Compiler Construction

Lecture 26: Code Optimization

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH Aachen University

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/cc10/

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Online Evaluation of CS Curricula

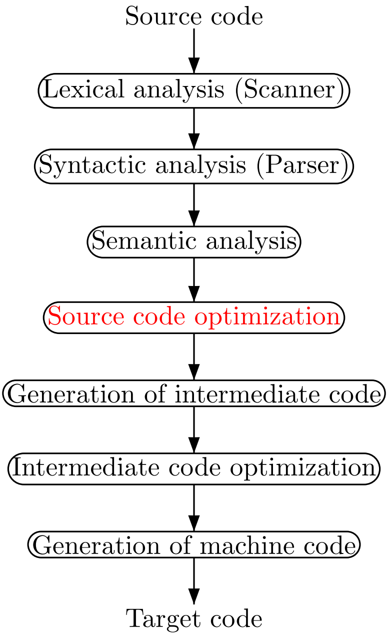
`http://www.campus.rwth-aachen.de/evasys/index.php?mca=online/index/`

- In German
- Losung: `autobahn`
- Advisory service, preparatory CS course
- Conditions of study
- Applied subject
- ...

Outline

1. Code Optimization
2. Preliminaries on Dataflow Analysis
3. Example: Available Expressions Analysis
4. Example: Live Variables Analysis
5. The Dataflow Analysis Framework

Conceptual Structure of a Compiler



Code Optimization

Goal: Make generated code faster and/or more compact

Common procedure:

- Gather information about program by performing some kind of analysis
- Exploit information to optimize code

Here: dataflow analysis

$\Longrightarrow$ attach properties to program statements
that hold every time when statement is executed

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Dataflow Analysis: the Approach

- Traditional form of program analysis
- Idea: describe how analysis information flows through program
- Distinctions:

direction of flow: forward vs. backward analyses
quantification over paths: may (union) vs. must (intersection) analyses
dependence on statement order: flow-sensitive vs. flow-insensitive analyses
procedures: interprocedural vs. intraprocedural analyses
distinction of procedure calls: context-sensitive vs. context-insensitive analyses

Labeled Programs

- Goal: localization of analysis information
- Dataflow information will be associated with
  - assignments
  - tests in conditionals (`if`) and loops (`while`)

  These constructs will be called blocks (denotation: Blk).
- Assume set of labels Lab with meta variable $l \in Lab$ (usually $Lab = \mathbb{N}$)

Definition 26.1 (Labeled WHILE programs)

The syntax of labeled WHILE programs is defined by the following context-free grammar:

$$
\begin{aligned}
A &::= z \mid I \mid A_1 \texttt{ + } A_2 \in AExp \\
B &::= A_1 \texttt{ < } A_2 \mid \texttt{not } B \mid B_1 \texttt{ and } B_2 \in BExp \\
C &::= [I \texttt{ := } A]^l \mid C_1 \texttt{;} C_2 \mid \\
&\qquad \texttt{if } [B]^l \texttt{ then } C_1 \texttt{ else } C_2 \mid \texttt{while } [B]^l \texttt{ do } C \in Cmd
\end{aligned}
$$

Here all labels in a statement $C \in Cmd$ are assumed to be distinct.

A WHILE Program with Labels

Example 26.2

```
x := 6;
y := 7;
z := 0;
while x > 0 do
  x := x - 1;
  v := y;
  while v > 0 do
    v := v - 1;
    z := z + 1;
```

Representing Control Flow I

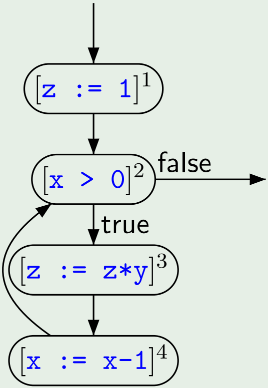
- Every (labeled) statement has a single entry (given by the initial label) and generally multiple exits (given by the final labels)
- Labels are connected via control-flow edges
- Formally:
  - initial label $\mathrm{init} : Cmd \to Lab$
  - final labels $\mathrm{final} : Cmd \to 2^{Lab}$
  - (control) flow relation $\mathrm{flow}(C) \subseteq Lab \times Lab$

Example 26.3

Visualization by flow graph:

```
C = [z := 1]^1;
    while [x > 0]^2 do
      [z := z*y]^3;
      [x := x-1]^4
```

$$\begin{aligned}\text{init}(C) &= 1\\ \text{final}(C) &= \{2\}\\ \text{flow}(C) &= \{(1,2),(2,3),(3,4),(4,2)\}\end{aligned}$$



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Goal of the Analysis

Available Expressions Analysis

The goal of Available Expressions Analysis is to determine, for each program point, which (complex) expressions *must* have been computed, and not later modified, on all paths to the program point.

- can be used to avoid recomputations of expressions
- only interesting for non-trivial (i.e., composite) arithmetic expressions

Example 26.4 (Available Expressions Analysis)

$[\texttt{x := a+b}]^1$;
$[\texttt{y := a*b}]^2$;
while $[\texttt{y > a+b}]^3$ do
  $[\texttt{a := a+1}]^4$;
  $[\texttt{x := a+b}]^5$

- a+b available at label 3
- a+b not available at label 5
- possible optimization:
  while $[\texttt{y > x}]^3$ do

Formalizing Available Expressions Analysis I

- Given $C \in Cmd$, $Lab_C/Blk_C/AExp_C$ denote the sets of all labels/blocks/complex arithmetic expressions occurring in C, respectively
- Given $A \in AExp_C$, $\mathrm{Var}(A)$ denotes the set of all variable identifiers occurring in A
- An expression A is killed in a block β if any of the variables in A is modified in β
- Formally: $\mathrm{kill}_{AE} : Blk_C \to 2^{AExp_C}$ is defined by

$$\mathrm{kill}_{AE}([I := A]^l) := \{A' \in AExp_C \mid I \in \mathrm{Var}(A')\}$$
$$\mathrm{kill}_{AE}([B]^l) := \emptyset$$

- An expression A is generated in a block β if it is evaluated in and none of its variables are modified by β
- Formally: $\mathrm{gen}_{AE} : Blk_C \to 2^{AExp_C}$ is defined by

$$\mathrm{gen}_{AE}([I := A]^l) := \begin{cases} \{A\} & \text{if } A \in AExp_C, I \notin \mathrm{Var}(A) \\ \emptyset & \text{otherwise} \end{cases}$$
$$\mathrm{gen}_{AE}([B]^l) := AExp_B$$

Example 26.5 ($\mathrm{kill}_{AE}/\mathrm{gen}_{AE}$ functions)

$C =$
```
[x := a+b]^1;
[y := a*b]^2;
while [y > a+b]^3 do
  [a := a+1]^4;
  [x := a+b]^5
```

- $AExp_C = \{$a+b, a*b, a+1$\}$
-

Lab_C	$\mathrm{kill}_{AE}(\beta^l)$	$\mathrm{gen}_{AE}(\beta^l)$
1	$\emptyset$	{a+b}
2	$\emptyset$	{a*b}
3	$\emptyset$	{a+b}
4	{a+b, a*b, a+1}	$\emptyset$
5	$\emptyset$	{a+b}

The Equation System I

- Analysis itself defined by setting up an equation system
- For each $l \in Lab_C$, $AE_l \subseteq AExp_C$ represents the set of available expressions at the entry of block β^l
- Formally:
$$AE_l = \begin{cases} \emptyset & \text{if } l = \text{init}(C) \\ \bigcap\{\varphi_{l'}(AE_{l'}) \mid (l', l) \in \text{flow}(C)\} & \text{otherwise} \end{cases}$$
where $\varphi_{l'} : 2^{AExp_C} \to 2^{AExp_C}$ denotes the transfer function of block $\beta^{l'}$, given by
$$\varphi_{l'}(A) := (A \setminus \text{kill}_{AE}(\beta^{l'})) \cup \text{gen}_{AE}(\beta^{l'})$$
- Characterization of analysis:

  forward: starts in $\text{init}(C)$ and proceeds downwards

  must: $\bigcap$ in equation for AE_l

  flow-sensitive: results depending on order of assignments
- In general: solution not necessarily unique

  $\Longrightarrow$ choose greatest one

The Equation System II

Reminder:

$$AE_l = \begin{cases} \emptyset & \text{if } l = \mathrm{init}(C) \\ \bigcap\{\varphi_{l'}(AE_{l'}) \mid (l', l) \in \mathrm{flow}(C)\} & \text{otherwise} \end{cases}$$

$$\varphi_{l'}(E) = (E \setminus \mathrm{kill}_{AE}(\beta^{l'})) \cup \mathrm{gen}_{AE}(\beta^{l'})$$

Example 26.6 (AE equation system)

$C =$ [x := a+b][1];
[y := a*b][2];
while [y > a+b][3] do
[a := a+1][4];
[x := a+b][5]

$l \in Lab_C$	$\mathrm{kill}_{AE}(\beta^l)$	$\mathrm{gen}_{AE}(\beta^l)$
1	$\emptyset$	{a+b}
2	$\emptyset$	{a*b}
3	$\emptyset$	{a+b}
4	{a+b, a*b, a+1}	$\emptyset$
5	$\emptyset$	{a+b}

Equations:

$$\begin{aligned} AE_1 &= \emptyset \\ AE_2 &= \varphi_1(AE_1) = AE_1 \cup \{\texttt{a+b}\} \\ AE_3 &= \varphi_2(AE_2) \cap \varphi_5(AE_5) \\ &= (AE_2 \cup \{\texttt{a*b}\}) \cap (AE_5 \cup \{\texttt{a+b}\}) \\ AE_4 &= \varphi_3(AE_3) = AE_3 \cup \{\texttt{a+b}\} \\ AE_5 &= \varphi_4(AE_4) = AE_4 \setminus \{\texttt{a+b}, \texttt{a*b}, \texttt{a+1}\} \end{aligned}$$

Solution:

$$\begin{aligned} AE_1 &= \emptyset \\ AE_2 &= \{\texttt{a+b}\} \\ AE_3 &= \{\texttt{a+b}\} \\ AE_4 &= \{\texttt{a+b}\} \\ AE_5 &= \emptyset \end{aligned}$$

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Goal of the Analysis

Live Variables Analysis

The goal of Live Variables Analysis is to determine, for each program point, which variables *may* be live at the exit from the point.

- A variable is called live at the exit of a block if there exists a path from the block to a use of the variable that does not re-define the variable
- All variables considered to be live at the end of the program (alternative: restriction to output variables)
- Can be used for Dead Code Elimination: remove assignments to non-live variables

Example 26.7 (Live Variables Analysis)

```
[x := 2]^1;
[y := 4]^2;
[x := 1]^3;
if [y > 0]^4 then
  [z := x]^5
else
  [z := y*y]^6;
[x := z]^7
```

- x not live at exit from label 1
- y live at exit from 2
- x live at exit from 3
- z live at exits from 5 and 6
- possible optimization: remove $[\texttt{x := 2}]^1$

Formalizing Live Variables Analysis I

- A variable on the left-hand side of an assignment is killed by the assignment; tests do not kill
- Formally: $\text{kill}_{LV} : Blk_C \to 2^{Var_C}$ is defined by
  $$\text{kill}_{LV}([I \ \texttt{:=}\ A]^l) := \{I\}$$
  $$\text{kill}_{LV}([B]^l) := \emptyset$$
- Every reading access generates a live variable
- Formally: $\text{gen}_{LV} : Blk_C \to 2^{Var_C}$ is defined by
  $$\text{gen}_{LV}([I \ \texttt{:=}\ A]^l) := \text{Var}(A)$$
  $$\text{gen}_{LV}([B]^l) := \text{Var}(B)$$

Example 26.8 ($\mathrm{kill}_{LV}/\mathrm{gen}_{LV}$ functions)

```
c = [x := 2]^1;
    [y := 4]^2;
    [x := 1]^3;
    if [y > 0]^4 then
      [z := x]^5
    else
      [z := y*y]^6;
    [x := z]^7
```

- $Var_c = \{x, y, z\}$
-

$l \in Lab_c$	$\mathrm{kill}_{LV}(\beta^l)$	$\mathrm{gen}_{LV}(\beta^l)$
1	$\{x\}$	$\emptyset$
2	$\{y\}$	$\emptyset$
3	$\{x\}$	$\emptyset$
4	$\emptyset$	$\{y\}$
5	$\{z\}$	$\{x\}$
6	$\{z\}$	$\{y\}$
7	$\{x\}$	$\{z\}$

The Equation System I

- For each $l \in Lab_C$, $LV_l \subseteq Var_c$ represents the set of live variables at the exit of block β^l
- Formally, for a program $C \in Cmd$ with isolated exits:
$$LV_l = \begin{cases} Var_C & \text{if } l \in \text{final}(C) \\ \bigcup\{\varphi_{l'}(LV_{l'}) \mid (l, l') \in \text{flow}(C)\} & \text{otherwise} \end{cases}$$
where $\varphi_{l'} : 2^{Var_C} \to 2^{Var_C}$ denotes the transfer function of block $\beta^{l'}$, given by
$$\varphi_{l'}(V) := (V \setminus \text{kill}_{LV}(\beta^{l'})) \cup \text{gen}_{LV}(\beta^{l'})$$
- Characterization of analysis:

  backward: starts in final(C) and proceeds upwards

  may: $\bigcup$ in equation for LV_l

  flow-sensitive: results depending on order of assignments
- In general: solution not necessarily unique

  $\Longrightarrow$ choose least one

The Equation System II

Reminder:

$$LV_l = \begin{cases} Var_C & \text{if } l \in \text{final}(C) \\ \bigcup\{\varphi_{l'}(LV_{l'}) \mid (l,l') \in \text{flow}(C)\} & \text{otherwise} \end{cases}$$

$$\varphi_{l'}(V) = (V \setminus \text{kill}_{LV}(\beta^{l'})) \cup \text{gen}_{LV}(\beta^{l'})$$

Example 26.9 (LV equation system)

$C =$
```
[x := 2]^1;[y := 4]^2;
[x := 1]^3;
if [y > 0]^4 then
  [z := x]^5
else
  [z := y*y]^6;
[x := z]^7
```

$l \in Lab_c$	$\text{kill}_{LV}(\beta^l)$	$\text{gen}_{LV}(\beta^l)$
1	{x}	$\emptyset$
2	{y}	$\emptyset$
3	{x}	$\emptyset$
4	$\emptyset$	{y}
5	{z}	{x}
6	{z}	{y}
7	{x}	{z}

$$\begin{aligned} LV_1 &= \varphi_2(LV_2) = LV_2 \setminus \{\mathtt{y}\} \\ LV_2 &= \varphi_3(LV_3) = LV_3 \setminus \{\mathtt{x}\} \\ LV_3 &= \varphi_4(LV_4) = LV_4 \cup \{\mathtt{y}\} \\ LV_4 &= \varphi_5(LV_5) \cup \varphi_6(LV_6) \\ &= ((LV_5 \setminus \{\mathtt{z}\}) \cup \{\mathtt{x}\}) \cup \\ &\quad ((LV_6 \setminus \{\mathtt{z}\}) \cup \{\mathtt{y}\}) \\ LV_5 &= \varphi_7(LV_7) = (LV_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\} \\ LV_6 &= \varphi_7(LV_7) = (LV_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\} \\ LV_7 &= \{\mathtt{x}, \mathtt{y}, \mathtt{z}\} \end{aligned}$$

Solution:

$$\begin{aligned} LV_1 &= \emptyset \\ LV_2 &= \{\mathtt{y}\} \\ LV_3 &= \{\mathtt{x}, \mathtt{y}\} \\ LV_4 &= \{\mathtt{x}, \mathtt{y}\} \\ LV_5 &= \{\mathtt{y}, \mathtt{z}\} \\ LV_6 &= \{\mathtt{y}, \mathtt{z}\} \\ LV_7 &= \{\mathtt{x}, \mathtt{y}, \mathtt{z}\} \end{aligned}$$

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Similarities between Analysis Problems

- **Observation:** the analyses presented so far have some similarities

$\Longrightarrow$ Look for underlying framework

- **Advantage:** possibility for designing (efficient) generic algorithms for solving dataflow equations
- **Overall pattern:** for $C \in Cmd$ and $l \in Lab_C$, the analysis information (AI) is described by equations of the form

$$AI_l = \begin{cases} \iota & \text{if } l \in E \\ \bigoplus\{\varphi_{l'}(AI_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

where

- ι specifies the initial analysis information
- E is $\{\text{init}(C)\}$ or $\text{final}(C)$
- $\bigoplus$ is $\bigcap$ or $\bigcup$
- $\varphi_{l'}$ denotes the transfer function of block $\beta^{l'}$
- F is $\text{flow}(C)$ or $\text{flow}^R(C)$ $(:= \{(l', l) \mid (l, l') \in \text{flow}(C)\})$

Characterization of Analyses

- **Direction of information flow:**
  - forward:
    - $E = \{\text{init}(C)\}$
    - c has isolated entry
    - $F = \text{flow}(C)$
    - AI_l concerns entry of β^l
  - backward:
    - $E = \text{final}(C)$
    - c has isolated exits
    - $F = \text{flow}^R(C)$
    - AI_l concerns exit of β^l
- **Quantification over paths:**
  - may:
    - $\bigoplus = \bigcup$
    - property satisfied by some path
    - interested in least solution (later)
  - must:
    - $\bigoplus = \bigcap$
    - property satisfied by all paths
    - interested in greatest solution (later)

Fixpoint Iteration I

Idea: use fixpoint iteration to solve dataflow equation system

1. For $C \in Cmd$ and $l \in Lab_C$, start with "initial" information AI_l $(AE_l = AExp_C,\ LV_l = \emptyset)$
2. Iteratively evaluate dataflow equations until fixpoint reached

Theoretical foundations:

- Analysis information D forms complete lattice $(D_{AE} = 2^{AExp_C},\ D_{LV} = 2^{Var_C})$
  - every subset of D has a least upper/greatest lower bound $\implies$ well-definedness of $\bigoplus$
- ... that satisfies the ascending chain condition
  - $d_1 \overset{\supseteq}{\subseteq} d_2 \overset{\supseteq}{\subseteq} \ldots \implies \exists n : d_n = d_{n+1} = \ldots$
- Combination operator and all transfer functions monotonic
  - $d_1 \overset{\supseteq}{\subseteq} d_2 \implies \varphi(d_1) \overset{\supseteq}{\subseteq} \varphi(d_2)$

$\implies$ Fixpoint effectively computable by iteration

Fixpoint Iteration II

Example 26.10 (Available Expressions; cf. Example 26.6)

Program:

$$
\begin{aligned}
C = {} & [\texttt{x := a+b}]^1; \\
& [\texttt{y := a*b}]^2; \\
& \texttt{while } [\texttt{y > a+b}]^3 \texttt{ do} \\
& \quad [\texttt{a := a+1}]^4; \\
& \quad [\texttt{x := a+b}]^5
\end{aligned}
$$

Equation system:

$$
\begin{aligned}
AE_1 &= \emptyset \\
AE_2 &= AE_1 \cup \{\texttt{a+b}\} \\
AE_3 &= (AE_2 \cup \{\texttt{a*b}\}) \cap (AE_5 \cup \{\texttt{a+b}\}) \\
AE_4 &= AE_3 \cup \{\texttt{a+b}\} \\
AE_5 &= AE_4 \setminus \{\texttt{a+b}, \texttt{a*b}, \texttt{a+1}\}
\end{aligned}
$$

Fixpoint iteration:

i	1	2	3	4	5
0	$AExp_c$	$AExp_c$	$AExp_c$	$AExp_c$	$AExp_c$
1	$\emptyset$	$AExp_c$	$AExp_c$	$AExp_c$	$\emptyset$
2	$\emptyset$	$\{\texttt{a+b}\}$	$\{\texttt{a+b}\}$	$AExp_c$	$\emptyset$
3	$\emptyset$	$\{\texttt{a+b}\}$	$\{\texttt{a+b}\}$	$\{\texttt{a+b}\}$	$\emptyset$
4	$\emptyset$	$\{\texttt{a+b}\}$	$\{\texttt{a+b}\}$	$\{\texttt{a+b}\}$	$\emptyset$

Example 26.11 (Live Variables; cf. Example 26.9)

Program:

```
[x := 2]1;
[y := 4]2;
[x := 1]3;
if [y > 0]4 then
  [z := x]5
else
  [z := y*y]6;
[x := z]7
```

Equation system:

$$
\begin{aligned}
LV_1 &= LV_2 \setminus \{\mathtt{y}\}\\
LV_2 &= LV_3 \setminus \{\mathtt{x}\}\\
LV_3 &= LV_4 \cup \{\mathtt{y}\}\\
LV_4 &= ((LV_5 \setminus \{\mathtt{z}\}) \cup \{\mathtt{x}\}) \cup ((LV_6 \setminus \{\mathtt{z}\}) \cup \{\mathtt{y}\})\\
LV_5 &= (LV_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\}\\
LV_6 &= (LV_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\}\\
LV_7 &= \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}
\end{aligned}
$$

Fixpoint iteration:

i	1	2	3	4	5	6	7
0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
1	$\emptyset$	$\emptyset$	{y}	{x, y}	{z}	{z}	{x, y, z}
2	$\emptyset$	{y}	{x, y}	{x, y}	{y, z}	{y, z}	{x, y, z}
3	$\emptyset$	{y}	{x, y}	{x, y}	{y, z}	{y, z}	{x, y, z}

Outlook

Summer Semester 2011: Static Program Analysis

- More on dataflow analysis
- Constraint-based analysis
- Abstract interpretation
- Pointer analysis