Exercise 1 (Characterisation of LL(1)): (4 Points)

In the lecture two characterizations of $LL(1)$ have been given:

- $G \in LL(1)$ iff for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \\ \Rightarrow_l w\gamma\alpha \end{cases}$$

  such that $\beta \neq \gamma$, it follows that $\mathrm{fi}(\beta\alpha) \cap \mathrm{fi}(\gamma\alpha) = \emptyset$.

- $G \in LL(1)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\mathrm{la}(A \to \beta) \cap \mathrm{la}(A \to \gamma) = \emptyset$$

a) Lift the second definition to $LL(k)$ for $k \in \mathbb{N}^+$. (The first definition was given for $k \in \mathbb{N}^+$ in the lecture.)

b) Show that the definitions are not equivalent by showing that the following grammar is in $LL(2)$ according to the first definition but not according to the second definition (also refered to as *strong* $LL(2)$ property).

$$\begin{aligned} S &\to aAab \mid bAbb \\ A &\to a \mid \varepsilon \end{aligned}$$

c) Explain (in a few words) why the definitions are not equivalent.

Exercise 2 (Find an equivalent LL(1) grammar): (4 Points)

Consider the following grammar G:

$$\begin{aligned} S &\to (L) \mid a \\ L &\to L, S \mid L, SS \mid S \mid SS \end{aligned}$$

a) Show that G is not an $LL(1)$ grammar.

b) Transform G into an equivalent grammar satisfying the $LL(1)$ property.

c) Prove that G has the $LL(1)$ property.

Exercise 3 (LL(1) grammars are never ambiguous): (2 Points)

Show that for every context-free grammar G the following holds:

$$G \text{ ambiguous } \Rightarrow G \notin LL(1)$$

Exercise 4 (Deterministic Top-Down Automaton): (4 Points)

Consider the grammar $G = (N, \Sigma, P, start)$ covering boolean expressions of WHILE with:

- $N := \{start, guard, rel\}$

- $\Sigma := \{AND, OR, ID, EQ, LEQ\}$
- $$\begin{array}{lcl} start & \rightarrow & guard \\ guard & \rightarrow & rel \mid guard\ AND\ guard \mid guard\ OR\ guard \\ rel & \rightarrow & ID\ EQ\ ID \mid ID\ LEQ\ ID \end{array}$$

a) Construct an equivalent grammar G' with $G' \in LL(1)$.

b) Specify the deterministic top-down parsing automaton of G'.

c) Provide a run of G' on the input $ID\ EQ\ ID\ AND\ ID\ LEQ\ ID$.