

Compiler Construction

Lecture 10: Syntax Analysis VI ($LR(0)$ Parsing)

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1 Repetition: $LR(0)$ Grammars

2 $LR(0)$ Parsing

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, **without any lookahead**.

Corollary (LR(0) grammar)

$G \in CFG_\Sigma$ has the **LR(0) property** if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

Goal: derive a **finite information** from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

LR(0) Items and Sets

Definition (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **LR(0) item** for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all **LR(0)** items for γ , called the **LR(0) set** (or: **LR(0) information**) of γ .
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary

- 1 For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.
- 2 $LR(0)(G)$ is finite.
- 3 The item $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$ indicates the possible **reduction** $(w, \alpha \beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \rightarrow \beta$ and $\gamma = \alpha \beta$.
- 4 The item $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$ indicates an incomplete handle β_1 (to be completed by shift operations or ϵ -steps).

Definition (LR(0) conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$ and $I \in \text{LR}(0)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

- I has a **reduce/reduce conflict** if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

Lemma

$G \in \text{LR}(0)$ iff no $I \in \text{LR}(0)(G)$ contains conflicting items.

Proof.

omitted □

Theorem (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

- ① $LR(0)(\varepsilon)$ is the least set such that
 - $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
 - if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.
- ② $LR(0)(\alpha Y)$ ($\alpha \in X^*$, $Y \in X$) is the least set such that
 - if $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
then $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ and
 - if $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$.

Example (cf. Example 9.6)

$G :$

- $S' \rightarrow S$
- $S \rightarrow B \mid C$
- $B \rightarrow aB \mid b$
- $C \rightarrow aC \mid c$

Example (cf. Example 9.6)

$G : \begin{array}{l} S' \rightarrow S \\ S \rightarrow B \mid C \\ B \rightarrow aB \mid b \\ C \rightarrow aC \mid c \end{array} \quad [S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S]$

Example (cf. Example 9.6)

$G : \begin{array}{l} S' \rightarrow S \\ S \rightarrow B \mid C \\ B \rightarrow aB \mid b \\ C \rightarrow aC \mid c \end{array}$

$[A \rightarrow \cdot B \gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$

$\implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B]$

Example (cf. Example 9.6)

$G : \begin{array}{l} S' \rightarrow S \\ S \rightarrow B \mid C \\ B \rightarrow aB \mid b \\ C \rightarrow aC \mid c \end{array}$

$[A \rightarrow \cdot B \gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$

$\implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C]$

Example (cf. Example 9.6)

$G :$ $S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$[A \rightarrow \cdot B \gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$
 $[B \rightarrow \cdot b]$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G :$ $S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$[A \rightarrow \cdot B \gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) :$

$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot B]$	$[S \rightarrow \cdot C]$	$[B \rightarrow \cdot aB]$
$[B \rightarrow \cdot b]$	$[C \rightarrow \cdot aC]$	$[C \rightarrow \cdot c]$	

Example (cf. Example 9.6)

$G : \quad S' \rightarrow S$
 $\quad S \rightarrow B \mid C$
 $\quad B \rightarrow aB \mid b$
 $\quad C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 &:= LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] & & [S \rightarrow \cdot B] & & [S \rightarrow \cdot C] & & [B \rightarrow \cdot aB] \\ & & [B \rightarrow \cdot b] & & [C \rightarrow \cdot aC] & & [C \rightarrow \cdot c] & & \\ I_1 &:= LR(0)(S) : & [S' \rightarrow S \cdot] & & & & & & \end{aligned}$$

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 &:= LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] & & [S \rightarrow \cdot B] & & [S \rightarrow \cdot C] & & [B \rightarrow \cdot aB] \\ & & [B \rightarrow \cdot b] & & [C \rightarrow \cdot aC] & & [C \rightarrow \cdot c] & & \\ I_1 &:= LR(0)(S) : & [S' \rightarrow S \cdot] & & & & & & \\ I_2 &:= LR(0)(B) : & [S \rightarrow B \cdot] & & & & & & \end{aligned}$$

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 &:= LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] & & [S \rightarrow \cdot B] & & [S \rightarrow \cdot C] & & [B \rightarrow \cdot aB] \\ & & [B \rightarrow \cdot b] & & [C \rightarrow \cdot aC] & & [C \rightarrow \cdot c] & & \\ I_1 &:= LR(0)(S) : & [S' \rightarrow S \cdot] & & & & & & \\ I_2 &:= LR(0)(B) : & [S \rightarrow B \cdot] & & & & & & \\ I_3 &:= LR(0)(C) : & [S \rightarrow C \cdot] & & & & & & \end{aligned}$$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G :$ $S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 &:= LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] & & [S \rightarrow \cdot B] & & [S \rightarrow \cdot C] & & [B \rightarrow \cdot aB] \\ & & [B \rightarrow \cdot b] & & [C \rightarrow \cdot aC] & & [C \rightarrow \cdot c] & & \\ I_1 &:= LR(0)(S) : & [S' \rightarrow S \cdot] & & & & & & \\ I_2 &:= LR(0)(B) : & [S \rightarrow B \cdot] & & & & & & \\ I_3 &:= LR(0)(C) : & [S \rightarrow C \cdot] & & & & & & \\ I_4 &:= LR(0)(a) : & [B \rightarrow a \cdot B] & & [C \rightarrow a \cdot C] & & & & \end{aligned}$$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G : \quad S' \rightarrow S$
 $\quad S \rightarrow B \mid C$
 $\quad B \rightarrow aB \mid b$
 $\quad C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB]$
 $\quad \quad \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$
 $I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$
 $I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$
 $I_4 := LR(0)(a) : \quad [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$
 $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$I_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$
 $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_5 := LR(0)(b) :$ $[B \rightarrow b \cdot]$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$I_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$
 $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_5 := LR(0)(b) :$ $[B \rightarrow b \cdot]$

$I_6 := LR(0)(c) :$ $[C \rightarrow c \cdot]$

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) :$
 $\begin{array}{llll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$

$I_1 := LR(0)(S) :$
 $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$
 $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$
 $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$
 $\begin{array}{llll} [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$

$I_5 := LR(0)(b) :$
 $[B \rightarrow b \cdot]$

$I_6 := LR(0)(c) :$
 $[C \rightarrow c \cdot]$

$I_7 := LR(0)(aB) :$
 $[B \rightarrow aB \cdot]$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$G : S' \rightarrow S$
 $S \rightarrow B \mid C$
 $B \rightarrow aB \mid b$
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$I_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$ $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$ $[B \rightarrow \cdot aB]$
 $[B \rightarrow \cdot b]$ $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$ $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$

$I_5 := LR(0)(b) :$ $[B \rightarrow b \cdot]$

$I_6 := LR(0)(c) :$ $[C \rightarrow c \cdot]$

$I_7 := LR(0)(aB) :$ $[B \rightarrow aB \cdot]$

$I_8 := LR(0)(aC) :$ $[C \rightarrow aC \cdot]$

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$$\begin{aligned} G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c \end{aligned}$$
$$I_0 := LR(0)(\varepsilon) : \quad \begin{array}{llll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$$
$$I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$$
$$I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$$
$$I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$$
$$I_4 := LR(0)(a) : \quad \begin{array}{llll} [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \end{array}$$
$$I_5 := LR(0)(b) : \quad [B \rightarrow b \cdot]$$
$$I_6 := LR(0)(c) : \quad [C \rightarrow c \cdot]$$
$$I_7 := LR(0)(aB) : \quad [B \rightarrow aB \cdot]$$
$$I_8 := LR(0)(aC) : \quad [C \rightarrow aC \cdot]$$

$(LR(0)(aa) = LR(0)(a) = I_4, LR(0)(ab) = LR(0)(b) = I_5,$
 $LR(0)(ac) = LR(0)(c) = I_6, \dots, I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

Computing $LR(0)$ Sets II

Example (cf. Example 9.6)

$$\begin{aligned} G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c \end{aligned}$$
$$I_0 := LR(0)(\varepsilon) : \quad \begin{array}{llll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$$
$$I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$$
$$I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$$
$$I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$$
$$I_4 := LR(0)(a) : \quad \begin{array}{llll} [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \end{array}$$
$$I_5 := LR(0)(b) : \quad [B \rightarrow b \cdot]$$
$$I_6 := LR(0)(c) : \quad [C \rightarrow c \cdot]$$
$$I_7 := LR(0)(aB) : \quad [B \rightarrow aB \cdot]$$
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$(LR(0)(aa) = LR(0)(a) = I_4, LR(0)(ab) = LR(0)(b) = I_5,$
 $LR(0)(ac) = LR(0)(c) = I_6, \dots, I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases)

no conflicts $\implies G \in LR(0)$ (but $G \notin LL(1)$)

Reduce/Reduce Conflicts

Example

$G :$

- $S' \rightarrow S$
- $S \rightarrow Aa \mid Bb$
- $A \rightarrow a$
- $B \rightarrow a$

Reduce/Reduce Conflicts

Example

$G : S' \rightarrow S$
 $S \rightarrow Aa \mid Bb$
 $A \rightarrow a$
 $B \rightarrow a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$
 $LR(0)(S) : [S' \rightarrow S \cdot]$
 $LR(0)(A) : [S \rightarrow A \cdot a]$
 $LR(0)(B) : [S \rightarrow B \cdot b]$
 $LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$
 $LR(0)(Aa) : [S \rightarrow Aa \cdot]$
 $LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Reduce/Reduce Conflicts

Example

$G : S' \rightarrow S$
 $S \rightarrow Aa \mid Bb$
 $A \rightarrow a$
 $B \rightarrow a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$
 $LR(0)(S) : [S' \rightarrow S \cdot]$
 $LR(0)(A) : [S \rightarrow A \cdot a]$
 $LR(0)(B) : [S \rightarrow B \cdot b]$
 $LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$
 $LR(0)(Aa) : [S \rightarrow Aa \cdot]$
 $LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Note: G is unambiguous

Example

$G : S' \rightarrow S$
 $S \rightarrow aS \mid a$

Example

$G : S' \rightarrow S$
 $S \rightarrow aS \mid a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a] \quad [S \rightarrow a \cdot]$

$LR(0)(aS) : [S \rightarrow aS \cdot]$

Shift/Reduce Conflicts

Example

$G : S' \rightarrow S$
 $S \rightarrow aS \mid a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a] \quad [S \rightarrow a \cdot]$

$LR(0)(aS) : [S \rightarrow aS \cdot]$

Note: G is unambiguous

1 Repetition: $LR(0)$ Grammars

2 $LR(0)$ Parsing

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 $\Rightarrow$ **new pushdown alphabet:** $LR(0)(G)$ in place of X

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 $\implies$ **new pushdown alphabet:** $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but **independent from** γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 $\implies$ **new pushdown alphabet:** $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but **independent from** γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

Definition 10.1 ($LR(0)$ goto function)

The function **goto** : $LR(0)(G) \times X \rightarrow LR(0)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

Example 10.2 (cf. Example 9.12)

$l_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$
 $[S \rightarrow \cdot B] \quad [S \rightarrow \cdot C]$
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$

$l_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$

$l_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C]$
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) :$ $[B \rightarrow b \cdot]$

$l_6 := LR(0)(c) :$ $[C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) :$ $[B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) :$ $[C \rightarrow aC \cdot]$

$l_9 := \emptyset$

The goto Function II

Example 10.2 (cf. Example 9.12)

$l_0 := LR(0)(\varepsilon) :$ $[S' \rightarrow \cdot S]$
 $[S \rightarrow \cdot B]$ $[S \rightarrow \cdot C]$
 $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$
 $l_1 := LR(0)(S) :$ $[S' \rightarrow S \cdot]$
 $l_2 := LR(0)(B) :$ $[S \rightarrow B \cdot]$
 $l_3 := LR(0)(C) :$ $[S \rightarrow C \cdot]$
 $l_4 := LR(0)(a) :$ $[B \rightarrow a \cdot B]$ $[C \rightarrow a \cdot C]$
 $[B \rightarrow \cdot aB]$ $[B \rightarrow \cdot b]$
 $[C \rightarrow \cdot aC]$ $[C \rightarrow \cdot c]$
 $l_5 := LR(0)(b) :$ $[B \rightarrow b \cdot]$
 $l_6 := LR(0)(c) :$ $[C \rightarrow c \cdot]$
 $l_7 := LR(0)(aB) :$ $[B \rightarrow aB \cdot]$
 $l_8 := LR(0)(aC) :$ $[C \rightarrow aC \cdot]$
 $l_9 := \emptyset$

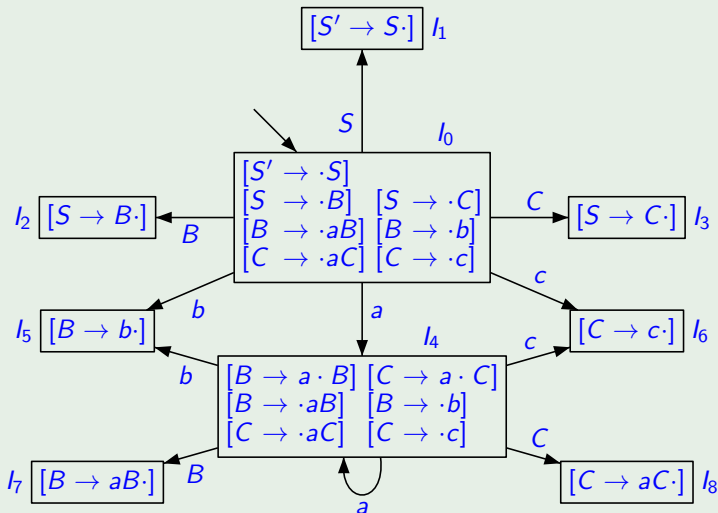
goto	S	B	C	a	b	c
l_0	l_1	l_2	l_3	l_4	l_5	l_6
l_1						
l_2						
l_3						
l_4		l_7	l_8	l_4	l_5	l_6
l_5						
l_6						
l_7						
l_8						
l_9						

(empty = l_9)

The goto Function III

Example 10.2 (continued)

Representation of `goto` function as finite automaton:



The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision.

(Reminder: $\pi_0 = S' \rightarrow S$)

The $LR(0)$ Action Function

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(Reminder: $\pi_0 = S' \rightarrow S$)

Definition 10.3 ($LR(0)$ action function)

The **$LR(0)$ action function**

$$\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision.

(Reminder: $\pi_0 = S' \rightarrow S$)

Definition 10.3 ($LR(0)$ action function)

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Corollary 10.4

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision.

(Reminder: $\pi_0 = S' \rightarrow S$)

Definition 10.3 ($LR(0)$ action function)

The $LR(0)$ **action function**

$$\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a \alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

Corollary 10.4

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

The $LR(0)$ Parsing Table

Example 10.5 (cf. Example 10.2)

$G :$ $S' \rightarrow S$ (0)
 $S \rightarrow B \mid C$ (1, 2)
 $B \rightarrow aB \mid b$ (3, 4)
 $C \rightarrow aC \mid c$ (5, 6)

$l_0 := LR(0)(\varepsilon) :$ $\begin{bmatrix} S' \rightarrow \cdot S \\ S \rightarrow \cdot B \\ B \rightarrow \cdot aB \\ C \rightarrow \cdot aC \end{bmatrix} \begin{bmatrix} S \rightarrow \cdot C \\ B \rightarrow \cdot b \\ C \rightarrow \cdot c \end{bmatrix}$

$l_1 := LR(0)(S) :$ $\begin{bmatrix} S' \rightarrow S \cdot \end{bmatrix}$

$l_2 := LR(0)(B) :$ $\begin{bmatrix} S \rightarrow B \cdot \end{bmatrix}$

$l_3 := LR(0)(C) :$ $\begin{bmatrix} S \rightarrow C \cdot \end{bmatrix}$

$l_4 := LR(0)(a) :$ $\begin{bmatrix} B \rightarrow a \cdot B \\ B \rightarrow \cdot aB \\ C \rightarrow \cdot aC \end{bmatrix} \begin{bmatrix} C \rightarrow a \cdot C \\ B \rightarrow \cdot b \\ C \rightarrow \cdot c \end{bmatrix}$

$l_5 := LR(0)(b) :$ $\begin{bmatrix} B \rightarrow b \cdot \end{bmatrix}$

$l_6 := LR(0)(c) :$ $\begin{bmatrix} C \rightarrow c \cdot \end{bmatrix}$

$l_7 := LR(0)(aB) :$ $\begin{bmatrix} B \rightarrow aB \cdot \end{bmatrix}$

$l_8 := LR(0)(aC) :$ $\begin{bmatrix} C \rightarrow aC \cdot \end{bmatrix}$

$l_9 := \emptyset$

The LR(0) Parsing Table

Example 10.5 (cf. Example 10.2)

$G : \quad S' \rightarrow S \quad (0)$
 $\quad S \rightarrow B \mid C \quad (1, 2)$
 $\quad B \rightarrow aB \mid b \quad (3, 4)$
 $\quad C \rightarrow aC \mid c \quad (5, 6)$

$LR(0)(G)$	act	goto						
		S	B	C	a	b	c	
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6	
l_1	accept							
l_2	red 1							
l_3	red 2							
l_4	shift		l_7	l_8	l_4	l_5	l_6	
l_5	red 4							
l_6	red 6							
l_7	red 3							
l_8	red 5							
l_9	error							

(empty = l_9)

$l_0 := LR(0)(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \\ [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \\ [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array}$
 $l_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$
 $l_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$
 $l_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$
 $l_4 := LR(0)(a) : \quad \begin{array}{l} [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \\ [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array}$
 $l_5 := LR(0)(b) : \quad [B \rightarrow b \cdot]$
 $l_6 := LR(0)(c) : \quad [C \rightarrow c \cdot]$
 $l_7 := LR(0)(aB) : \quad [B \rightarrow aB \cdot]$
 $l_8 := LR(0)(aC) : \quad [C \rightarrow aC \cdot]$
 $l_9 := \emptyset$

The $LR(0)$ Parsing Automaton I

Definition 10.6 ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of G is defined by the following components.

- Input alphabet Σ
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [\rho] \cup \{0, \text{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration (w, l_0, ε) where $l_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

shift: $(aw, \alpha l, z) \vdash (w, \alpha l', z)$ if $\text{act}(l) = \text{shift}$ and $\text{goto}(l, a) = l'$

reduce: $(w, \alpha l_1 \dots l_n, z) \vdash (w, \alpha l', z_i)$ if $\text{act}(l_n) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$,
and $\text{goto}(l, A) = l'$

accept: $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(l) = \text{accept}$

error: $(w, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(l) = \text{error}$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

(*aac*, *l*₀, ϵ)

LR(0)(G)	act	goto					
		<i>S</i>	<i>B</i>	<i>C</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>l</i> ₀	shift	<i>l</i> ₁	<i>l</i> ₂	<i>l</i> ₃	<i>l</i> ₄	<i>l</i> ₅	<i>l</i> ₆
<i>l</i> ₁	accept						
<i>l</i> ₂	red 1						
<i>l</i> ₃	red 2						
<i>l</i> ₄	shift		<i>l</i> ₇	<i>l</i> ₈	<i>l</i> ₄	<i>l</i> ₅	<i>l</i> ₆
<i>l</i> ₅	red 4						
<i>l</i> ₆	red 6						
<i>l</i> ₇	red 3						
<i>l</i> ₈	red 5						
<i>l</i> ₉	error						

(empty = *l*₉)

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$

LR(0)(G)	act	goto					
		<i>S</i>	<i>B</i>	<i>C</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>l</i> ₀	shift	<i>l</i> ₁	<i>l</i> ₂	<i>l</i> ₃	<i>l</i> ₄	<i>l</i> ₅	<i>l</i> ₆
<i>l</i> ₁	accept						
<i>l</i> ₂	red 1						
<i>l</i> ₃	red 2						
<i>l</i> ₄	shift		<i>l</i> ₇	<i>l</i> ₈	<i>l</i> ₄	<i>l</i> ₅	<i>l</i> ₆
<i>l</i> ₅	red 4						
<i>l</i> ₆	red 6						
<i>l</i> ₇	red 3						
<i>l</i> ₈	red 5						
<i>l</i> ₉	error						

(empty = *l*₉)

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$
 $\vdash (c, l_0 l_4 l_6, \varepsilon)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ϵ)
 $\vdash (ac, l_0 l_4, \epsilon)$
 $\vdash (c, l_0 l_4 l_4, \epsilon)$
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ϵ)
 $\vdash (ac, l_0 l_4, \epsilon)$
 $\vdash (c, l_0 l_4 l_4, \epsilon)$
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$
 $\vdash (\epsilon, l_0 l_4 l_4 l_8, 6)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ϵ)
 $\vdash (ac, l_0 l_4, \epsilon)$
 $\vdash (c, l_0 l_4 l_4, \epsilon)$
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$
 $\vdash (\epsilon, l_0 l_4 l_4 l_8, 6)$
 $(*)$
 $\vdash (\epsilon, l_0 l_4 l_8, 65)$
 $\vdash (\epsilon, l_0 l_3, 655)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$
 $\vdash (\varepsilon, l_0 l_3, 655)$
 $\vdash (\varepsilon, l_0 l_1, 6552)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$
 $\vdash (\varepsilon, l_0 l_3, 655)$
 $\vdash (\varepsilon, l_0 l_1, 6552)$
 $\vdash (\varepsilon, \varepsilon, 65520)$

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0 l_4, \varepsilon)$
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$
 $\vdash (\varepsilon, l_0 l_3, 655)$
 $\vdash (\varepsilon, l_0 l_1, 6552)$
 $\vdash (\varepsilon, \varepsilon, 65520)$

Check by rightmost derivation
(on the board)

The LR(0) Parsing Automaton II

Example 10.7 (cf. Example 10.5)

$$\begin{aligned}
 G : \quad S' &\rightarrow S & (0) \\
 S &\rightarrow B \mid C & (1, 2) \\
 B &\rightarrow aB \mid b & (3, 4) \\
 C &\rightarrow aC \mid c & (5, 6)
 \end{aligned}$$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

LR(0) parsing of *aac*:

$$\begin{aligned}
 &(aac, l_0, \varepsilon) \\
 \vdash &(ac, l_0 l_4, \varepsilon) \\
 \vdash &(c, l_0 l_4 l_4, \varepsilon) \\
 \vdash &(\varepsilon, l_0 l_4 l_4 l_6, \varepsilon) \\
 \vdash &(\varepsilon, l_0 l_4 l_4 l_8, 6) \\
 (*) & \\
 \vdash &(\varepsilon, l_0 l_4 l_8, 65) \\
 \vdash &(\varepsilon, l_0 l_3, 655) \\
 \vdash &(\varepsilon, l_0 l_1, 6552) \\
 \vdash &(\varepsilon, \varepsilon, 65520)
 \end{aligned}$$

Check by rightmost derivation
(on the board)

Remark: in the corresponding computation of $NBA(G)$, $(*)$ is nondeterministic

The $LR(0)$ Parsing Automaton III

Theorem 10.8 (Correctness of $LR(0)$ Parsing Automaton)

If $G \in LR(0)$, then the $LR(0)$ parsing automaton of G is deterministic, and for every $w \in \Sigma^*$ and $z \in \{0, \dots, p\}^*$:

$$(w, l_0, \varepsilon) \vdash^* (\varepsilon, \varepsilon, z) \quad \text{iff} \quad \overleftarrow{z} \text{ is a rightmost analysis of } w$$

Proof.

omitted □