Compiler Construction

Lecture 15: Semantic Analysis I (Attribute Grammars)

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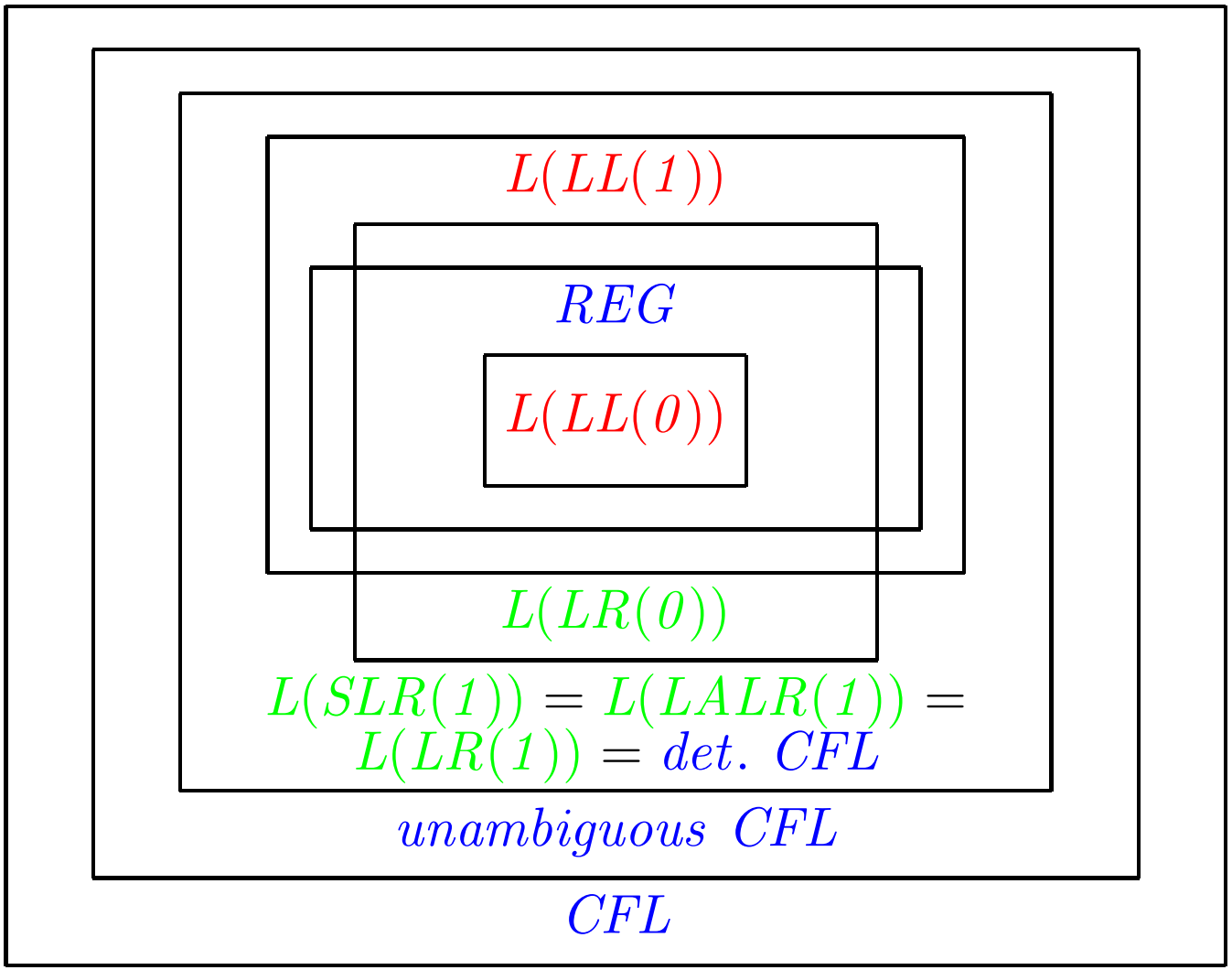
Outline

Repetition: Overview of Language Classes

(cf. O. Mayer: *Syntaxanalyse*, BI-Verlag, 1978, p. 409ff)



Moreover:

- $L(LL(k)) \subsetneq L(LL(k+1)) \subsetneq L(LR(1))$ for every $k \in \mathbb{N}$
- $L(LR(k)) = L(LR(1))$ for every $k \geq 1$

Why $REG \not\subseteq L(LR(0))$?

Definition (cf. Exercise 6.2)

A language $L \subseteq \Sigma^*$ is called prefix-free if $L \cap L \cdot \Sigma^+ = \emptyset$, i.e., if no proper prefix of an element of L is again in L.

Lemma (cf. Exercise 6.2)

$G \in LR(0) \implies L(G)$ *prefix-free*

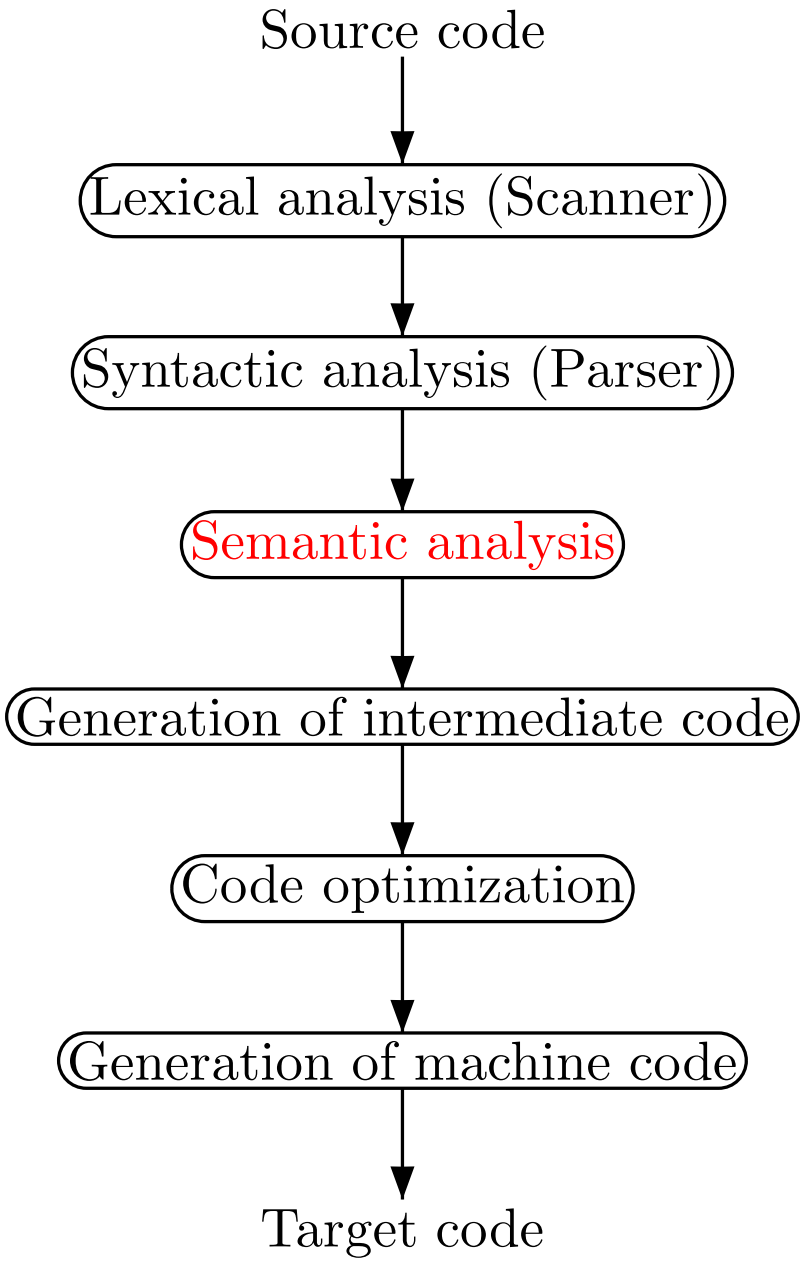
Corollary

$\{a, aa\} \in REG \setminus L(LR(0))$

Conjecture: $L \in REG \setminus L(LR(0)) \implies L(G)$ not prefix-free?
(cf. Exercise 7)

Outline

Conceptual Structure of a Compiler



Outline

Beyond Syntax

To generate (efficient) code, the compiler needs to answer many questions:

- Are there identifiers that are not declared? Declared but not used?
- Is `x` a scalar, an array, or a procedure? Of which type?
- Which declaration of `x` is used by each reference?
- Is `x` defined before it is used?
- Is the expression `3 * x + y` type consistent?
- Where should the value of `x` be stored (register/stack/heap)?
- Do `p` and `q` refer to the same memory location (aliasing)?
- ...

These cannot be expressed using context-free grammars!
(e.g., $\{ww \mid w \in \Sigma^*\} \notin CFL_\Sigma$)

Static Semantics

Static semantics refers to properties of program constructs

- which are true for every occurrence of this construct in every program execution (static) and
- can be decided at compile time
- but are context-sensitive and thus not expressible using context-free grammars (semantics).

Example properties:

Static: type or declaredness of an identifier, number of registers required to evaluate an expression, ...

Dynamic: value of an expression, size of runtime stack, ...

Among others, these properties are determined by

Scope rules: defines part of program where a declaration is valid

Typing rules: defines type consistency of expressions, statements, ...

Outline

Attribute Grammars I

Goal: compute context-dependent but runtime-independent properties of a given program

Idea: enrich context-free grammar by semantic rules which annotate syntax tree with attribute values

$\Longrightarrow$ Semantic analysis = attribute evaluation

Result: attributed syntax tree

In greater detail:

- With every nonterminal a set of attributes is associated.
- Two types of attributes are distinguished:

  Synthesized: bottom-up computation (from the leaves to the root)

  Inherited: top-down computation (from the root to the leaves)
- With every production a set of semantic rules is associated.

Attribute Grammars II

Advantage: attribute grammars provide a very flexible and broadly applicable mechanism for transporting information throught the syntax tree ("syntax-directed translation")

- Attribute values: symbol tables, data types, code, error flags, ...
- Application in Compiler Construction:
  - static semantics
  - program analysis for optimization
  - code generation
  - error handling
- Automatic attribute evaluation by compiler generators (cf. `yacc`'s synthesized attributes)
- Originally designed by D. Knuth for defining the semantics of context-free languages (Math. Syst. Theory 2 (1968), pp. 127–145)

Example: Knuth's Binary Numbers I

Example 15.1 (only synthesized attributes)

Binary numbers (with fraction):

G_B :	Numbers	$S \to L$	$v.0$	$=$	$v.1$
		$S \to L\,.\,L$	$v.0$	$=$	$v.1 + v.3/2^{l.3}$
	Lists	$L \to B$	$v.0$	$=$	$v.1$
			$l.0$	$=$	1
		$L \to LB$	$v.0$	$=$	$2 * v.1 + v.2$
			$l.0$	$=$	$l.1 + 1$
	Bits	$B \to$ `0`	$v.0$	$=$	0
	Bits	$B \to$ `1`	$v.0$	$=$	1

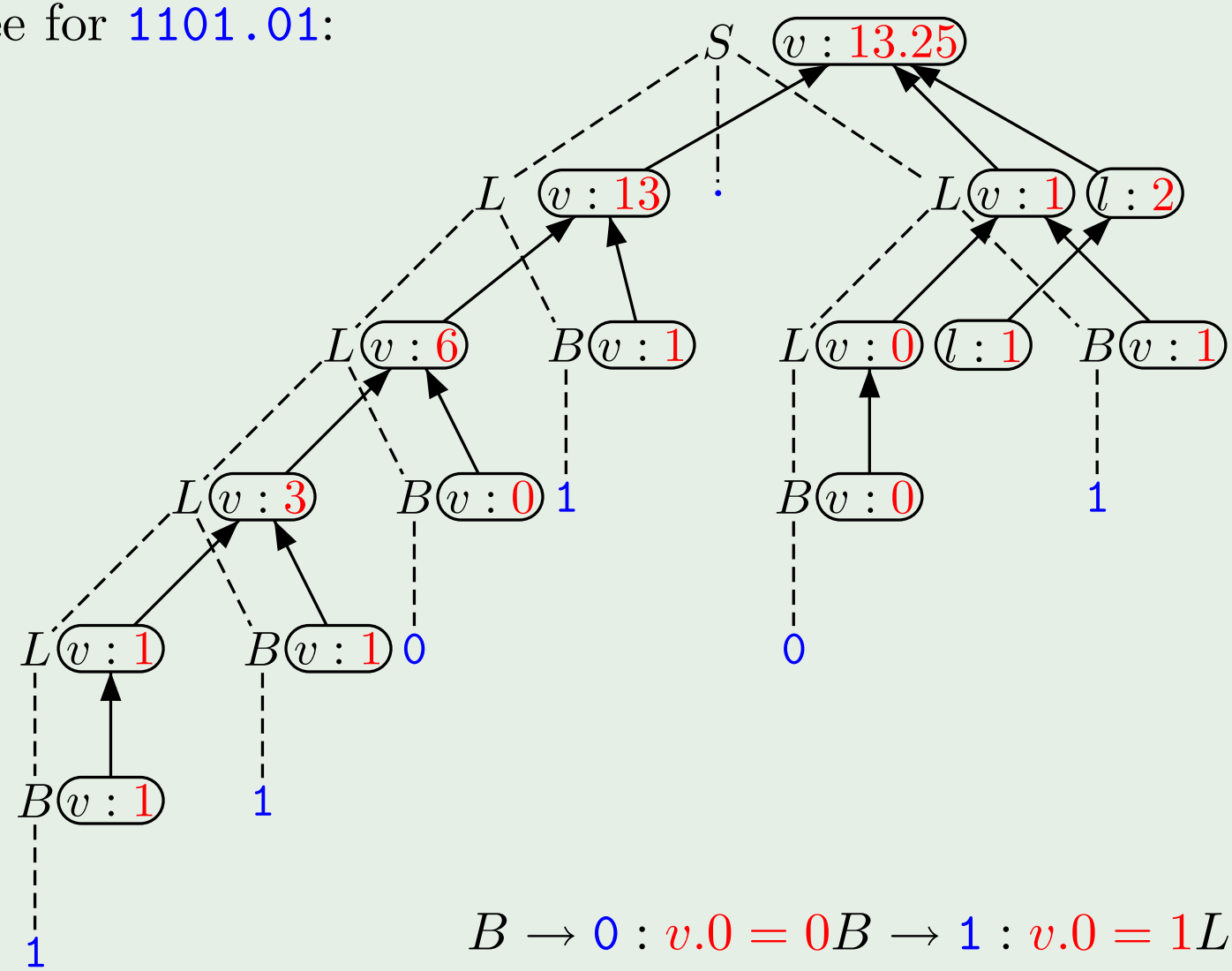
Synthesized attributes of S, L, B: v (value; domain: $V^v := \mathbb{Q}$)
of L: l (length; domain: $V^l := \mathbb{N}$)

Semantic rules: equations with attribute variables
(index = position of symbol; 0 = left-hand side)

Example: Knuth's Binary Numbers II

Example 15.1 (continued)

Syntax tree for 1101.01:



$B \to 0 : v.0 = 0 \quad B \to 1 : v.0 = 1 \quad L \to B : v.0 = v.1 \quad L \to B : l.0 = 1 \quad L \to LB : v.0 = 2 * v.1 + v.2 \quad L \to LB : l.0 =$

Outline

Adding Inherited Attributes I

Example 15.2 (synthesized + inherited attributes)

Binary numbers (with fraction):

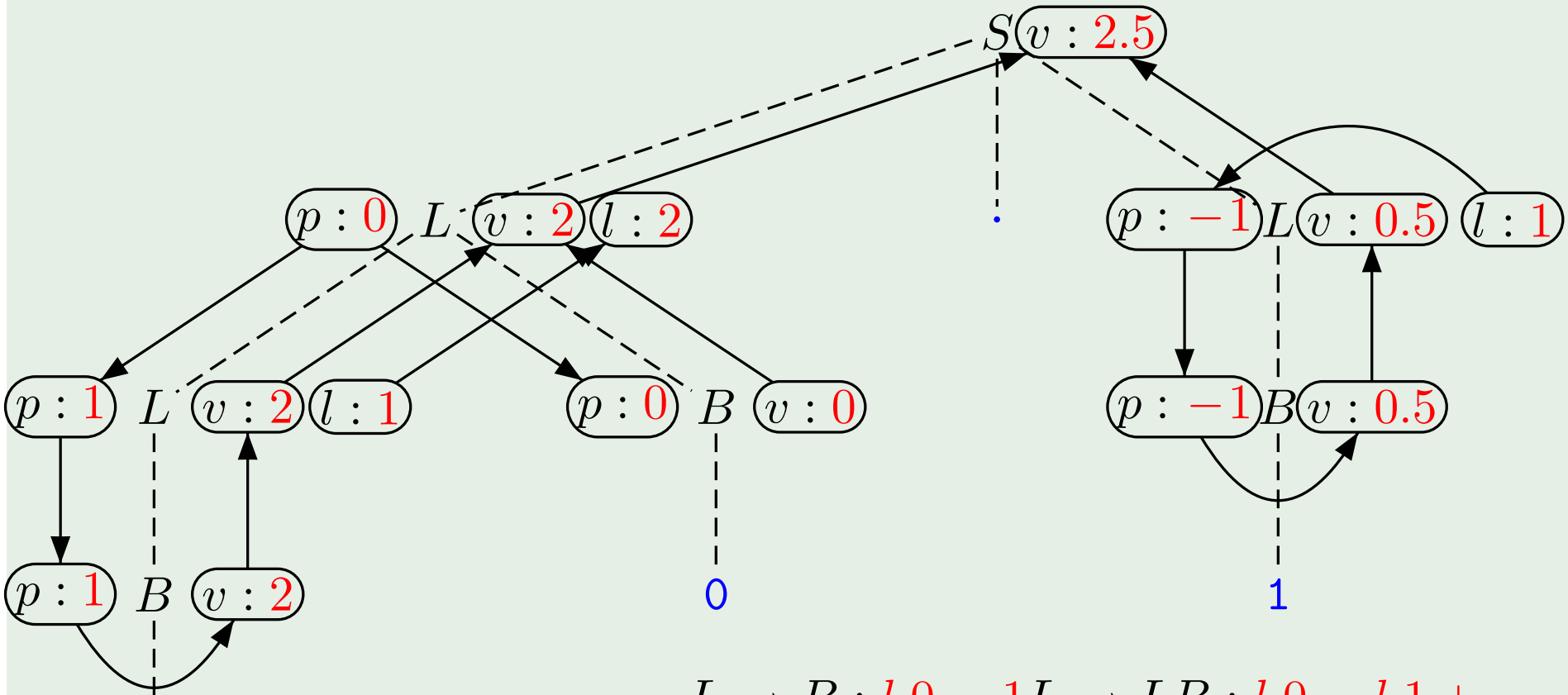
G'_B :	Numbers	$S \to L$	$v.0$	$=$	$v.1$
			$p.1$	$=$	0
		$S \to L\,.\,L$	$v.0$	$=$	$v.1 + v.3$
			$p.1$	$=$	0
			$p.3$	$=$	$-\,l.3$
	Lists	$L \to B$	$v.0$	$=$	$v.1$
			$l.0$	$=$	1
			$p.1$	$=$	$p.0$
		$L \to LB$	$v.0$	$=$	$v.1 + v.2$
			$l.0$	$=$	$l.1 + 1$
			$p.1$	$=$	$p.0 + 1$
			$p.2$	$=$	$p.0$
	Bits	$B \to$ `0`	$v.0$	$=$	0
	Bits	$B \to$ `1`	$v.0$	$=$	$2^{p.0}$

Synthesized attributes of S, L, B: v (value; domain: $V^v := \mathbb{Q}$)
of L: l (length; domain: $V^l := \mathbb{N}$)
Inherited attribute of L, B: p (position; domain: $V^p := \mathbb{Z}$)

Adding Inherited Attributes II

Example 15.2 (continued)

Syntax tree for 10.1:



$L \to B : l.0 = 1 \quad L \to LB : l.0 = l.1 +$

$1 \quad S \to L.L : p.1 = 0 \quad S \to L.L : p.3 = -l.3 \quad L \to LB : p.1 = p.0 + 1 \quad L \to$

$LB : p.2 = p.0 \quad L \to B : p.1 = p.0 \quad B \to 0 : v.0 = 0 \quad B \to 1 : v.0 = 2^{p.0} \quad L \to$

Outline

Formal Definition of Attribute Grammars I

Definition 15.3 (Attribute grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.
- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.
- Every production $\pi = Y_0 \to Y_1 \dots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \dots, r\}\}$$
of attribute variables of π with the subsets of inner and outer variables:
$$In_\pi := \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\}$$
$$Out_\pi := Var_\pi \setminus In_\pi$$
- A semantic rule of π is an equation of the form
$$\alpha.i = f(\alpha_1.i_1, \dots, \alpha_n.i_n)$$
where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \to V^\alpha$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every inner variable of π, and let $E := (E_\pi \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V \rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Formal Definition of Attribute Grammars II

Example 15.4 (cf. Example 15.2)

$\mathfrak{A}_B \in AG$ for binary numbers:

- Attributes: $Att = Syn \uplus Inh$ with $Syn = \{v, l\}$ and $Inh = \{p\}$
- Value sets: $V^v = \mathbb{Q},\ V^l = \mathbb{N},\ V^p = \mathbb{Z}$
- Attribute assignment:

$Y \in X$	S	L	B	0	1
$\mathrm{syn}(Y)$	$\{v\}$	$\{v, l\}$	$\{v\}$	$\emptyset$	$\emptyset$
$\mathrm{inh}(Y)$	$\emptyset$	$\{p\}$	$\{p\}$	$\emptyset$	$\emptyset$

- Attribute variables:

$\pi \in P$	$S \to L$	$S \to L.L$	$L \to B$
In_π	$\{v.0, p.1\}$	$\{v.0, p.1, p.3\}$	$\{v.0, l.0, p.1\}$
Out_π	$\{v.1, l.1\}$	$\{v.1, l.1, v.3, l.3\}$	$\{v.1, p.0\}$

$\pi \in P$	$L \to LB$	$B \to 0$	$B \to 1$
In_π	$\{v.0, l.0, p.1, p.2\}$	$\{v.0\}$	$\{v.0\}$
Out_π	$\{v.1, v.2, l.1, p.0\}$	$\{p.0\}$	$\{p.0\}$

- Semantic rules: see Example 15.2
  (e.g., $E_{S \to L} = \{v.0 = v.1, p.1 = 0\}$)