

Compiler Construction

Lecture 22:

Evaluation of Strongly Noncircular Attribute Grammars

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Winter semester 2010/11

- 1 Repetition: Strongly Noncircular Attribute Grammars
- 2 (Strongly) Noncircular Attribute Grammars
- 3 Evaluation of Strongly Noncircular Attribute Grammars

Simplifying the Circularity Test

Idea: to simplify the circularity test, do not distinguish between attribute dependences which are caused by **different syntax trees**

Definition (Attribute dependence (modified))

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- Reminder: if t is a syntax tree with root label $A \in N$ and root node k , $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \xrightarrow{A} \alpha$).
- For every $A \in N$,

$$\begin{aligned} IS'(A) &:= \{(\beta, \alpha) \mid \beta \xrightarrow{A} \alpha \text{ in some syntax tree with root label } A\} \\ &\subseteq \text{Inh} \times \text{Syn} \end{aligned}$$

The Strong Circularity Test

Algorithm (Strong circularity test for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$

Procedure: ① for every $A \in N$, *iteratively construct* $IS'(A)$ as follows:

- ① if $\pi = A \rightarrow w \in P$, then $is[\pi] \subseteq IS'(A)$
- ② if $\pi = A \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$, then $is[\pi; IS'(A_1), \dots, IS'(A_r)] \subseteq IS'(A)$

② test whether there exists

$\pi = A \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ such that the following relation is *cyclic*:

$$\rightarrow_\pi \cup \bigcup_{i=1}^r \{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in IS'(A_i)\}$$

(where $p_i := \sum_{j=1}^i |w_{j-1}| + i$)

Output: “yes” or “no”

Example

on the board

Strongly Noncircular Attribute Grammars

Definition (Strong noncircularity)

An attribute grammar is called **strongly noncircular** if Algorithm 21.3 yields the answer “no”.

Lemma

*The time complexity of the strong circularity test is **polynomial** in the size of the attribute grammar (= maximal length of right-hand sides of productions).*

Proof.

omitted



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Lemma 22.1

- ① *Every strongly noncircular attribute grammar is noncircular.*
- ② *There are noncircular attribute grammars which are not strongly noncircular.*

(Strongly) Noncircular Attribute Grammars

Lemma 22.1

- ① *Every strongly noncircular attribute grammar is noncircular.*
- ② *There are noncircular attribute grammars which are not strongly noncircular.*

Proof.

- ① Clear since $is \subseteq IS'(A)$ for every $A \in N$ and $is \in IS(A)$
- ② The attribute grammar in Example 21.4 is noncircular but not strongly noncircular (on the board).



- 1 Repetition: Strongly Noncircular Attribute Grammars
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- 3 Evaluation of Strongly Noncircular Attribute Grammars

Attribute Evaluation Methods

- Given:
- (strongly) noncircular attribute grammar
 $\mathfrak{A} = \langle G, E, V \rangle \in AG$
 - syntax tree t of G
 - valuation $v : Syn_{\Sigma} \rightarrow V$ where
 $Syn_{\Sigma} := \{\alpha.k \mid k \text{ labelled by } a \in \Sigma, \alpha \in \text{syn}(a)\} \subseteq Var_t$

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- Methods:
- ① Topological sorting of D_t :
 - ① start with attribute variables which depend at most on synthesized attributes of terminals (Syn_{Σ})
 - ② proceed by successive substitution

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- Methods:
- 1 Topological sorting of D_t :
 - 1 start with attribute variables which depend at most on synthesized attributes of terminals (Syn_{Σ})
 - 2 proceed by successive substitution
 - 2 **Recursive functions** (for strongly noncircular AGs):
 - 1 for every $A \in N$ and $\alpha \in \text{syn}(A)$, define evaluation function $g_{A,\alpha}$ with the following parameters:
 - the node of t where α has to be evaluated and
 - all inherited attributes of A on which α (potentially) depends
 - 2 for every $\alpha \in \text{syn}(S)$, evaluate $g_{S,\alpha}(k_0)$ where k_0 denotes the root of t

Attribute Evaluation Methods

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 - 2 for every $\alpha \in \text{syn}(S)$, evaluate $g_{S,\alpha}(k_0)$ where k_0 denotes the root of t
 - 3 Special cases: S-attributed grammars (yacc), L-attributed grammars

Attribute Evaluation by Recursive Functions

Restriction: only for **strongly noncircular** attribute grammars

Attribute Evaluation by Recursive Functions

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- Principle:
- ① for every $A \in N$ and $\alpha \in \text{syn}(A)$, define **evaluation function** $g_{A,\alpha}$ with the following parameters:
 - the **node of** t where α has to be evaluated (which is labelled by A) and
 - all **inherited attributes of** A on which α (potentially) depends (that is, $\{\beta \in \text{inh}(A) \mid (\beta, \alpha) \in IS'(A)\}$)
 - ② given a syntax tree t with root k_0 , **evaluate** $g_{S,\alpha}(k_0)$ for every $\alpha \in \text{syn}(S)$

Attribute Evaluation by Recursive Functions

Restriction: only for **strongly noncircular attribute grammars**

Principle: ① for every $A \in N$ and $\alpha \in \text{syn}(A)$, define **evaluation function** $g_{A,\alpha}$ with the following parameters:

- the **node of** t where α has to be evaluated (which is labelled by A) and
- all **inherited attributes of** A on which α (potentially) depends (that is, $\{\beta \in \text{inh}(A) \mid (\beta, \alpha) \in IS'(A)\}$)

② given a syntax tree t with root k_0 , **evaluate** $g_{S,\alpha}(k_0)$ for every $\alpha \in \text{syn}(S)$

Result: evaluates synthesized attribute variables at root of t and all attribute variables on which they actually depend (according to E_t)

Definition of Evaluation Functions I

For every $A \in N$ and $\alpha \in \text{syn}(A)$, let

- $IS'(A) \subseteq \text{inh}(A) \times \text{syn}(A)$ as computed by strong circularity test (Algorithm 21.3)
- $\text{inh}(A, \alpha) := \{\beta \in \text{inh}(A) \mid (\beta, \alpha) \in IS'(A)\}$
- $A \rightarrow \delta_1 \mid \dots \mid \delta_m$ all A -productions in P

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Then $g_{A,\alpha}$ is given by

```
 $g_{A,\alpha}(k_0, \text{inh}(A, \alpha)) :=$  case production applied at  $k_0$  of  
     $\vdots$   
     $A \rightarrow \delta_j : \text{eval}(\alpha.0)$   
     $\vdots$   
end
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Then $g_{A,\alpha}$ is given by

$g_{A,\alpha}(k_0, \text{inh}(A, \alpha)) :=$ **case** production applied at k_0 **of**

$\vdots$
 $A \rightarrow \delta_j : \text{eval}(\alpha.0)$
 $\vdots$
end

with

$$\text{eval}(\gamma.i) := \begin{cases} \gamma & \text{if } \gamma \in \text{Inh}, i = 0 \\ f(\text{eval}(\gamma_1.i_1), \dots, \text{eval}(\gamma_n.i_n)) & \text{if } \gamma.i \in \text{In}_{A \rightarrow \delta_j}, \gamma.i = f(\gamma_1.i_1, \dots, \gamma_n.i_n) \in E_{A \rightarrow \delta_j} \\ g_{Y_i, \gamma}(k_i, \text{eval}(\beta_1.i), \dots, \text{eval}(\beta_l.i)) & \text{if } \gamma \in \text{Syn}, i > 0, Y_i \in N, \\ & \text{inh}(Y_i, \gamma) = \{\beta_1, \dots, \beta_l\} \\ v(\gamma.i) & \text{if } \gamma \in \text{Syn}, i > 0, Y_i \in \Sigma \end{cases}$$

where $\delta_j = Y_1 \dots Y_r$, and where k_i denotes the i th successor of k_0

Definition of Evaluation Functions II

Example 22.2 (cf. Example 15.2)

G'_B :

$S \rightarrow L \quad v.0 = v.1$

$p.1 = 0$

$S \rightarrow L.L \quad v.0 = v.1 + v.3$

$p.1 = 0$

$p.3 = -l.3$

$L \rightarrow B \quad v.0 = v.1$

$l.0 = 1$

$p.1 = p.0$

$L \rightarrow LB \quad v.0 = v.1 + v.2$

$l.0 = l.1 + 1$

$p.1 = p.0 + 1$

$p.2 = p.0$

$B \rightarrow 0 \quad v.0 = 0$

$B \rightarrow 1 \quad v.0 = 2^{p.0}$

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$l.0 = l.1 + 1$

$p.1 = p.0 + 1$

$p.2 = p.0$

$B \rightarrow 0 \quad v.0 = 0$

$B \rightarrow 1 \quad v.0 = 2^{p.0}$

$A \in N$	S	L	B
$IS'(A)$	$\emptyset$	$\{(p, v)\}$	$\{(p, v)\}$

Definition of Evaluation Functions II

Example 22.2 (cf. Example 15.2)

$G'_B:$		$g_{S,v}(k_0) = \text{case production}(k_0) \text{ of}$
$S \rightarrow L$	$v.0 = v.1$	$S \rightarrow L : g_{L,v}(k_1, 0)$
	$p.1 = 0$	$S \rightarrow L.L : g_{L,v}(k_1, 0) +$
$S \rightarrow L.L$	$v.0 = v.1 + v.3$	$g_{L,v}(k_3, -g_{L,l}(k_3))$
	$p.1 = 0$	end
	$p.3 = -l.3$	$g_{L,v}(k_0, p) = \text{case production}(k_0) \text{ of}$
$L \rightarrow B$	$v.0 = v.1$	$L \rightarrow B : g_{B,v}(k_1, p)$
	$l.0 = 1$	$L \rightarrow LB : g_{L,v}(k_1, p + 1)$
	$p.1 = p.0$	$+ g_{B,v}(k_2, p)$
$L \rightarrow LB$	$v.0 = v.1 + v.2$	end
	$l.0 = l.1 + 1$	$g_{L,l}(k_0) = \text{case production}(k_0) \text{ of}$
	$p.1 = p.0 + 1$	$L \rightarrow B : 1$
	$p.2 = p.0$	$L \rightarrow LB : g_{L,l}(k_1) + 1$
$B \rightarrow 0$	$v.0 = 0$	end
$B \rightarrow 1$	$v.0 = 2^{p.0}$	$g_{B,v}(k_0, p) = \text{case production}(k_0) \text{ of}$
		$B \rightarrow 0 : 0$
		$B \rightarrow 1 : 2^p$
		end

$A \in N$	S	L	B
$IS'(A)$	$\emptyset$	$\{(p, v)\}$	$\{(p, v)\}$

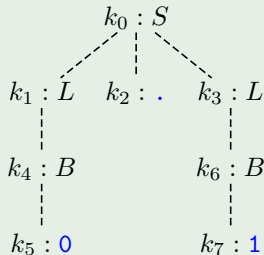
Example 22.2 (continued)

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 $g_{S,v}(k_0) = \text{case production}(k_0) \text{ of}$   
     $S \rightarrow L : g_{L,v}(k_1, 0)$   
     $S \rightarrow L \cdot \bar{L} : g_{L,v}(k_1, 0) +$   
         $g_{L,v}(k_3, -g_{L,l}(k_3))$   
    end  
 $g_{L,v}(k_0, p) = \text{case production}(k_0) \text{ of}$   
     $L \rightarrow B : g_{B,v}(k_1, p)$   
     $L \rightarrow LB : g_{L,v}(k_1, p + 1)$   
         $+ g_{B,v}(k_2, p)$   
    end  
 $g_{L,l}(k_0) = \text{case production}(k_0) \text{ of}$   
     $L \rightarrow B : 1$   
     $L \rightarrow LB : g_{L,l}(k_1) + 1$   
    end  
 $g_{B,v}(k_0, p) = \text{case production}(k_0) \text{ of}$   
     $B \rightarrow 0 : 0$   
     $B \rightarrow 1 : 2^p$   
    end
```


Example 22.2 (continued)

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 $g_{S,v}(k_0) = \text{case production}(k_0) \text{ of}$   
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         $g_{L,v}(k_3, -g_{L,l}(k_3))$   
     $\text{end}$   
 $g_{L,v}(k_0, p) = \text{case production}(k_0) \text{ of}$   
     $L \rightarrow B : g_{B,v}(k_1, p)$   
     $L \rightarrow LB : g_{L,v}(k_1, p + 1)$   
         $+ g_{B,v}(k_2, p)$   
     $\text{end}$   
 $g_{L,l}(k_0) = \text{case production}(k_0) \text{ of}$   
     $L \rightarrow B : 1$   
     $L \rightarrow LB : g_{L,l}(k_1) + 1$   
     $\text{end}$   
 $g_{B,v}(k_0, p) = \text{case production}(k_0) \text{ of}$   
     $B \rightarrow 0 : 0$   
     $B \rightarrow 1 : 2^p$   
     $\text{end}$ 
```

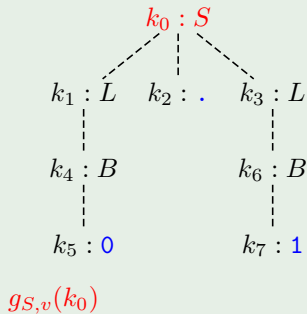
Syntax tree t :



Example 22.2 (continued)

$g_{S,v}(k_0) = \text{case production}(k_0) \text{ of}$
 $S \rightarrow L : g_{L,v}(k_1, 0)$
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 $g_{L,v}(k_3, -g_{L,l}(k_3))$
 end
 $g_{L,v}(k_0, p) = \text{case production}(k_0) \text{ of}$
 $L \rightarrow B : g_{B,v}(k_1, p)$
 $L \rightarrow LB : g_{L,v}(k_1, p + 1)$
 $+ g_{B,v}(k_2, p)$
 end
 $g_{L,l}(k_0) = \text{case production}(k_0) \text{ of}$
 $L \rightarrow B : 1$
 $L \rightarrow LB : g_{L,l}(k_1) + 1$
 end
 $g_{B,v}(k_0, p) = \text{case production}(k_0) \text{ of}$
 $B \rightarrow 0 : 0$
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 end

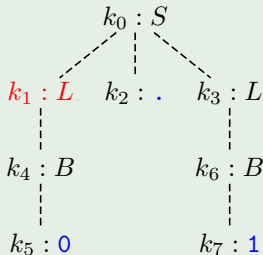
Syntax tree t :



Example 22.2 (continued)

$$\begin{aligned}
 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad S \rightarrow L : g_{L,v}(k_1, 0) \\
 &\quad S \rightarrow L \cdot L : g_{L,v}(k_1, 0) + \\
 &\quad \quad g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &\text{end} \\
 g_{L,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : g_{B,v}(k_1, p) \\
 &\quad L \rightarrow LB : g_{L,v}(k_1, p+1) \\
 &\quad \quad + g_{B,v}(k_2, p) \\
 &\text{end} \\
 g_{L,l}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : 1 \\
 &\quad L \rightarrow LB : g_{L,l}(k_1) + 1 \\
 &\text{end} \\
 g_{B,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad B \rightarrow 0 : 0 \\
 &\quad B \rightarrow 1 : 2^p \\
 &\text{end}
 \end{aligned}$$

Syntax tree t :

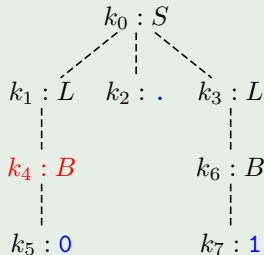


$$\begin{aligned}
 g_{S,v}(k_0) \\
 &= g_{L,v}(k_1, 0) + g_{L,v}(k_3, -g_{L,l}(k_3))
 \end{aligned}$$

Example 22.2 (continued)

$$\begin{aligned}
 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
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 g_{L,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
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 &\quad L \rightarrow LB : g_{L,v}(k_1, p+1) \\
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 g_{B,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad B \rightarrow 0 : 0 \\
 &\quad B \rightarrow 1 : 2^p \\
 &\text{end}
 \end{aligned}$$

Syntax tree t :

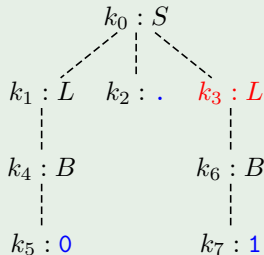


$$\begin{aligned}
 g_{S,v}(k_0) &= g_{L,v}(k_1, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= g_{B,v}(k_4, 0) + g_{L,v}(k_3, -g_{L,l}(k_3))
 \end{aligned}$$

Example 22.2 (continued)

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 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
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Syntax tree t :

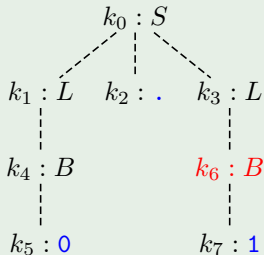


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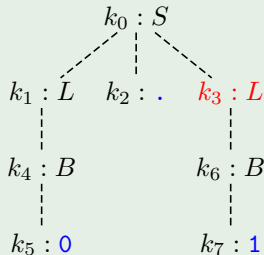


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 &= g_{B,v}(k_4, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{B,v}(k_6, -g_{L,l}(k_3))
 \end{aligned}$$

Example 22.2 (continued)

$$\begin{aligned}
 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad S \rightarrow L : g_{L,v}(k_1, 0) \\
 &\quad S \rightarrow L \cdot L : g_{L,v}(k_1, 0) + \\
 &\quad \quad g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &\text{end} \\
 g_{L,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : g_{B,v}(k_1, p) \\
 &\quad L \rightarrow LB : g_{L,v}(k_1, p+1) \\
 &\quad \quad + g_{B,v}(k_2, p) \\
 &\text{end} \\
 g_{L,l}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : 1 \\
 &\quad L \rightarrow LB : g_{L,l}(k_1) + 1 \\
 &\text{end} \\
 g_{B,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad B \rightarrow 0 : 0 \\
 &\quad B \rightarrow 1 : 2^p \\
 &\text{end}
 \end{aligned}$$

Syntax tree t :

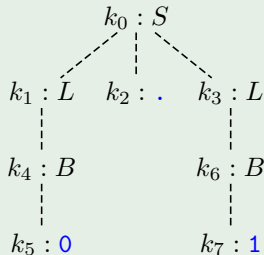


$$\begin{aligned}
 g_{S,v}(k_0) &= g_{L,v}(k_1, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= g_{B,v}(k_4, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{B,v}(k_6, -g_{L,l}(k_3)) \\
 &= 0 + 2^{-g_{L,l}(k_3)}
 \end{aligned}$$

Example 22.2 (continued)

$$\begin{aligned}
 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad S \rightarrow L : g_{L,v}(k_1, 0) \\
 &\quad S \rightarrow L \cdot L : g_{L,v}(k_1, 0) + \\
 &\quad \quad g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &\quad \text{end} \\
 g_{L,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
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 &\quad \quad + g_{B,v}(k_2, p) \\
 &\quad \text{end} \\
 g_{L,l}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : 1 \\
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 &\quad \text{end} \\
 g_{B,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad B \rightarrow 0 : 0 \\
 &\quad B \rightarrow 1 : 2^p \\
 &\quad \text{end}
 \end{aligned}$$

Syntax tree t :

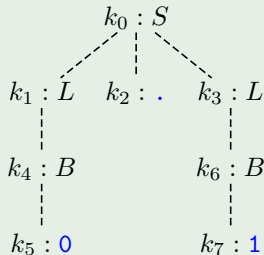


$$\begin{aligned}
 g_{S,v}(k_0) &= g_{L,v}(k_1, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
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 &= 0 + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{B,v}(k_6, -g_{L,l}(k_3)) \\
 &= 0 + 2^{-g_{L,l}(k_3)} \\
 &= 0 + 2^{-1}
 \end{aligned}$$

Example 22.2 (continued)

$$\begin{aligned}
 g_{S,v}(k_0) &= \text{case production}(k_0) \text{ of} \\
 &\quad S \rightarrow L : g_{L,v}(k_1, 0) \\
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 g_{L,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad L \rightarrow B : g_{B,v}(k_1, p) \\
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 &\text{end} \\
 g_{B,v}(k_0, p) &= \text{case production}(k_0) \text{ of} \\
 &\quad B \rightarrow 0 : 0 \\
 &\quad B \rightarrow 1 : 2^p \\
 &\text{end}
 \end{aligned}$$

Syntax tree t :



$$\begin{aligned}
 g_{S,v}(k_0) &= g_{L,v}(k_1, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= g_{B,v}(k_4, 0) + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{L,v}(k_3, -g_{L,l}(k_3)) \\
 &= 0 + g_{B,v}(k_6, -g_{L,l}(k_3)) \\
 &= 0 + 2^{-g_{L,l}(k_3)} \\
 &= 0 + 2^{-1} \\
 &= 0.5
 \end{aligned}$$

Why **Strong** Noncircularity?

If the attribute grammar is not strongly noncircular, then the construction of the evaluation functions fails.

Example 22.3 (cf. Example 21.4)

$$\begin{array}{ll} S \rightarrow A & \alpha.0 = \alpha_2.1 \\ & \beta_1.1 = \alpha_1.1 \\ & \beta_2.1 = \alpha_2.1 \\ A \rightarrow \mathbf{a} & \alpha_1.0 = \beta_2.0 \\ & \alpha_2.0 = 2 \\ A \rightarrow \mathbf{b} & \alpha_1.0 = 1 \\ & \alpha_2.0 = \beta_1.0 \end{array}$$

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If the attribute grammar is not strongly noncircular, then the construction of the evaluation functions fails.

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$$S \rightarrow A \quad \alpha.0 = \alpha_2.1$$
$$\beta_1.1 = \alpha_1.1$$
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$$\alpha_2.0 = 2$$
$$A \rightarrow \mathbf{b} \quad \alpha_1.0 = 1$$
$$\alpha_2.0 = \beta_1.0$$

From Example 21.4:

$$IS'(A) = \{(\beta_2, \alpha_1), (\beta_1, \alpha_2)\}$$

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$\alpha_2.0 = 2$

$A \rightarrow \mathbf{b}$ $\alpha_1.0 = 1$

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Definition of $g_{S,\alpha}$:

$g_{S,\alpha}(k_0)$

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Definition of $g_{S,\alpha}$:

$g_{S,\alpha}(k_0)$

$= \text{eval}(\alpha.0)$

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$$\begin{aligned} g_{S,\alpha}(k_0) &= \text{eval}(\alpha.0) \\ &= \text{eval}(\alpha_2.1) \\ &= g_{A,\alpha_2}(k_1, \text{eval}(\beta_1.1)) \end{aligned}$$

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Definition of $g_{S,\alpha}$:

$g_{S,\alpha}(k_0)$

$= \text{eval}(\alpha.0)$

$= \text{eval}(\alpha_2.1)$

$= g_{A,\alpha_2}(k_1, \text{eval}(\beta_1.1))$

$= g_{A,\alpha_2}(k_1, \text{eval}(\alpha_1.1))$

$= g_{A,\alpha_2}(k_1, g_{A,\alpha_1}(k_1, \text{eval}(\beta_2.1)))$

From Example 21.4:

$IS'(A) = \{(\beta_2, \alpha_1), (\beta_1, \alpha_2)\}$

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Definition of $g_{S,\alpha}$:

$$\begin{aligned} g_{S,\alpha}(k_0) &= \text{eval}(\alpha.0) \\ &= \text{eval}(\alpha_2.1) \\ &= g_{A,\alpha_2}(k_1, \text{eval}(\beta_1.1)) \\ &= g_{A,\alpha_2}(k_1, \text{eval}(\alpha_1.1)) \\ &= g_{A,\alpha_2}(k_1, g_{A,\alpha_1}(k_1, \text{eval}(\beta_2.1))) \\ &= g_{A,\alpha_2}(k_1, g_{A,\alpha_1}(k_1, \text{eval}(\alpha_2.1))) \end{aligned}$$

$\Rightarrow$ **does not terminate!**