

Compiler Construction

Lecture 4: Lexical Analysis III (First-Longest-Match Analysis)

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- 1 Repetition: The Extended Matching Problem
- 2 First-Longest-Match Analysis
- 3 Implementation of FLM Analysis
- 4 First-Longest-Match Analysis with NFA

The Extended Matching Problem I

Definition

Let $n \geq 1$ and $\alpha_1, \dots, \alpha_n \in RE_\Omega$ with $\varepsilon \notin \llbracket \alpha_i \rrbracket \neq \emptyset$ for every $i \in [n]$ ($= \{1, \dots, n\}$). Let $\Sigma := \{T_1, \dots, T_n\}$ be an alphabet of corresponding **tokens** and $w \in \Omega^+$. If $w_1, \dots, w_k \in \Omega^+$ such that

- $w = w_1 \dots w_k$ and
- for every $j \in [k]$ there exists $i_j \in [n]$ such that $w_j \in \llbracket \alpha_{i_j} \rrbracket$,

then

- $(w_1, \dots, w_k)$ is called a **decomposition** and
- $(T_{i_1}, \dots, T_{i_k})$ is called an **analysis**

of w w.r.t. $\alpha_1, \dots, \alpha_n$.

Problem (Extended matching problem)

Given $\alpha_1, \dots, \alpha_n \in RE_\Omega$ and $w \in \Omega^+$, decide whether there exists a decomposition of w w.r.t. $\alpha_1, \dots, \alpha_n$ and determine a corresponding analysis.

The Extended Matching Problem II

Observation: neither the decomposition nor the analysis are uniquely determined

Example

- ❶ $\alpha = a^+, w = aa$
 $\implies$ two decompositions (aa) and (a, a) with unique analysis each
- ❷ $\alpha_1 = a \mid b, \alpha_2 = a \mid c, w = a$
 $\implies$ unique decomposition (a) but two analyses (T_1) and (T_2)

Goal: make both unique $\implies$ deterministic scanning

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Two principles:

- ① **Principle of the longest match** (“maximal munch tokenization”)
 - for uniqueness of decomposition
 - make lexemes as long as possible
 - motivated by applications: e.g., every (non-empty) prefix of an identifier is also an identifier

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- ❶ **Principle of the longest match** (“maximal munch tokenization”)
 - for uniqueness of decomposition
 - make lexemes as long as possible
 - motivated by applications: e.g., every (non-empty) prefix of an identifier is also an identifier
- ❷ **Principle of the first match**
 - for uniqueness of analysis
 - choose first matching regular expression (in the given order)
 - therefore: arrange keywords before identifiers (if keywords protected)

Principle of the Longest Match

Definition 4.1 (Longest-match decomposition)

A decomposition $(w_1, \dots, w_k)$ of $w \in \Omega^+$ w.r.t. $\alpha_1, \dots, \alpha_n \in RE_\Omega$ is called a **longest-match decomposition (LM decomposition)** if, for every $i \in [k]$, $x \in \Omega^+$, and $y \in \Omega^*$,

$$w = w_1 \dots w_i x y \implies \text{there is no } j \in [n] \text{ such that } w_i x \in \llbracket \alpha_j \rrbracket$$

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Corollary 4.2

Given w and $\alpha_1, \dots, \alpha_n$,

- at most one LM decomposition of w exists (clear by definition) and*

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Corollary 4.2

Given w and $\alpha_1, \dots, \alpha_n$,

- *at most one LM decomposition of w exists (clear by definition) and*
- *it is possible that w has a decomposition but no LM decomposition (see following example).*

Example 4.3

$$w = aab, \alpha_1 = a^+, \alpha_2 = ab$$

$\implies (a, ab)$ is a decomposition but no LM decomposition exists

Principle of the First Match

Problem: a (unique) LM decomposition can have **several associated analyses** (since $\llbracket \alpha_i \rrbracket \cap \llbracket \alpha_j \rrbracket \neq \emptyset$ with $i \neq j$ is possible; cf. keyword/identifier problem)

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Definition 4.4 (First-longest-match analysis)

Let $(w_1, \dots, w_k)$ be the LM decomposition of $w \in \Omega^+$ w.r.t. $\alpha_1, \dots, \alpha_n \in RE_\Omega$. Its **first-longest-match analysis (FLM analysis)** $(T_{i_1}, \dots, T_{i_k})$ is determined by

$$i_j := \min\{l \in [n] \mid w_j \in \llbracket \alpha_l \rrbracket\} \text{ for every } j \in [k].$$

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Corollary 4.5

Given w and $\alpha_1, \dots, \alpha_n$, there is at most one FLM analysis of w . It exists iff the LM decomposition of w exists.

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Algorithm 4.6 (FLM analysis—overview)

Input: *expressions* $\alpha_1, \dots, \alpha_n \in RE_\Omega$, *tokens* $\{T_1, \dots, T_n\}$,
input word $w \in \Omega^+$

Implementation of FLM Analysis

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Input: *expressions* $\alpha_1, \dots, \alpha_n \in RE_\Omega$, *tokens* $\{T_1, \dots, T_n\}$,
input word $w \in \Omega^+$

Procedure: ❶ *for every* $i \in [n]$, *construct* $\mathfrak{A}_i \in DFA_\Omega$ *such that*
 $L(\mathfrak{A}_i) = \llbracket \alpha_i \rrbracket$ (see *DFA method*; Alg. ??)

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- ❷ construct the *product automaton* $\mathfrak{A} \in DFA_\Omega$ such that $L(\mathfrak{A}) = \bigcup_{i=1}^n \llbracket \alpha_i \rrbracket$

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Output: FLM analysis of w (if existing)

(2) The Product Automaton

Definition 4.7 (Product automaton)

Let $\mathfrak{A}_i = \langle Q_i, \Omega, \delta_i, q_0^{(i)}, F_i \rangle \in DFA_\Omega$ for every $i \in [n]$. The **product automaton** $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_\Omega$ is defined by

- $Q := Q_1 \times \dots \times Q_n$
- $q_0 := (q_0^{(1)}, \dots, q_0^{(n)})$
- $\delta((q^{(1)}, \dots, q^{(n)}), a) := (\delta_1(q^{(1)}, a), \dots, \delta_n(q^{(n)}, a))$
- $(q^{(1)}, \dots, q^{(n)}) \in F$ iff there ex. $i \in [n]$ such that $q^{(i)} \in F_i$

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Lemma 4.8

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Remark: similar construction for intersection ($F := F_1 \times \dots \times F_n$)

(3) Partitioning the Final States

Definition 4.9 (Partition of final states)

Let $\mathcal{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_{\Omega}$ be the product automaton as constructed before. Its set of final states is **partitioned** into

$F = \bigsqcup_{i=1}^n F^{(i)}$ by the requirement

$(q^{(1)}, \dots, q^{(n)}) \in F^{(i)}$ iff $q^{(i)} \in F_i$ and $\forall j \in [i-1] : q^{(j)} \notin F_j$

(or: $F^{(i)} := (Q_1 \setminus F_1) \times \dots \times (Q_{i-1} \setminus F_{i-1}) \times F_i \times Q_{i+1} \times \dots \times Q_n$)

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Corollary 4.10

The above construction yields ($w \in \Omega^+$, $i \in [n]$):

$\hat{\delta}(q_0, w) \in F^{(i)}$ iff $w \in \llbracket \alpha_i \rrbracket$ and $w \notin \bigcup_{j=1}^{i-1} \llbracket \alpha_j \rrbracket$.

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Definition 4.11 (Productive states)

Given $\mathfrak{A}$ as above, a state $q \in Q$ is called **productive** if there exists $w \in \Omega^*$ such that $\hat{\delta}(q, w) \in F$. The set of productive states of $\mathfrak{A}$ is denoted by P (and thus $F \subseteq P$).

(4) The Backtracking DFA I

Goal: extend $\mathfrak{A}$ to the backtracking DFA $\mathfrak{B}$ with output by equipping the input tape with two pointers: a **backtracking head** for marking the last encountered match, and a **lookahead** for determining the longest match.

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A configuration of $\mathfrak{B}$ has three components
(remember: $\Sigma := \{T_1, \dots, T_n\}$ denotes the set of tokens):

- 1 a **mode** $m \in \{N\} \uplus \Sigma$:
 - $m = N$ (“normal”): look for first match (no final state reached yet)
 - $m = T \in \Sigma$: token T has been recognized, looking for possible longer match

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- ② an **input tape** $vgw \in \Omega^* \cdot Q \cdot \Omega^*$:
 - v : lookahead part of input ($v \neq \varepsilon \implies m \in \Sigma$)
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- ❸ an **output tape** $W \in \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$:
 - Σ^* : sequence of tokens recognized so far
 - **lexerr**: a lexical error has occurred (i.e., a non-productive state was entered or the suffix of the input is not a valid lexeme)

(4) The Backtracking DFA II

Definition 4.12 (Backtracking DFA)

- The set of **configurations** of $\mathfrak{B}$ is given by
$$(\{N\} \uplus \Sigma) \times \Omega^* \cdot Q \cdot \Omega^* \times \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$$
- The **initial configuration** for an input word $w \in \Omega^+$ is (N, q_0w, ε) .

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- The **transitions** of $\mathfrak{B}$ are defined as follows (where $q' := \delta(q, a)$):
 - normal mode: look for a match

$$(N, qaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} \\ (N, q'w, W) & \text{if } q' \in P \setminus F \\ \text{output: } W \cdot \text{lexerr} & \text{if } q' \notin P \end{cases}$$

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- backtrack mode: look for longest match

$$(T, vqaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} \\ (T, vaq'w, W) & \text{if } q' \in P \setminus F \\ (N, q_0vaw, WT) & \text{if } q' \notin P \end{cases}$$

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- end of input

$$\begin{aligned} (T, q, W) &\vdash \mathbf{output: } WT && \text{if } q \in F \\ (N, q, W) &\vdash \mathbf{output: } W \cdot \text{lexerr} && \text{if } q \in P \setminus F \\ (T, vaq, W) &\vdash (N, q_0va, WT) && \text{if } q \in P \setminus F \end{aligned}$$

(4) The Backtracking DFA III

Lemma 4.13

Given the backtracking DFA $\mathfrak{B}$ as before and $w \in \Omega^+$,

$$(N, q_0w, \varepsilon) \vdash^* \begin{cases} W \in \Sigma^* & \text{iff } W \text{ is the FLM analysis of } w \\ W \cdot \text{lexerr} & \text{iff no FLM analysis of } w \text{ exists} \end{cases}$$

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Example 4.14

$\alpha = (ab)^+$, $w = abaa$ (on the board)

Remarks:

- **Time complexity:** $\mathcal{O}(|w|^2)$ in worst case

Example 4.15

$\alpha_1 = a$, $\alpha_2 = a^*b$, $w = a^m$ requires $\mathcal{O}(m^2)$

(4) The Backtracking DFA IV

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$\alpha_1 = a$, $\alpha_2 = a^*b$, $w = a^m$ requires $\mathcal{O}(m^2)$

- Improvement by **tabular method** (similar to Knuth-Morris-Pratt Algorithm for pattern matching in strings)

Literature: Th. Reps: “Maximal-Munch” *Tokenization in Linear Time*, ACM TOPLAS 20(2), 1998, 259–273