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Exercise 1 (Attributed Grammars and Equality Systems): (4 Points)

Consider the language $L = \{a^{2^n} | n \in \mathbb{N}\}$. Notice that L is not context-free! Provide an attributed grammar for the language given by $\{a\}^*$. Determine – via a synthesized boolean attribute at the starting symbol – whether a derived word is contained in L. For this purpose do only use plain functions like addition, multiplication or equality testing, but not testing if a function is a power of two etc.
Generate the corresponding equality system for the input word a^4 and solve it.

Exercise 2 (Circularity Test): (8 Points)

Consider the following grammar $G = (N, \Sigma, P, S)$ with inherited attributes $i1, i2$ and synthesised attributes $s1, s2$.

$$
\begin{array}{lcll}
S' & \rightarrow & S & i1.0 = 1 \\
 & & & i2.0 = 2 \\
 & & & i1.1 = s1.1 \\
 & & & i2.1 = i1.0 \\
 & & & s2.0 = s2.1 \\
S & \rightarrow & AA & i1.1 = s1.1 \\
 & & & i2.1 = i1.0 \\
 & & & i1.2 = 0 \\
 & & & i2.2 = i2.0 \\
 & & & s1.0 = s2.1 \\
 & & & s2.0 = s2.2 \\
S & \rightarrow & A & i1.1 = 0 \\
 & & & i2.1 = i2.0 \\
 & & & s2.0 = s2.1 \\
A & \rightarrow & a & s1.0 = 0 \\
 & & & s2.0 = i1.0 \\
A & \rightarrow & b & s2.0 = 0 \\
 & & & s1.0 = i2.0
\end{array}
$$

a) Provide the dependency graph for each production in G.

b) Apply the circularity test from the lecture to G.

1. Calculate the set $IS(A)$ for all $A \in N$.
2. Is G circular? Justify your answer.

c) To simplify the circularity test we want to consider so-called strong circularity. To this aim we modify our circularity test in a way, that attribute dependencies caused by different syntax trees are not distinguished anymore. Thus $IS(A)$, $A \in N$, is now defined as follows:

$$IS(A) = \{(\beta, \alpha) | \beta \stackrel{A}{\hookrightarrow} \alpha \text{ in some syntax tree t with root label A}\} \subseteq Inh \times Syn$$

with $\beta \in inh(A), \alpha \in syn(A)$.

Hint: $IS(A)$ is not a system of attribute dependence sets anymore, but a union!

If we adapt the circularity test from the lecture to this definition of $IS(A)$, does it provide the result that G is strongly non-circular? Argue why!