Compiler Construction

Lecture 16: Semantic Analysis II (Circularity of Attribute Grammars)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH Aachen University

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/cc10/

Winter semester 2010/11

Outline

Attribute Grammars I

Goal: compute context-dependent but runtime-independent properties of a given program

Idea: enrich context-free grammar by semantic rules which annotate syntax tree with attribute values

$\Longrightarrow$ Semantic analysis = attribute evaluation

Result: attributed syntax tree

In greater detail:

- With every nonterminal a set of attributes is associated.
- Two types of attributes are distinguished:

  Synthesized: bottom-up computation (from the leaves to the root)

  Inherited: top-down computation (from the root to the leaves)
- With every production a set of semantic rules is associated.

Example (synthesized + inherited attributes)

Binary numbers (with fraction):

$$
\begin{array}{llcll}
G'_B: & \text{Numbers} & S \to L & v.0 = v.1 & \\
 & & & p.1 = 0 & \\
 & & S \to L\,.\,L & v.0 = v.1 + v.3 & \\
 & & & p.1 = 0 & \\
 & & & p.3 = -l.3 & \\
 & \text{Lists} & L \to B & v.0 = v.1 & \\
 & & & l.0 = 1 & \\
 & & & p.1 = p.0 & \\
 & & L \to LB & v.0 = v.1 + v.2 & \\
 & & & l.0 = l.1 + 1 & \\
 & & & p.1 = p.0 + 1 & \\
 & & & p.2 = p.0 & \\
 & \text{Bits} & B \to 0 & v.0 = 0 & \\
 & \text{Bits} & B \to 1 & v.0 = 2^{p.0} &
\end{array}
$$

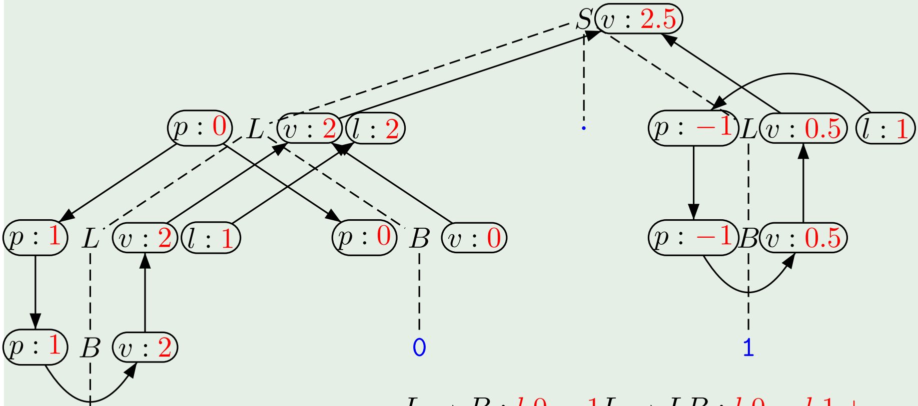
Synthesized attributes of S, L, B: v (value; domain: $V^v := \mathbb{Q}$)

of L: l (length; domain: $V^l := \mathbb{N}$)

Inherited attribute of L, B: p (position; domain: $V^p := \mathbb{Z}$)

Example (continued)

Syntax tree for `10.1`:



$L \to B : l.0 = 1 L \to LB : l.0 = l.1 +$
$1 S \to L \,.\, L : p.1 = 0 S \to L \,.\, L : p.3 = -l.3 L \to LB : p.1 = p.0 + 1 L \to$
$LB : p.2 = p.0 L \to B : p.1 = p.0 B \to 0 : v.0 = 0 B \to 1 : v.0 = 2^{p.0} L \to$

Formal Definition of Attribute Grammars

Definition (Attribute grammar)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.
- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.
- Every production $\pi = Y_0 \to Y_1 \ldots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \ldots, r\}\}$$
of attribute variables of π with the subsets of inner and outer variables:
$$In_\pi := \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\}$$
$$Out_\pi := Var_\pi \setminus In_\pi$$
- A semantic rule of π is an equation of the form
$$\alpha.i = f(\alpha_1.i_1, \ldots, \alpha_n.i_n)$$
where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \ldots \times V^{\alpha_n} \to V^\alpha$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every inner variable of π, and let $E := (E_\pi \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V\rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Outline

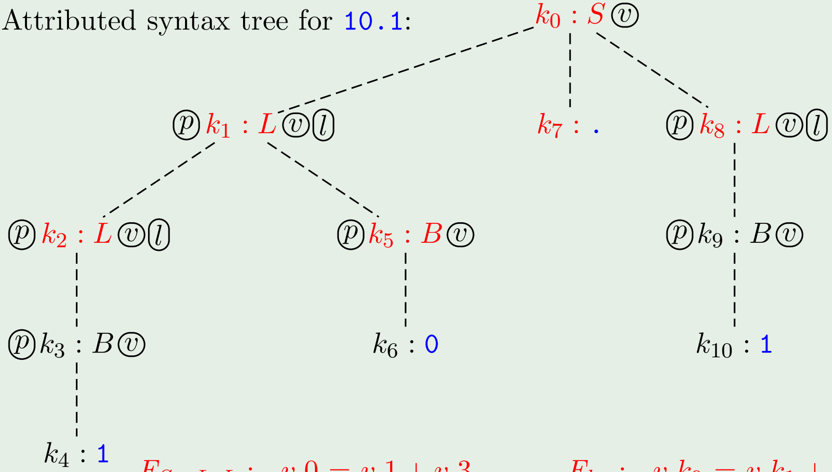
Definition 16.1 (Attribution of syntax trees)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K.

- K determines the set of attribute variables of t:
  $$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \text{att}(Y)\}.$$
- Let $k_0 \in K$ be an (inner) node where production $\pi = Y_0 \to Y_1 \ldots Y_r \in P$ is applied, and let $k_1, \ldots, k_r \in K$ be the corresponding successor nodes. The attribute equation system E_{k_0} of k_0 is obtained from E_π by substituting every attribute index $i \in \{0, \ldots, r\}$ by k_i.
- The attribute equation system of t is given by
  $$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Example 16.2 (cf. Example 16.1)

Attributed syntax tree for `10.1`:



$$E_{S \to L.L} : \quad \begin{aligned} v.0 &= v.1 + v.3 \\ p.1 &= 0 \\ p.3 &= -l.3 \end{aligned} \quad \xrightarrow{\text{subst}} \quad E_{k_0} : \quad \begin{aligned} v.k_0 &= v.k_1 + v.k_8 \\ p.k_1 &= 0 \\ p.k_8 &= -l.k_8 \end{aligned}$$

$$E_{L \to LB} : \quad v.0 = v.1 + v.2 \qquad E_{k_1} : \quad v.k_1 = v.k_2 + v.k_5$$

Attribution of Syntax Trees III

Corollary 16.3

For each $\alpha.k \in Var_t$ except the inherited attribute variables at the root and the synthesized attribute variables at the leaves of t, E_t contains exactly one equation with left-hand side $\alpha.k$.

Assumptions:

- The start symbol does not have inherited attributes: $\text{inh}(S) = \emptyset$.
- Synthesized attributes of terminal symbols are provided by the scanner.

Outline

Solvability of Attribute Equation System I

Definition 16.4 (Solution of attribute equation system)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G. A solution of E_t is a mapping

$$v : Var_t \to V$$

such that, for every $\alpha.k \in Var_t$ and $\alpha.k = f(\alpha.k_1, \ldots, \alpha.k_n) \in E_t$,

$$v(\alpha.k) = f(v(\alpha.k_1), \ldots, v(\alpha.k_n)).$$

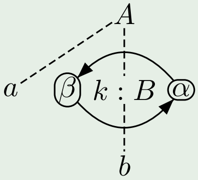
In general, the attribute equation system E_t of a given syntax tree t can have

- no solution,
- exactly one solution, or
- several solutions.

Example 16.5

- $A \to aB, B \to b \in P$
- $\alpha \in \mathrm{syn}(B)$, $\beta \in \mathrm{inh}(B)$
- $\beta.2 = f(\alpha.2) \in E_{A \to aB}$
- $\alpha.0 = g(\beta.0) \in E_{B \to b}$

$\Longrightarrow$ cyclic dependency:



$$E_t : \quad \begin{aligned} \beta.k &= f(\alpha.k) \\ \alpha.k &= g(\beta.k) \end{aligned}$$

$\Longrightarrow$ for $V^\alpha := V^\beta := \mathbb{N}$, $g(x) := x$, and

- $f(x) := x + 1$: no solution
- $f(x) := 2x$: exactly one solution $(v(\alpha.k) = v(\beta.k) = 0)$
- $f(x) := x$: infinitely many solutions $(v(\alpha.k) = v(\beta.k) = y$ for any $y \in \mathbb{N})$

Circularity of Attribute Grammars

Goal: unique solvability of equation system
$\Longrightarrow$ avoid cyclic dependencies

Definition 16.6 (Circularity)

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is called circular if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called noncircular.

Remark: because of the division of Var_π into In_π and Out_π, cyclic dependencies cannot occur at production level (see Corollary 16.8 later).

Outline

Attribute Dependency Graphs I

Goal: graphic representation of attribute dependencies

Definition 16.7 (Production dependency graph)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$. Every production $\pi \in P$ determines the dependency graph $D_\pi := \langle Var_\pi, \rightarrow_\pi \rangle$ where the set of edges $\rightarrow_\pi \subseteq Var_\pi \times Var_\pi$ is given by

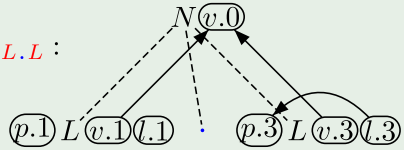
$$x \rightarrow_\pi y \quad \text{iff} \quad y = f(\ldots, x, \ldots) \in E_\pi.$$

Corollary 16.8

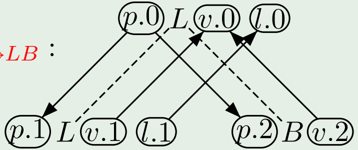
The dependency graph of a production is acyclic (since $\rightarrow_\pi \subseteq Out_\pi \times In_\pi$).

Example 16.9 (cf. Example 16.1)

1. $N \to L.L$:
$$\begin{aligned} v.0 &= v.1 + v.3 \\ p.1 &= 0 \\ p.3 &= -l.3 \end{aligned}$$
$\Longrightarrow D_{N\to L.L}$:



2. $L \to LB$:
$$\begin{aligned} v.0 &= v.1 + v.2 \\ l.0 &= l.1 + 1 \\ p.1 &= p.0 + 1 \\ p.2 &= p.0 \end{aligned}$$
$\Longrightarrow D_{N\to LB}$:



Attribute Dependency Graphs III

Just as the attribute equation system E_t of a syntax tree t is obtained from the semantic rules of the contributing productions, the dependency graph of t is obtained by "glueing together" the dependency graphs of the productions.

Definition 16.10 (Tree dependency graph)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G.

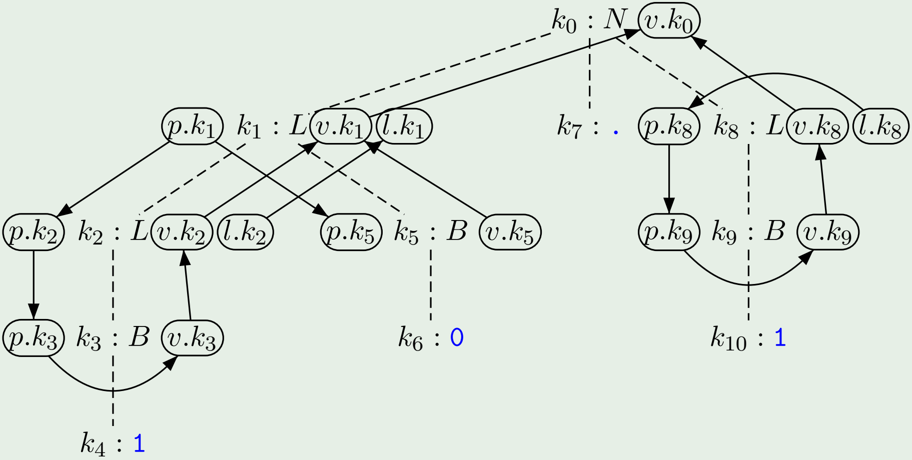
- The dependency graph of t is defined by $D_t := \langle Var_t, \to_t \rangle$ where the set of edges $\to_t \subseteq Var_t \times Var_t$ is given by
$$x \to_t y \quad \text{iff} \quad y = f(\ldots, x, \ldots) \in E_t.$$
- D_t is called cyclic if there exists $x \in Var_t$ such that $x \to_t^+ x$.

Corollary 16.11

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is circular iff there exists a syntax tree t of G such that D_t is cyclic.

Example 16.12 (cf. Example 16.1)

(Acyclic) dependency graph of the syntax tree for `10.1`:



Outline

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a "cover" production $\pi = A_0 \to w_0 A_1 w_1 \ldots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the "upper end" of the cycle and
- for at least one $i \in [r]$, some attributes in $\mathrm{syn}(A_i)$ depend on attributes in $\mathrm{inh}(A_i)$.

Example 16.13

on the board

To identify such "critical" situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\mathrm{syn}(A_i)$ can depend on attributes in $\mathrm{inh}(A_i)$.

Attribute Dependency Graphs and Circularity II

Definition 16.14 (Attribute dependence)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).
- For every syntax tree t with root label $A \in N$,
$$is(A, t) := \{(\beta, \alpha) \in \text{inh}(A) \times \text{syn}(A) \mid \beta \overset{A}{\hookrightarrow} \alpha \text{ in } t\}.$$
- For every $A \in N$,
$$IS(A) := \{is(A, t) \mid t \text{ syntax tree with root label A}\} \subseteq 2^{Inh \times Syn}.$$

Remark: it is important that $IS(A)$ is a system of attribute dependence sets, not a union (later: strong noncircularity).

Example 16.15

on the board

Outline

In the circularity test, the dependency systems $IS(A)$ are iteratively computed. It employs the following notation:

Definition 16.16

Given $\pi = A \to w_0 A_1 w_1 \dots A_r w_r \in P$ and $is_i \subseteq \text{inh}(A_i) \times \text{syn}(A_i)$ for every $i \in [r]$, let

$$is[\pi; is_1, \dots, is_r] \subseteq \text{inh}(A) \times \text{syn}(A)$$

be given by

$$is[\pi; is_1, \dots, is_r] :=$$
$$\left\{(\beta, \alpha) \mid (\beta.0, \alpha.0) \in (\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta'.p_i, \alpha'.p_i) \mid (\beta', \alpha') \in is_i\})^+\right\}$$

where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$.

Example 16.17

on the board

Algorithm 16.18 (Circularity test for attribute grammars)

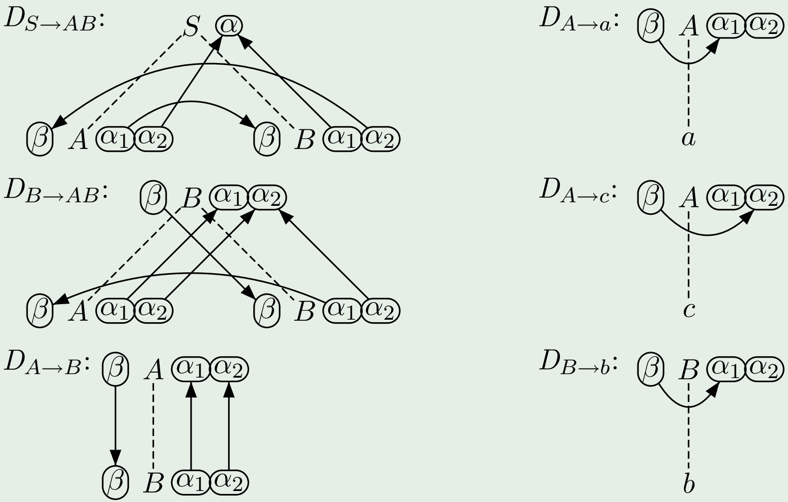
Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

Procedure:

1. *for every* $A \in N$, *iteratively construct* $IS(A)$ *as follows:*
   1. *if* $\pi = A \to w \in P$, *then* $is[\pi] \in IS(A)$
   2. *if* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$, *then* $is[\pi; is_1, \ldots, is_r] \in IS(A)$
2. *test whether* $\mathfrak{A}$ *is circular by checking if there exist* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$ *such that the following relation is cyclic:*
   $$\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in is_i\}$$
   (where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$*)*

Output: *"yes" or "no"*

Example 16.19



Application of Algorithm 16.18: on the board

Outline

Correctness and Complexity of Circularity Test

Theorem 16.20 (Correctness of circularity test)

An attribute grammar is circular iff Algorithm 16.18 yields the answer "yes".

Proof.

by induction on the syntax tree t with cyclic D_t □

Lemma 16.21

The time complexity of the circularity test is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

Proof.

by reduction of the word problem of alternating Turing machines (see M. Jazayeri: *A Simpler Construction for Showing the Intrinsically Exponential Complexity of the Circularity Problem for Attribute Grammars*, Comm. of the ACM 28(4), 1981, pp. 715–720) □