Compiler Construction

Lecture 6: Syntactic Analysis I (Introduction)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH Aachen University

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/cc10/

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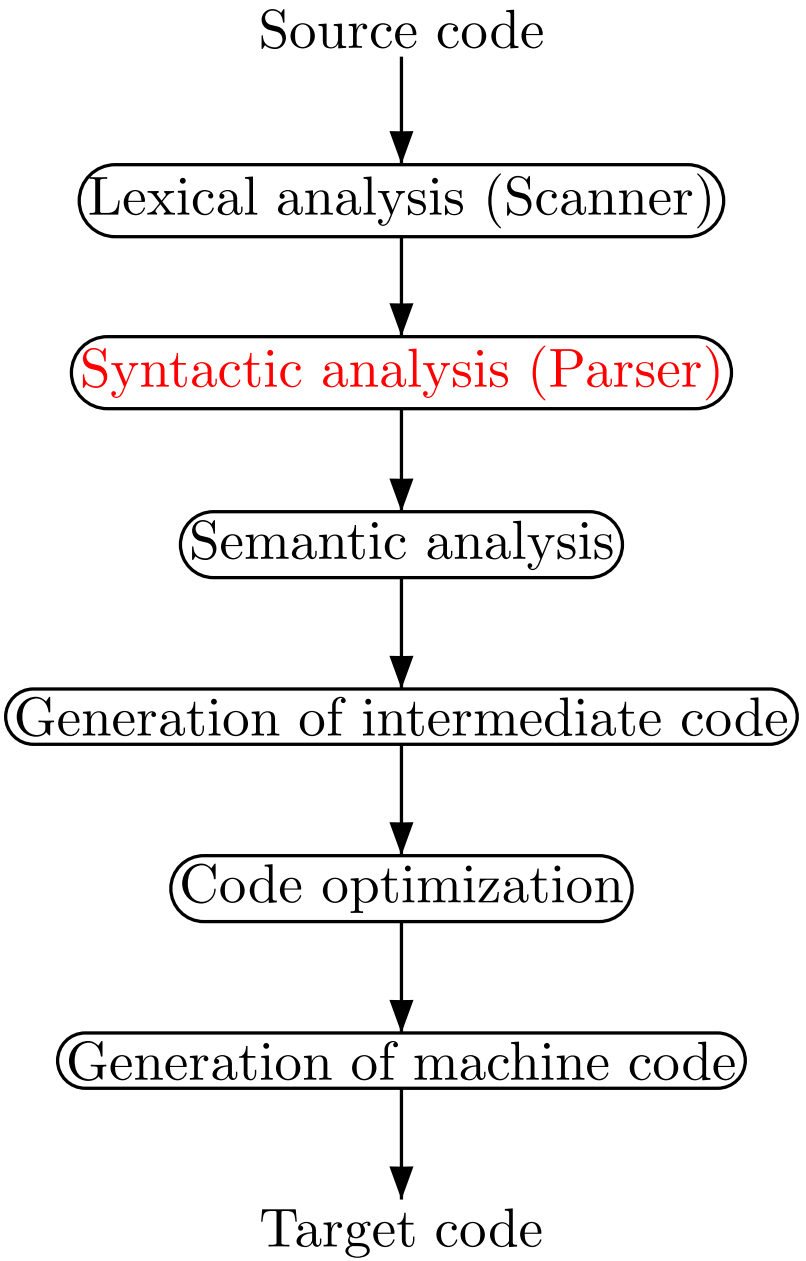
Conceptual Structure of a Compiler



Outline

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  - Σ (finite) set of tokens (= syntactic atoms; terminals)
    (e.g., $\{\mathsf{id}, \mathsf{if}, \mathsf{int}, \ldots\}$)
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  atomic: keywords, variable/type/procedure/... identifiers, numerals, arithmetic/Boolean operators, ...

  complex: declarations, arithmetic/Boolean expressions, statements, ...
- **Observation:** the hierarchical structure of syntactic units can be described by context-free grammars

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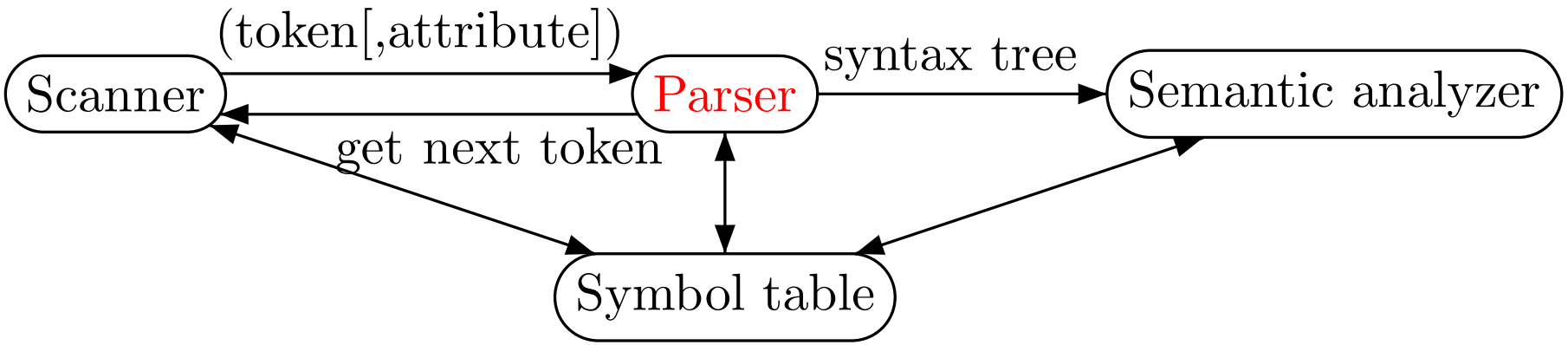

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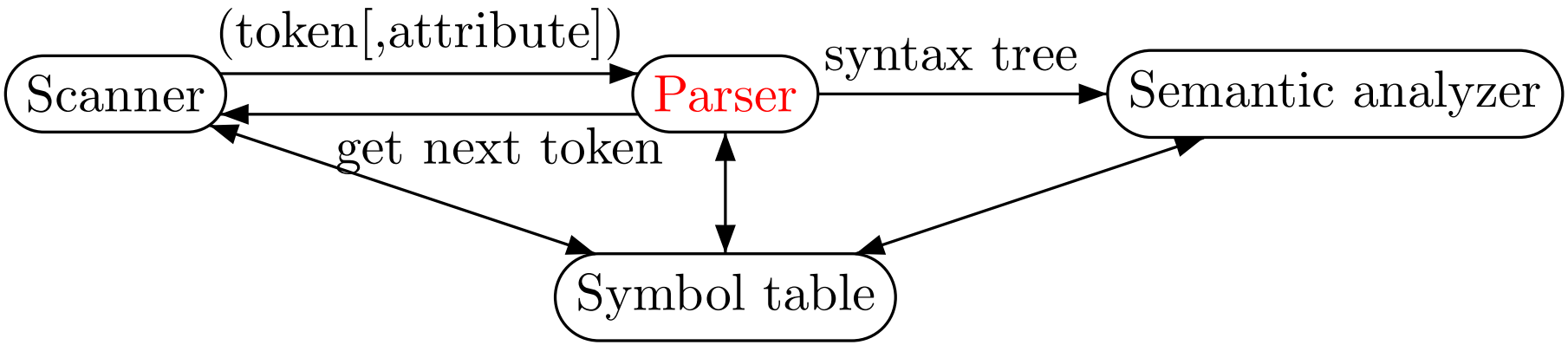
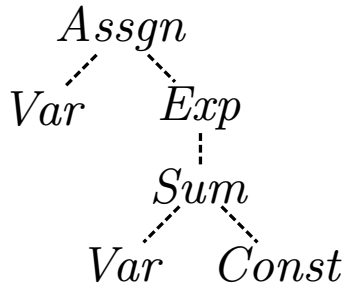


Example: ...␣x1␣:=y2+␣1␣;␣...

↓ Scanner

$\ldots(\mathsf{id}, p_1)(\mathsf{gets},)(\mathsf{id}, p_2)(\mathsf{plus},)(\mathsf{int}, 1)(\mathsf{sem},)\ldots$ $\xrightarrow{\text{Parser}}$



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Context-Free Grammars I

Definition 6.2 (Syntax of context-free grammars)

A context-free grammar (CFG) (over Σ) is a quadruple

$$G = \langle N, \Sigma, P, S \rangle$$

where

- N is a finite set of nonterminal symbols,
- Σ is a (finite) alphabet of terminal symbols (disjoint from N),
- P is a finite set of production rules of the form $A \to \alpha$ where $A \in N$ and $\alpha \in X^*$ for $X := N \cup \Sigma$, and
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Remarks: as denotations we generally use

- $A, B, C, \ldots \in N$ for nonterminal symbols
- $a, b, c, \ldots \in \Sigma$ for terminal symbols
- $u, v, w, \ldots \in \Sigma^*$ for terminal words
- $\alpha, \beta, \gamma, \ldots \in X^*$ for sentences

Context-Free Grammars II

Context-free grammars generate context-free languages:

Definition 6.3 (Semantics of context-free grammars)

Let $G = \langle N, \Sigma, P, S \rangle$ be a context-free grammar.

- The derivation relation $\Rightarrow \subseteq X^* \times X^*$ of G is defined by

$$\alpha \Rightarrow \beta \text{ iff there exist } \alpha_1, \alpha_2 \in X^*, A \to \gamma \in P \text{ such that } \alpha = \alpha_1 A \alpha_2 \text{ and } \beta = \alpha_1 \gamma \alpha_2.$$

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Remark: obviously,

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow_l^* w\} = \{w \in \Sigma^* \mid S \Rightarrow_r^* w\}$$

Example 6.4

The grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ over $\Sigma := \{a, b\}$, given by the productions

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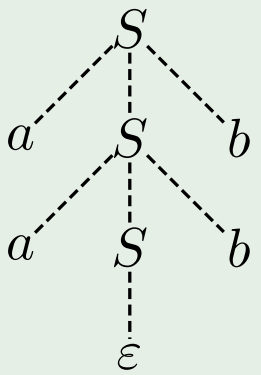
The example derivation

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

can be represented by the following syntax tree for $aabb$:



Syntax Trees, Derivations, and Words

Remark: the connection between derivations, syntax trees, and generated words is not unique

1. A syntax tree generally represents several derivations.
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However:

1. Every syntax tree yields exactly one word (= concatenation of leafs).
2. Every syntax tree corresponds to exactly one leftmost derivation, and vice versa.
3. Every syntax tree corresponds to exactly one rightmost derivation, and vice versa.

(Un-)Ambiguity of CFGs and CFLs

Definition 6.6 (Ambiguity)

- A context-free grammar $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

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on the board

Corollary 6.8

A grammar $G \in CFG_\Sigma$ is unambiguous
iff every word $w \in L(G)$ has exactly one leftmost derivation
iff every word $w \in L(G)$ has exactly one rightmost derivation.