Compiler Construction

Lecture 21: Error Handling in Top-Down Parsing & Strongly Noncircular Attribute Grammars

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Outline

Lookahead Sets

Definition (Lookahead set)

Given $\pi = A \to \beta \in P$,

$$\mathrm{la}(\pi) := \mathrm{fi}(\beta \cdot \mathrm{fo}(A)) \subseteq \Sigma_\varepsilon$$

is called the lookahead set of π (where $\mathrm{fi}(\Gamma) := \bigcup_{\gamma \in \Gamma} \mathrm{fi}(\gamma)$).

Corollary

1. *For all* $a \in \Sigma$,
$$a \in \mathrm{la}(A \to \beta) \textit{ iff } a \in \mathrm{fi}(\beta) \textit{ or } (\beta \Rightarrow^* \varepsilon \textit{ and } a \in \mathrm{fo}(A))$$
2. $\varepsilon \in \mathrm{la}(A \to \beta)$ *iff* $\beta \Rightarrow^* \varepsilon$ *and* $\varepsilon \in \mathrm{fo}(A)$

Characterization of $LL(1)$

Theorem (Characterization of $LL(1)$)

$G \in LL(1)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\mathrm{la}(A \to \beta) \cap \mathrm{la}(A \to \gamma) = \emptyset.$$

Proof.

on the board □

Remark: the above theorem generally does not hold if $k > 1$ (cf. exercises)

Definition (Deterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S\rangle \in LL(1)$. The deterministic top-down parsing automaton of G, DTA(G), is defined by the following components.

- Input alphabet Σ, pushdown alphabet X, output alphabet $[p]$
- Configurations $\Sigma^* \times X^* \times [p]^*$, initial configuration (w, S, ε), final configurations $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$ (as NTA(G))
- Action function

$$\mathrm{act} : \Sigma_\varepsilon \times X_\varepsilon \to \{(\alpha, i) \mid \pi_i = A \to \alpha\} \cup \{\mathsf{pop}, \mathsf{accept}, \mathsf{error}\}$$

$$\begin{aligned} \text{with } \mathrm{act}(x, A) &:= (\alpha, i) \text{ if } \pi_i = A \to \alpha \text{ and } x \in \mathrm{la}(\pi_i) \\ \mathrm{act}(a, a) &:= \mathsf{pop} \\ \mathrm{act}(\varepsilon, \varepsilon) &:= \mathsf{accept} \\ \mathrm{act}(x, y) &:= \mathsf{error} \qquad \text{otherwise} \end{aligned}$$

- Transitions for $x \in \Sigma_\varepsilon$, $w \in \Sigma^*$, $Y \in X$, $\beta \in X^*$, and $z \in [p]^*$:

$$(xw, Y\beta, z) \vdash \begin{cases} (xw, \alpha\beta, zi) & \text{if } \mathrm{act}(x, Y) = (\alpha, i) \\ (w, \beta, z) & \text{if } \mathrm{act}(x, Y) = \mathsf{pop} \end{cases}$$

Outline

Error Handling

Error configurations of DTA(G):

- $(aw, A\alpha, z)$ where $a \notin \bigcup_{A\to\beta\in P} \mathrm{la}(A \to \beta)$ $(\implies \mathrm{act}(a, A) = \mathsf{error})$
- $(aw, b\alpha, z)$ where $a \neq b$ $(\implies \mathrm{act}(a, b) = \mathsf{error})$
- $(\varepsilon, A\alpha, z)$ where $\varepsilon \notin \bigcup_{A\to\beta\in P} \mathrm{la}(A \to \beta)$ $(\implies \mathrm{act}(\varepsilon, A) = \mathsf{error})$
- $(\varepsilon, b\alpha, z)$ $(\implies \mathrm{act}(\varepsilon, b) = \mathsf{error})$
- (aw, ε, z) $(\implies \mathrm{act}(a, \varepsilon) = \mathsf{error})$

Error Handling

Error configurations of $\mathrm{DTA}(G)$:

- $(aw, A\alpha, z)$ where $a \notin \bigcup_{A\to\beta\in P} \mathrm{la}(A \to \beta)$ $(\implies \mathsf{act}(a, A) = \mathsf{error})$
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- (aw, ε, z) $(\implies \mathsf{act}(a, \varepsilon) = \mathsf{error})$

Observation: correct prefix property of LL parsing, i.e., syntactic errors are detected at the earliest possible position (every input prefix which does not produce an error can be extended to a word $w \in L(G)$)

Does not mean: error is recognized at the position where it is caused!

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Example: assignment `a := b * c - (d + e));`
Possible corrections:

- remove closing bracket: `a := b * c - (d + e);`
- insert opening bracket: `a := b * (c - (d + e));`

The General Problem

- Let $w = xy \in \Sigma^*$ be the input word such that x is the longest prefix of a word in $L(G)$ (i.e., the error is detected at the first symbol of y) and $w \notin L(G)$.

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- Parser makes assumption about error type and corrects w accordingly:
  - Assumes prefix x' of x to be correct
  - Correct prefix property
    $\implies$ there exists $z \in \Sigma^*$ such that $x'z \in L(G)$
  - Parser chooses prefix z' of z and suffix y' of y
  - Parsing resumed with input $w' := x'z'y'$ (at first symbol of z') (error recovery)

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- Desirable properties of correction:
  - At least one symbol of y' can be processed before next error occurs (if $y' \neq \varepsilon$)
  - Preserve as many symbols of w as possible (i.e., x' and y' "long" and z' "short")

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- Desirable properties of correction:
  - At least one symbol of y' can be processed before next error occurs (if $y' \neq \varepsilon$)
  - Preserve as many symbols of w as possible (i.e., x' and y' "long" and z' "short")
- $x' \neq x$ hard to implement, therefore usually $x' := x$

Aspects of Error Handling

Further criteria for "good" error handling:

- Continuation of parsing in any case, independent of severity of error
- High probability of correct error diagnosis
- Suppression of subsequent errors
- Complexity of analyzing correct inputs not impaired

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Observation: no “best method” available

- correction not unique
- experience of programmer
- peculiarities of (programming) language

$\Longrightarrow$ employ heuristics

Panic Mode

Simplest form of error handling: panic mode

Upon occurrence of an error,

- skip input symbols ...
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Example: suitable synchronizing tokens in imperative languages for

- assignments: "`;`"
- declarations: "`;`" or "`,`"
- control structures: `fi` or `od`
- blocks: `end`

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Example: suitable synchronizing tokens in imperative languages for

- assignments: "`;`"
- declarations: "`;`" or "`,`"
- control structures: `fi` or `od`
- blocks: `end`

Challenge: choose set of synchronizing tokens such that

- parser recovers quickly from errors that are likely to occur and
- not too much input is overread

(see Aho/Lam/Sethi/Ullman: *Compilers: Principles, Techniques, and Tools*, 2nd ed., pp. 228)

Error Handling Example I

Example 21.1 (cf. Example 9.3)

$$
G'_{AE}: \quad \begin{array}{lcll}
E & \rightarrow & TE' & (1) \\
E' & \rightarrow & \texttt{+}TE' \mid \varepsilon & (2,3) \\
T & \rightarrow & FT' & (4) \\
T' & \rightarrow & \texttt{*}FT' \mid \varepsilon & (5,6) \\
F & \rightarrow & \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (7,8,9)
\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E	$\{\varepsilon, \texttt{)}\}$
E'	$\{\varepsilon, \texttt{)}\}$
T	$\{\texttt{+}, \varepsilon, \texttt{)}\}$
T'	$\{\texttt{+}, \varepsilon, \texttt{)}\}$
F	$\{\texttt{*}, \texttt{+}, \varepsilon, \texttt{)}\}$

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 & T' & \rightarrow \mathtt{*}FT' \mid \varepsilon & (5,6) \\
 & F & \rightarrow \mathtt{(}E\mathtt{)} \mid \mathtt{a} \mid \mathtt{b} & (7,8,9)
\end{array}
$$

$A \in N$	$\mathrm{fo}(A)$
E	$\{\varepsilon, \mathtt{)}\}$
E'	$\{\varepsilon, \mathtt{)}\}$
T	$\{\mathtt{+}, \varepsilon, \mathtt{)}\}$
T'	$\{\mathtt{+}, \varepsilon, \mathtt{)}\}$
F	$\{\mathtt{*}, \mathtt{+}, \varepsilon, \mathtt{)}\}$

With synchronizing tokens from fo sets:

$\mathrm{act} : \Sigma_\varepsilon \times X_\varepsilon \rightarrow \{(\alpha, i) \mid \pi_i = A \rightarrow \alpha\} \cup \{\mathsf{pop}, \mathsf{accept}, \mathsf{error}, \mathrm{sync}\}$ (empty $=$ error)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop	sync	sync	sync	sync	sync	
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$	sync	pop	sync	sync	sync	sync	
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT', 5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE', 2)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\text{act}(x, Y) = \text{sync}$
  $\implies$ pop Y and resume parsing
- $\text{act}(x, A) =$ error
  $\implies$ skip x and resume parsing

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		(a, 8)	pop	sync	sync	sync	sync	sync	
b	$(TE', 1)$		$(FT', 4)$		(b, 9)	sync	pop	sync	sync	sync	sync	
(	$(TE', 1)$		$(FT', 4)$		((E), 7)	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(*FT', 5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(+TE', 2)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	sync	accept

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(+a**+b, E , ε)

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		(a, 8)	pop	sync	sync	sync	sync	sync	
b	$(TE', 1)$		$(FT', 4)$		(b, 9)	sync	pop	sync	sync	sync	sync	
(	$(TE', 1)$		$(FT', 4)$		((E), 7)	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	pop	sync	sync	
*				(*FT', 5)	sync	sync	sync	sync	sync	pop	sync	
+		(+TE', 2)	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	sync	accept

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- act(x, Y) = sync
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(+a**+b, E, ε)
$\vdash$ (a**+b, E, ε)

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\texttt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\texttt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\texttt{(}E\texttt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\texttt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\texttt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\mathrm{act}(x,Y) = \mathrm{sync}$
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$$
\begin{array}{ll}
 & (\texttt{+a**+b}, E, \varepsilon) \\
\vdash & (\texttt{a**+b}, E, \varepsilon) \\
\vdash & (\texttt{a**+b}, TE', 1)
\end{array}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	`a`	`b`	`(`	`)`	`*`	`+`	ε
`a`	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop	sync	sync	sync	sync	sync	
`b`	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$	sync	pop	sync	sync	sync	sync	
`(`	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$	sync	sync	pop	sync	sync	sync	
`)`	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	pop	sync	sync	
`*`				$(\mathtt{*}FT', 5)$	sync	sync	sync	sync	sync	pop	sync	
`+`		$(\mathtt{+}TE', 2)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	sync	accept

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- $\text{act}(x, Y) = \text{sync}$
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$$
\begin{aligned}
& (\mathtt{+a{*}{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{*}{+}b}, TE', 1) \\
\vdash\ & (\mathtt{a{*}{*}{+}b}, FT'E', 14)
\end{aligned}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

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- $\mathrm{act}(x,Y) = \mathrm{sync}$
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$$\begin{array}{ll}
 & (\mathtt{+a{*}{*}+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a{*}{*}+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a{*}{*}+b},\ TE',\ 1) \\
\vdash & (\mathtt{a{*}{*}+b},\ FT'E',\ 14) \\
\vdash & (\mathtt{a{*}{*}+b},\ \mathtt{a}T'E',\ 148)
\end{array}$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop	sync	sync	sync	sync	sync	
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$	sync	pop	sync	sync	sync	sync	
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT', 5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE', 2)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon, 3)$	sync	$(\varepsilon, 6)$	sync	sync	sync	sync	sync	sync	sync	accept

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- act(x, Y) = sync
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$$
\begin{array}{ll}
 & (\mathtt{+a{*}{*}{+}b}, E, \varepsilon) \\
\vdash & (\mathtt{a{*}{*}{+}b}, E, \varepsilon) \\
\vdash & (\mathtt{a{*}{*}{+}b}, TE', 1) \\
\vdash & (\mathtt{a{*}{*}{+}b}, FT'E', 14) \\
\vdash & (\mathtt{a{*}{*}{+}b}, \mathtt{a}T'E', 148) \\
\vdash & (\mathtt{{*}{+}b}, T'E', 148)
\end{array}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

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- $\mathrm{act}(x,Y) = \mathrm{sync}$
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  $\implies$ skip x and resume parsing

$$
\begin{array}{ll}
 & (\mathtt{+a*+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a*+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a*+b},\ TE',\ 1) \\
\vdash & (\mathtt{a*+b},\ FT'E',\ 14) \\
\vdash & (\mathtt{a*+b},\ \mathtt{a}T'E',\ 148) \\
\vdash & (\mathtt{*+b},\ T'E',\ 148) \\
\vdash & (\mathtt{*+b},\ \mathtt{*}FT'E',\ 1485)
\end{array}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop	sync	sync	sync	sync	sync	
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$	sync	pop	sync	sync	sync	sync	
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$	sync	sync	pop	sync	sync	sync	
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\begin{aligned}
& (\mathtt{+a*+b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a*+b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a*+b}, TE', 1) \\
\vdash\ & (\mathtt{a*+b}, FT'E', 14) \\
\vdash\ & (\mathtt{a*+b}, \mathtt{a}T'E', 148) \\
\vdash\ & (\mathtt{*+b}, T'E', 148) \\
\vdash\ & (\mathtt{*+b}, \mathtt{*}FT'E', 1485) \\
\vdash\ & (\mathtt{+b}, FT'E', 1485)
\end{aligned}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\text{act}(x,Y) = \text{sync}$
  $\implies$ pop Y and resume parsing
- $\text{act}(x,A) =$ error
  $\implies$ skip x and resume parsing

$$
\begin{array}{ll}
& (\mathtt{+a**+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a**+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a**+b},\ TE',\ 1) \\
\vdash & (\mathtt{a**+b},\ FT'E',\ 14) \\
\vdash & (\mathtt{a**+b},\ \mathtt{a}T'E',\ 148) \\
\vdash & (\mathtt{**+b},\ T'E',\ 148) \\
\vdash & (\mathtt{**+b},\ \mathtt{*}FT'E',\ 1485) \\
\vdash & (\mathtt{+b},\ FT'E',\ 1485) \\
\vdash & (\mathtt{+b},\ T'E',\ 1485)
\end{array}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- act(x, Y) = sync
  $\Longrightarrow$ pop Y and resume parsing
- act(x, A) = error
  $\Longrightarrow$ skip x and resume parsing

$$
\begin{aligned}
& (\mathtt{+a*+b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a*+b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a*+b}, TE', 1) \\
\vdash\ & (\mathtt{a*+b}, FT'E', 14) \\
\vdash\ & (\mathtt{a*+b}, \mathtt{a}T'E', 148) \\
\vdash\ & (\mathtt{*+b}, T'E', 148) \\
\vdash\ & (\mathtt{*+b}, \mathtt{*}FT'E', 1485) \\
\vdash\ & (\mathtt{+b}, FT'E', 1485) \\
\vdash\ & (\mathtt{+b}, T'E', 1485) \\
\vdash\ & (\mathtt{+b}, E', 14856)
\end{aligned}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		(a, 8)	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		(b, 9)	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		((E), 7)	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				(*FT', 5)	sync	sync	sync	sync	sync	pop	sync	
+		(+TE', 2)	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- act(x, Y) = sync
  $\implies$ pop Y and resume parsing
- act(x, A) = error
  $\implies$ skip x and resume parsing

(+a*+b, E, ε)
$\vdash$ (a*+b, E, ε)
$\vdash$ (a*+b, TE', 1)
$\vdash$ (a*+b, $FT'E'$, 14)
$\vdash$ (a*+b, a$T'E'$, 148)

$\vdash$ (*+b, $T'E'$, 148)
$\vdash$ (*+b, *$FT'E'$, 1485)
$\vdash$ (+b, $FT'E'$, 1485)
$\vdash$ (+b, $T'E'$, 1485)
$\vdash$ (+b, E', 14856)
$\vdash$ (+b, +TE', 148562)

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\mathrm{act}(x, Y) = \text{sync}$
  $\implies$ pop Y and resume parsing
- $\mathrm{act}(x, A) = \text{error}$
  $\implies$ skip x and resume parsing

$$
\begin{aligned}
& (\mathtt{+a{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{+}b}, TE', 1) \\
\vdash\ & (\mathtt{a{*}{+}b}, FT'E', 14) \\
\vdash\ & (\mathtt{a{*}{+}b}, \mathtt{a}T'E', 148) \\
\vdash\ & (\mathtt{{*}{+}b}, T'E', 148) \\
\vdash\ & (\mathtt{{*}{+}b}, \mathtt{*}FT'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, FT'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, T'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, E', 14856) \\
\vdash\ & (\mathtt{{+}b}, \mathtt{+}TE', 148562) \\
\vdash\ & (\mathtt{b}, TE', 148562)
\end{aligned}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\mathrm{act}(x,Y) = \mathrm{sync}$
  $\Longrightarrow$ pop Y and resume parsing
- $\mathrm{act}(x,A) =$ error
  $\Longrightarrow$ skip x and resume parsing

$$
\begin{array}{llll}
 & (\mathtt{+a{*}{+}b}, & E & , \varepsilon) \\
\vdash & (\mathtt{a{*}{+}b}, & E & , \varepsilon) \\
\vdash & (\mathtt{a{*}{+}b}, & TE' & , 1) \\
\vdash & (\mathtt{a{*}{+}b}, & FT'E' & , 14) \\
\vdash & (\mathtt{a{*}{+}b}, & \mathtt{a}T'E' & , 148) \\
\vdash & (\mathtt{{*}{+}b}, & T'E' & , 148) \\
\vdash & (\mathtt{{*}{+}b}, & \mathtt{*}FT'E' & , 1485) \\
\vdash & (\mathtt{{+}b}, & FT'E' & , 1485) \\
\vdash & (\mathtt{{+}b}, & T'E' & , 1485) \\
\vdash & (\mathtt{{+}b}, & E' & , 14856) \\
\vdash & (\mathtt{{+}b}, & \mathtt{+}TE' & , 148562) \\
\vdash & (\mathtt{b}, & TE' & , 148562) \\
\vdash & (\mathtt{b}, & FT'E' & , 1485624)
\end{array}
$$

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\text{act}(x, Y) = \text{sync}$
  $\Longrightarrow$ pop Y and resume parsing
- $\text{act}(x, A) = \text{error}$
  $\Longrightarrow$ skip x and resume parsing

$$
\begin{array}{ll}
 & (\mathtt{+a*+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a*+b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{a*+b},\ TE',\ 1) \\
\vdash & (\mathtt{a*+b},\ FT'E',\ 14) \\
\vdash & (\mathtt{a*+b},\ \mathtt{a}T'E',\ 148) \\
\vdash & (\mathtt{*+b},\ T'E',\ 148) \\
\vdash & (\mathtt{*+b},\ \mathtt{*}FT'E',\ 1485) \\
\vdash & (\mathtt{+b},\ FT'E',\ 1485) \\
\vdash & (\mathtt{+b},\ T'E',\ 1485) \\
\vdash & (\mathtt{+b},\ E',\ 14856) \\
\vdash & (\mathtt{+b},\ \mathtt{+}TE',\ 148562) \\
\vdash & (\mathtt{b},\ TE',\ 148562) \\
\vdash & (\mathtt{b},\ FT'E',\ 1485624) \\
\vdash & (\mathtt{b},\ \mathtt{b}T'E',\ 14856249)
\end{array}
$$

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\mathrm{act}(x,Y) = \mathrm{sync}$
  $\Longrightarrow$ pop Y and resume parsing
- $\mathrm{act}(x,A) = \mathsf{error}$
  $\Longrightarrow$ skip x and resume parsing

$$
\begin{array}{rll}
 & (\mathtt{+a*+b}, E & , \varepsilon) \\
\vdash & (\mathtt{a*+b}, E & , \varepsilon) \\
\vdash & (\mathtt{a*+b}, TE' & , 1) \\
\vdash & (\mathtt{a*+b}, FT'E' & , 14) \\
\vdash & (\mathtt{a*+b}, \mathtt{a}T'E' & , 148) \\
\vdash & (\mathtt{*+b}, T'E' & , 148) \\
\vdash & (\mathtt{*+b}, \mathtt{*}FT'E' & , 1485) \\
\vdash & (\mathtt{+b}, FT'E' & , 1485) \\
\vdash & (\mathtt{+b}, T'E' & , 1485) \\
\vdash & (\mathtt{+b}, E' & , 14856) \\
\vdash & (\mathtt{+b}, \mathtt{+}TE' & , 148562) \\
\vdash & (\mathtt{b}, TE' & , 148562) \\
\vdash & (\mathtt{b}, FT'E' & , 1485624) \\
\vdash & (\mathtt{b}, \mathtt{b}T'E' & , 14856249) \\
\vdash & (\varepsilon, T'E' & , 14856249)
\end{array}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- $\text{act}(x,Y) = \text{sync}$
  $\implies$ pop Y and resume parsing
- $\text{act}(x,A) = \text{error}$
  $\implies$ skip x and resume parsing

$$
\begin{aligned}
& (\mathtt{+a{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{+}b}, E, \varepsilon) \\
\vdash\ & (\mathtt{a{*}{+}b}, TE', 1) \\
\vdash\ & (\mathtt{a{*}{+}b}, FT'E', 14) \\
\vdash\ & (\mathtt{a{*}{+}b}, \mathtt{a}T'E', 148) \\
\vdash\ & (\mathtt{{*}{+}b}, T'E', 148) \\
\vdash\ & (\mathtt{{*}{+}b}, \mathtt{*}FT'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, FT'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, T'E', 1485) \\
\vdash\ & (\mathtt{{+}b}, E', 14856) \\
\vdash\ & (\mathtt{{+}b}, \mathtt{+}TE', 148562) \\
\vdash\ & (\mathtt{b}, TE', 148562) \\
\vdash\ & (\mathtt{b}, FT'E', 1485624) \\
\vdash\ & (\mathtt{b}, \mathtt{b}T'E', 14856249) \\
\vdash\ & (\varepsilon, T'E', 14856249) \\
\vdash\ & (\varepsilon, E', 148562496)
\end{aligned}
$$

Error Handling Example II

Example 21.1 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop	sync	sync	sync	sync	sync	
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$	sync	pop	sync	sync	sync	sync	
(	$(TE',1)$		$(FT',4)$		$(\mathtt{(}E\mathtt{)},7)$	sync	sync	pop	sync	sync	sync	
)	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	pop	sync	sync	
*				$(\mathtt{*}FT',5)$	sync	sync	sync	sync	sync	pop	sync	
+		$(\mathtt{+}TE',2)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	pop	
ε	sync	$(\varepsilon,3)$	sync	$(\varepsilon,6)$	sync	sync	sync	sync	sync	sync	sync	accept

Meaning of table entries:

- act(x,Y) = sync
  $\Longrightarrow$ pop Y and resume parsing
- act(x,A) = error
  $\Longrightarrow$ skip x and resume parsing

$$
\begin{aligned}
& (\mathtt{+a*+b},\ E,\ \varepsilon) \\
\vdash\ & (\mathtt{a*+b},\ E,\ \varepsilon) \\
\vdash\ & (\mathtt{a*+b},\ TE',\ 1) \\
\vdash\ & (\mathtt{a*+b},\ FT'E',\ 14) \\
\vdash\ & (\mathtt{a*+b},\ \mathtt{a}T'E',\ 148) \\
\vdash\ & (\mathtt{*+b},\ T'E',\ 148) \\
\vdash\ & (\mathtt{*+b},\ \mathtt{*}FT'E',\ 1485) \\
\vdash\ & (\mathtt{+b},\ FT'E',\ 1485) \\
\vdash\ & (\mathtt{+b},\ T'E',\ 1485) \\
\vdash\ & (\mathtt{+b},\ E',\ 14856) \\
\vdash\ & (\mathtt{+b},\ \mathtt{+}TE',\ 148562) \\
\vdash\ & (\mathtt{b},\ TE',\ 148562) \\
\vdash\ & (\mathtt{b},\ FT'E',\ 1485624) \\
\vdash\ & (\mathtt{b},\ \mathtt{b}T'E',\ 14856249) \\
\vdash\ & (\varepsilon,\ T'E',\ 14856249) \\
\vdash\ & (\varepsilon,\ E',\ 148562496) \\
\vdash\ & (\varepsilon,\ \varepsilon,\ 1485624963)
\end{aligned}
$$

Outline

Formal Definition of Attribute Grammars

Definition (Attribute grammar)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.
- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.
- Every production $\pi = Y_0 \to Y_1 \ldots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \ldots, r\}\}$$
of attribute variables of π with the subsets of inner and outer variables:
$$\begin{aligned} In_\pi &:= \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\} \\ Out_\pi &:= Var_\pi \setminus In_\pi \end{aligned}$$
- A semantic rule of π is an equation of the form
$$\alpha.i = f(\alpha_1.i_1, \ldots, \alpha_n.i_n)$$
where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \ldots \times V^{\alpha_n} \to V^\alpha$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every inner variable of π, and let $E := (E_\pi \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V\rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Attribution of Syntax Trees I

Definition (Attribution of syntax trees)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K.

- K determines the set of attribute variables of t:
$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \text{att}(Y)\}.$$
- Let $k_0 \in K$ be an (inner) node where production $\pi = Y_0 \to Y_1 \dots Y_r \in P$ is applied, and let $k_1, \dots, k_r \in K$ be the corresponding successor nodes. The attribute equation system E_{k_0} of k_0 is obtained from E_π by substituting every attribute index $i \in \{0, \dots, r\}$ by k_i.
- The attribute equation system of t is given by
$$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Circularity of Attribute Grammars

Goal: unique solvability of equation system
$\Longrightarrow$ avoid cyclic dependencies

Definition (Circularity)

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is called circular if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called noncircular.

Remark: because of the division of Var_π into In_π and Out_π, cyclic dependencies cannot occur at production level (see Corollary 16.8).

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a "cover" production $\pi = A_0 \to w_0 A_1 w_1 \dots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the "upper end" of the cycle and
- for at least one $i \in [r]$, some attributes in $\mathrm{syn}(A_i)$ depend on attributes in $\mathrm{inh}(A_i)$.

Example

on the board

To identify such "critical" situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\mathrm{syn}(A_i)$ can depend on attributes in $\mathrm{inh}(A_i)$.

Definition (Attribute dependence)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \to_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).
- For every syntax tree t with root label $A \in N$,
$$is(A,t) := \{(\beta,\alpha) \in \text{inh}(A) \times \text{syn}(A) \mid \beta \overset{A}{\hookrightarrow} \alpha \text{ in } t\}.$$
- For every $A \in N$,
$$IS(A) := \{is(A,t) \mid t \text{ syntax tree with root label A}\} \subseteq 2^{Inh \times Syn}.$$

Remark: it is important that $IS(A)$ is a system of attribute dependence sets, not a union (later: strong noncircularity).

Example

on the board

Outline

Simplifying the Circularity Test

Idea: to simplify the circularity test, do not distinguish between attribute dependences which are caused by different syntax trees

Simplifying the Circularity Test

Idea: to simplify the circularity test, do not distinguish between attribute dependences which are caused by different syntax trees

Definition 21.2 (Attribute dependence (modified))

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- Reminder: if t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).

Idea: to simplify the circularity test, do not distinguish between attribute dependences which are caused by different syntax trees

Definition 21.2 (Attribute dependence (modified))

Let $\mathfrak{A} = \langle G, E, V\rangle \in AG$ with $G = \langle N, \Sigma, P, S\rangle$.

- Reminder: if t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).
- For every $A \in N$,

$$\begin{aligned} IS'(A) &:= \{(\beta, \alpha) \mid \beta \overset{A}{\hookrightarrow} \alpha \text{ in some syntax tree with root label A}\} \\ &\subseteq Inh \times Syn \end{aligned}$$

Algorithm 21.3 (Strong circularity test for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

Algorithm 21.3 (Strong circularity test for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

Procedure:

1. *for every* $A \in N$, *iteratively construct* $IS'(A)$ *as follows:*
   1. *if* $\pi = A \to w \in P$, *then* $is[\pi] \subseteq IS'(A)$
   2. *if* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$, *then* $is[\pi; IS'(A_1), \ldots, IS'(A_r)] \subseteq IS'(A)$

The Strong Circularity Test

Algorithm 21.3 (Strong circularity test for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

Procedure:

1. *for every* $A \in N$, *iteratively construct* $IS'(A)$ *as follows:*
   1. *if* $\pi = A \to w \in P$, *then* $is[\pi] \subseteq IS'(A)$
   2. *if* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$, *then* $is[\pi; IS'(A_1), \ldots, IS'(A_r)] \subseteq IS'(A)$
2. *test whether there exists* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *such that the following relation is cyclic:*

   $$\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in IS'(A_i)\}$$

   (where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$*)*

The Strong Circularity Test

Algorithm 21.3 (Strong circularity test for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

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Output: *"yes" or "no"*

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Example 21.4

on the board

Strongly Noncircular Attribute Grammars

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omitted □