarrow * \cdot R, \varepsilon] \\ [R \rightarrow \cdot L, =] \quad [R \rightarrow \cdot L, \varepsilon] \end{array}$$

$$\quad \begin{array}{l} [L \rightarrow \cdot *R, =] \quad [L \rightarrow \cdot a, =] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_5'(a) : [L \rightarrow a \cdot, =] \quad [L \rightarrow a \cdot, \varepsilon]$$

$$I_6'(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_7'(*R) : [L \rightarrow *R \cdot, =] \quad [L \rightarrow *R \cdot, \varepsilon]$$

$$I_8'(*L) : [R \rightarrow L \cdot, =] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I_9'(L=R) : [S \rightarrow L=R \cdot, \varepsilon]$$

$$I_{10}'(L=L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I_{11}'(L=*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_{12}'(L=a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I_{13}'(L=*R) : [L \rightarrow *R \cdot, \varepsilon]$$

LR(0) Equivalence

Example 13.15 (cf. Example 13.1/13.7)

$$G_{LR} : \quad \begin{array}{l} S' \rightarrow S \quad S \rightarrow L=R \mid R \\ L \rightarrow *R \mid a \quad R \rightarrow L \end{array}$$

$$LR(0)(G_{LR}) :$$

$$I_0(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \\ [S \rightarrow \cdot R] \quad [L \rightarrow \cdot *R] \\ [L \rightarrow \cdot a] \quad [R \rightarrow \cdot L] \end{array}$$

$$I_1(S) : [S' \rightarrow S \cdot]$$

$$I_2(L) : [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$$

$$I_3(R) : [S \rightarrow R \cdot]$$

$$I_4(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_5(a) : [L \rightarrow a \cdot]$$

$$I_6(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_7(*R) : [L \rightarrow * \cdot R]$$

$$I_8(*L) : [R \rightarrow L \cdot]$$

$$I_9(L=R) : [S \rightarrow L=R \cdot]$$

$$\Rightarrow \quad \begin{array}{l} I_4' \sim_0 I_{11}' \\ I_5' \sim_0 I_{12}' \\ I_7' \sim_0 I_{13}' \end{array}$$

$$LR(1)(G_{LR}) :$$

$$I_0'(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \\ [S \rightarrow \cdot R, \varepsilon] \quad [L \rightarrow \cdot *R, =] \\ [L \rightarrow \cdot a, =] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_1'(S) : [S' \rightarrow S \cdot, \varepsilon]$$

$$I_2'(L) : [S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I_3'(R) : [S \rightarrow R \cdot, \varepsilon]$$

$$I_4'(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, =] \quad [L \rightarrow * \cdot R, \varepsilon] \\ [R \rightarrow \cdot L, =] \quad [R \rightarrow \cdot L, \varepsilon] \end{array}$$

$$\quad \begin{array}{l} [L \rightarrow \cdot *R, =] \quad [L \rightarrow \cdot a, =] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_5'(a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I_6'(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_7'(*R) : [L \rightarrow * \cdot R, \varepsilon]$$

$$I_8'(*L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I_9'(L=R) : [S \rightarrow L=R \cdot, \varepsilon]$$

$$I_{10}'(L=L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I_{11}'(L=*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I_{12}'(L=a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I_{13}'(L=*R) : [L \rightarrow * \cdot R, \varepsilon]$$

Example 13.15 (cf. Example 13.1/13.7)

$$G_{LR} : \quad \begin{array}{l} S' \rightarrow S \quad S \rightarrow L=R \mid R \\ L \rightarrow *R \mid a \quad R \rightarrow L \end{array}$$

$$LR(0)(G_{LR}) :$$

$$I_0(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \\ [S \rightarrow \cdot R] \quad [L \rightarrow \cdot *R] \\ [L \rightarrow \cdot a] \quad [R \rightarrow \cdot L] \end{array}$$

$$I_1(S) : [S' \rightarrow S \cdot]$$

$$I_2(L) : [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$$

$$I_3(R) : [S \rightarrow R \cdot]$$

$$I_4(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_5(a) : [L \rightarrow a \cdot]$$

$$I_6(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \\ [L \rightarrow * \cdot R] \quad [L \rightarrow \cdot a] \end{array}$$

$$I_7(*R) : [L \rightarrow *R \cdot]$$

$$I_8(*L) : [R \rightarrow L \cdot]$$

$$I_9(L=R) : [S \rightarrow L=R \cdot]$$

$$\Rightarrow \quad \begin{array}{l} I'_4 \sim_0 I'_{11} \\ I'_5 \sim_0 I'_{12} \\ I'_7 \sim_0 I'_{13} \\ I'_8 \sim_0 I'_{10} \end{array}$$

$$LR(1)(G_{LR}) :$$

$$I'_0(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \\ [S \rightarrow \cdot R, \varepsilon] \quad [L \rightarrow \cdot *R, =] \\ [L \rightarrow \cdot a, =] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_1(S) : [S' \rightarrow S \cdot, \varepsilon]$$

$$I'_2(L) : [S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$$

$$I'_3(R) : [S \rightarrow R \cdot, \varepsilon]$$

$$I'_4(*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, =] \quad [L \rightarrow * \cdot R, \varepsilon] \\ [R \rightarrow \cdot L, =] \quad [R \rightarrow \cdot L, \varepsilon] \end{array}$$

$$\quad \begin{array}{l} [L \rightarrow \cdot *R, =] \quad [L \rightarrow \cdot a, =] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_5(a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I'_6(L=) : \quad \begin{array}{l} [S \rightarrow L= \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_7(*R) : [L \rightarrow *R \cdot, \varepsilon]$$

$$I'_8(*L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I'_9(L=R) : [S \rightarrow L=R \cdot, \varepsilon]$$

$$I'_{10}(L=L) : [R \rightarrow L \cdot, \varepsilon]$$

$$I'_{11}(L=*) : \quad \begin{array}{l} [L \rightarrow * \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \\ [L \rightarrow \cdot *R, \varepsilon] \quad [L \rightarrow \cdot a, \varepsilon] \end{array}$$

$$I'_{12}(L=a) : [L \rightarrow a \cdot, \varepsilon]$$

$$I'_{13}(L=*R) : [L \rightarrow *R \cdot, \varepsilon]$$

Corollary 13.16

For every $G \in CFG_{\Sigma}$, $|LR(1)(G) / \sim_0| = |LR(0)(G)|$.

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For every $G \in CFG_{\Sigma}$, $|LR(1)(G) / \sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$ -equivalent $LR(1)$ sets (maintaining the lookahead information, but possibly introducing conflicts)

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For every $G \in CFG_{\Sigma}$, $|LR(1)(G) / \sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$ -equivalent $LR(1)$ sets (maintaining the lookahead information, but possibly introducing conflicts)

Definition 13.17 ($LALR(1)$ sets)

Let $G \in CFG_{\Sigma}$.

- An information $I \in LR(1)(G)$ determines the $LALR(1)$ set
$$\bigcup [I]_{\sim_0} = \bigcup \{I' \in LR(1)(G) \mid I' \sim_0 I\}.$$
- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

Corollary 13.16

For every $G \in CFG_{\Sigma}$, $|LR(1)(G) / \sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$ -equivalent $LR(1)$ sets (maintaining the lookahead information, but possibly introducing conflicts)

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- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

Remark: by Corollary 13.16, $|LALR(1)(G)| = |LR(0)(G)|$
(but $LALR(1)$ sets provide additional lookahead information)

Example 13.18 (cf. Example 13.15)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid \mathbf{a} \quad R \rightarrow L$

$LR(0)(G_{LR}) :$

$I_0(\varepsilon) :$ $[S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \quad [L \rightarrow \cdot *R] \quad [R \rightarrow \cdot L]$
 $[S \rightarrow \cdot R] \quad [L \rightarrow \cdot \mathbf{a}]$

$I_1(S) :$ $[S' \rightarrow S \cdot]$

$I_2(L) :$ $[S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$

$I_3(R) :$ $[S \rightarrow R \cdot]$

$I_4(*) :$ $[L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \quad [L \rightarrow *R \cdot] \quad [L \rightarrow \cdot \mathbf{a}]$

$I_5(\mathbf{a}) :$ $[L \rightarrow \mathbf{a} \cdot]$

$I_6(L=) :$ $[S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \quad [L \rightarrow *R \cdot] \quad [L \rightarrow \cdot \mathbf{a}]$

$I_7(*R) :$ $[L \rightarrow *R \cdot]$

$I_8(*L) :$ $[R \rightarrow L \cdot]$

$I_9(L=R) :$ $[S \rightarrow L=R \cdot]$

$LALR(1)(G_{LR}) :$

$I_0'' := I_0' :$ $[S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \quad [S \rightarrow \cdot R, \varepsilon] \quad [L \rightarrow \cdot *R, =/\varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, =/\varepsilon] \quad [R \rightarrow \cdot L, \varepsilon]$

$I_1'' := I_1' :$ $[S' \rightarrow S \cdot, \varepsilon]$

$I_2'' := I_2' :$ $[S \rightarrow L \cdot =R, \varepsilon] \quad [R \rightarrow L \cdot, \varepsilon]$

$I_3'' := I_3' :$ $[S \rightarrow R \cdot, \varepsilon]$

$I_4'' := I_4' \cup I_{11}' :$ $[L \rightarrow * \cdot R, =/\varepsilon] \quad [R \rightarrow \cdot L, =/\varepsilon] \quad [L \rightarrow *R \cdot, =/\varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, =/\varepsilon]$

$I_5'' := I_5' \cup I_{12}' :$ $[L \rightarrow \mathbf{a} \cdot, =/\varepsilon]$

$I_6'' := I_6' :$ $[S \rightarrow L= \cdot R, \varepsilon] \quad [R \rightarrow \cdot L, \varepsilon] \quad [L \rightarrow *R \cdot, \varepsilon] \quad [L \rightarrow \cdot \mathbf{a}, \varepsilon]$

$I_7'' := I_7' \cup I_{13}' :$ $[L \rightarrow *R \cdot, =/\varepsilon]$

$I_8'' := I_8' \cup I_{10}' :$ $[R \rightarrow L \cdot, =/\varepsilon]$

$I_9'' := I_9' :$ $[S \rightarrow L=R \cdot, \varepsilon]$

The $LALR(1)$ Action Function

The $LALR(1)$ action function is defined in analogy to the $LR(1)$ case (Definition 13.8).

Definition 13.19 ($LALR(1)$ action function)

The $LALR(1)$ action function

$$\text{act} : LALR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } \pi(i) = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $LALR(1)$ Action Function

The $LALR(1)$ action function is defined in analogy to the $LR(1)$ case (Definition 13.8).

Definition 13.19 ($LALR(1)$ action function)

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Definition 13.20 ($LALR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $LALR(1)$ property (notation: $G \in LALR(1)$) if its $LALR(1)$ action function is well defined.

The $LALR(1)$ goto Function

Example 13.21 (cf. Example 13.18)

$G_{LR} \in LALR(1)$

The $LALR(1)$ goto Function

Example 13.21 (cf. Example 13.18)

$$G_{LR} \in LALR(1)$$

Also the $LR(1)$ goto function (Definition 13.10) carries over to the $LALR(1)$ case. Reason:

Lemma 13.22

Let $G \in CFG_{\Sigma}$ and $I_1, I_2 \in LR(1)(G)$ such that $I_1 \sim_0 I_2$. Then, for every $Y \in X$, $\text{goto}(I_1, Y) \sim_0 \text{goto}(I_2, Y)$.

The $LALR(1)$ goto Function

Example 13.21 (cf. Example 13.18)

$$G_{LR} \in LALR(1)$$

Also the $LR(1)$ goto function (Definition 13.10) carries over to the $LALR(1)$ case. Reason:

Lemma 13.22

Let $G \in CFG_{\Sigma}$ and $I_1, I_2 \in LR(1)(G)$ such that $I_1 \sim_0 I_2$. Then, for every $Y \in X$, $\text{goto}(I_1, Y) \sim_0 \text{goto}(I_2, Y)$.

Again, act and goto form the $LALR(1)$ parsing table of G .

The $LALR(1)$ Parsing Table

Example 13.23 (cf. Example 13.18)

$LALR(1)(G_{LR})$	act/goto Σ_ϵ				goto N		
	*	=	a	ϵ	S	L	R
I_0''	shift/ I_4''		shift/ I_5''		I_1''	I_2''	I_3''
I_1''				accept			
I_2''		shift/ I_6''		red 5			
I_3''				red 2			
I_4''	shift/ I_4''		shift/ I_5''			I_8''	I_7''
I_5''		red 4		red 4			
I_6''	shift/ I_4''		shift/ I_5''			I_8''	I_9''
I_7''		red 3		red 3			
I_8''		red 5		red 5			
I_9''				red 1			

(empty = error/ $\emptyset$)