

Compiler Construction

Lecture 11: Syntax Analysis VII ($SLR(1)$ and $LR(1)$ Parsing)

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1 Repetition: $LR(0)$ Parsing

2 $SLR(1)$ Parsing

3 $LR(1)$ Parsing

The goto Function

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 $\implies$ **new pushdown alphabet:** $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but **independent from** γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

Definition ($LR(0)$ goto function)

The function **goto** : $LR(0)(G) \times X \rightarrow LR(0)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision.

(Reminder: $\pi_0 = S' \rightarrow S$)

Definition ($LR(0)$ action function)

The $LR(0)$ **action function**

$$\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a \alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

Corollary

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

The $LR(0)$ Parsing Automaton I

Definition ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of G is defined by the following components.

- Input alphabet Σ
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [p] \cup \{0, \text{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration (w, l_0, ε) where $l_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

shift: $(aw, \alpha l, z) \vdash (w, \alpha l', z)$ if $\text{act}(l) = \text{shift}$ and $\text{goto}(l, a) = l'$

reduce: $(w, \alpha l_1 \dots l_n, z) \vdash (w, \alpha l', zi)$ if $\text{act}(l_n) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$,
and $\text{goto}(l, A) = l'$

accept: $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(l) = \text{accept}$

error: $(w, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(l) = \text{error}$

- 1 Repetition: $LR(0)$ Parsing
- 2 $SLR(1)$ Parsing
- 3 $LR(1)$ Parsing

Removing Conflicts in $LR(0)$ Parsing

In practice: often $G \notin LR(0)$

Example 11.1

$G_{AE} :$ $E' \rightarrow E$ $E \rightarrow E+T \mid T$
 $T \rightarrow T*F \mid F$ $F \rightarrow (E) \mid a \mid b$

Removing Conflicts in $LR(0)$ Parsing

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$LR(0)(G_{AE})$ with conflicts:

l_0 : $[E' \rightarrow \cdot E]$	$[E \rightarrow \cdot E+T]$	l_1 : $[E' \rightarrow E \cdot]$	$[E \rightarrow E \cdot +T]$
$[E \rightarrow \cdot T]$	$[T \rightarrow \cdot T*F]$	l_2 : $[E \rightarrow T \cdot]$	$[T \rightarrow T \cdot *F]$
$[T \rightarrow \cdot F]$	$[F \rightarrow \cdot (E)]$	l_3 : $[T \rightarrow F \cdot]$	
$[F \rightarrow \cdot a]$	$[F \rightarrow \cdot b]$		
l_4 : $[F \rightarrow (\cdot E)]$	$[E \rightarrow \cdot E+T]$	l_5 : $[F \rightarrow a \cdot]$	
$[E \rightarrow \cdot T]$	$[T \rightarrow \cdot T*F]$	l_6 : $[F \rightarrow b \cdot]$	
$[T \rightarrow \cdot F]$	$[F \rightarrow \cdot (E)]$	l_7 : $[E \rightarrow E+ \cdot T]$	$[T \rightarrow \cdot T*F]$
$[F \rightarrow \cdot a]$	$[F \rightarrow \cdot b]$	$[T \rightarrow \cdot F]$	$[F \rightarrow \cdot (E)]$
		$[F \rightarrow \cdot a]$	$[F \rightarrow \cdot b]$
l_8 : $[T \rightarrow T* \cdot F]$	$[F \rightarrow \cdot (E)]$	l_9 : $[F \rightarrow (E \cdot)]$	$[E \rightarrow E \cdot +T]$
$[F \rightarrow \cdot a]$	$[F \rightarrow \cdot b]$	l_{10} : $[E \rightarrow E+T \cdot]$	$[T \rightarrow T \cdot *F]$
l_{11} : $[T \rightarrow T*F \cdot]$		l_{12} : $[F \rightarrow (E) \cdot]$	

Goal: resolving conflicts by considering next input symbol

Adding Lookahead I

Goal: resolving conflicts by considering next input symbol

Observations:

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
 $\Rightarrow S' \Rightarrow_r^* \alpha Aw \Rightarrow_r$

$\alpha\beta_1 a\beta_2 w$
pushdown next input symbol

Thus: shift only on lookahead a

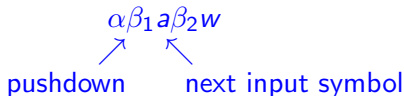
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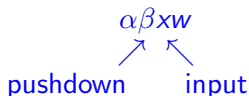
$$\Rightarrow S' \Rightarrow_r^* \alpha A w \Rightarrow_r$$



Thus: shift only on lookahead a

- $[A \rightarrow \beta \cdot] \in LR(0)(\alpha\beta)$

$$\Rightarrow S' \Rightarrow_r^* \alpha A x w \Rightarrow_r$$



$$\Rightarrow x \in \text{fo}(A) \subseteq \Sigma_\epsilon \text{ (} x = \epsilon \text{ only if } w = \epsilon \text{)}$$

Thus: reduce with $A \rightarrow \beta$ only if lookahead $x \in \text{fo}(A)$

Adding Lookahead II

Example 11.2 (cf. Example 11.1)

$G_{AE} :$

$E' \rightarrow E$	(0)
$E \rightarrow E+T \mid T$	(1, 2)
$T \rightarrow T*F \mid F$	(3, 4)
$F \rightarrow (E) \mid a \mid b$	(5, 6, 7)

$A \in N$	$\text{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

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$A \in N$	$fo(A)$
E'	$\{\varepsilon\}$
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- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot + T]\}$:
 - accept on lookahead ε
 - shift on lookahead $+$

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- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 2 on lookahead $+ /) / \varepsilon$
 - shift on lookahead $*$

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 - accept on lookahead ε
 - shift on lookahead $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 2 on lookahead $+ /) / \varepsilon$
 - shift on lookahead $*$
- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 1 on lookahead $+ /) / \varepsilon$
 - shift on lookahead $*$

Adding Lookahead II

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- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 1 on lookahead $+ /) / \varepsilon$
 - shift on lookahead $*$

$\Rightarrow$ **SLR(1) parsing** (Simple **LR(1)**)

The $SLR(1)$ Action Function

Definition 11.3 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\text{act} : LR(0)(G) \times \Sigma_\epsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \\ & \text{and } x \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } x = \epsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $SLR(1)$ Action Function

Definition 11.3 ($SLR(1)$ action function)

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is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \\ & \text{and } x \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } x = \epsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition 11.4 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

The $SLR(1)$ Action Function

Definition 11.3 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\text{act} : LR(0)(G) \times \Sigma_\epsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \\ & \text{and } x \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } x = \epsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition 11.4 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

Together, act and the $LR(0)$ goto function (cf. Definition 10.1) form the $SLR(1)$ parsing table of G .

The SLR(1) Parsing Table

Example 11.5 (cf. Example 11.1)

$I_0 : \begin{bmatrix} E' \rightarrow \cdot E \\ E \rightarrow \cdot T \\ T \rightarrow \cdot F \\ F \rightarrow \cdot a \end{bmatrix} \quad \begin{bmatrix} E \rightarrow \cdot E+T \\ T \rightarrow \cdot T*F \\ F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix} \quad I_1 : \begin{bmatrix} E' \rightarrow E \cdot \\ E \rightarrow T \cdot \\ T \rightarrow F \cdot \end{bmatrix} \quad \begin{bmatrix} E \rightarrow E \cdot +T \\ T \rightarrow T \cdot *F \end{bmatrix}$
 $I_4 : \begin{bmatrix} F \rightarrow (\cdot E) \\ E \rightarrow \cdot T \\ T \rightarrow \cdot F \\ F \rightarrow \cdot a \end{bmatrix} \quad \begin{bmatrix} E \rightarrow \cdot E+T \\ T \rightarrow \cdot T*F \\ F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix} \quad I_5 : \begin{bmatrix} F \rightarrow a \cdot \\ F \rightarrow b \cdot \end{bmatrix} \quad I_6 : \begin{bmatrix} F \rightarrow a \cdot \\ F \rightarrow b \cdot \end{bmatrix}$
 $I_8 : \begin{bmatrix} T \rightarrow T* \cdot F \\ F \rightarrow \cdot a \end{bmatrix} \quad \begin{bmatrix} F \rightarrow \cdot (E) \\ F \rightarrow \cdot b \end{bmatrix} \quad I_9 : \begin{bmatrix} F \rightarrow (E \cdot) \\ E \rightarrow E \cdot +T \\ T \rightarrow T \cdot *F \end{bmatrix}$
 $I_{11} : \begin{bmatrix} T \rightarrow T*F \cdot \end{bmatrix} \quad I_{12} : \begin{bmatrix} F \rightarrow (E) \cdot \end{bmatrix}$

$A \in N$	$fo(A)$
E'	$\{\epsilon\}$
E	$\{+,), \epsilon\}$
T	$\{+, *,), \epsilon\}$
F	$\{+, *,), \epsilon\}$

The SLR(1) Parsing Table

Example 11.5 (cf. Example 11.1)

l_0 :	$[E' \rightarrow \cdot E]$ $[E \rightarrow \cdot E+T]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$	$[E \rightarrow \cdot E+T]$ $[T \rightarrow \cdot T*F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$	l_1 :	$[E' \rightarrow E \cdot]$ $[E \rightarrow T \cdot]$ $[T \rightarrow F \cdot]$	$[E \rightarrow E \cdot +T]$ $[T \rightarrow T \cdot *F]$
l_4 :	$[F \rightarrow (\cdot E)]$ $[E \rightarrow \cdot T]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$	$[E \rightarrow \cdot E+T]$ $[T \rightarrow \cdot T*F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$	l_5 :	$[F \rightarrow a \cdot]$ $[F \rightarrow b \cdot]$ l_7 :	$[F \rightarrow E+ \cdot T]$ $[T \rightarrow \cdot T*F]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot a]$ $[F \rightarrow \cdot b]$
l_8 :	$[T \rightarrow T* \cdot F]$ $[F \rightarrow \cdot a]$	$[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot b]$	l_9 :	$[F \rightarrow (E \cdot)]$ l_{10} :	$[E \rightarrow E \cdot +T]$ $[T \rightarrow T \cdot *F]$
l_{11} :	$[T \rightarrow T*F \cdot]$		l_{12} :	$[F \rightarrow (E) \cdot]$	

$A \in N$	$fo(A)$
E'	$\{\epsilon\}$
E	$\{+,), \epsilon\}$
T	$\{+, *,), \epsilon\}$
F	$\{+, *,), \epsilon\}$

$LR(0)(G_{AE})$	act							goto								
	+	*	(	)	a	b	ϵ	E	T	F	+	*	(	)	a	b
l_0			shift		shift	shift		l_1	l_2	l_3			l_4		l_5	l_6
l_1	shift						accept									
l_2	red 2	shift		red 2			red 2									
l_3	red 4	red 4		red 4			red 4									
l_4			shift		shift	shift		l_9	l_2	l_3			l_4		l_5	l_6
l_5	red 6	red 6		red 6			red 6									
l_6	red 7	red 7		red 7			red 7									
l_7			shift		shift	shift										
l_8			shift		shift	shift		l_{10}	l_3				l_4		l_5	l_6
l_9	shift			shift									l_4		l_5	l_6
l_{10}	red 1	shift		red 1			red 1									
l_{11}	red 3	red 3		red 3			red 3									
l_{12}	red 5	red 5		red 5			red 5									

The $SLR(1)$ Parsing Automaton

Definition 11.6 ($SLR(1)$ parsing automaton)

The $SLR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 10.6), except for the transition relation:

shift: $(aw, \alpha l, z) \vdash (w, \alpha l', z)$ if $\text{act}(l, a) = \text{shift}$ and $\text{goto}(l, a) = l'$

reduce _{a} : $(aw, \alpha l_1 \dots l_n, z) \vdash (aw, \alpha l', zi)$ if $\text{act}(l_n, a) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(l, A) = l'$

reduce _{ϵ} : $(\epsilon, \alpha l_1 \dots l_n, z) \vdash (\epsilon, \alpha l', zi)$ if $\text{act}(l_n, \epsilon) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(l, A) = l'$

accept: $(\epsilon, l_0 l, z) \vdash (\epsilon, \epsilon, z 0)$ if $\text{act}(l, \epsilon) = \text{accept}$

error _{a} : $(aw, \alpha l, z) \vdash (\epsilon, \epsilon, z \text{error})$ if $\text{act}(l, a) = \text{error}$

error _{ϵ} : $(\epsilon, \alpha l, z) \vdash (\epsilon, \epsilon, z \text{error})$ if $\text{act}(l, \epsilon) = \text{error}$

- 1 Repetition: $LR(0)$ Parsing
- 2 $SLR(1)$ Parsing
- 3 $LR(1)$ Parsing

SLR(1) Conflicts

Problem: not all conflicts can be resolved using fo sets

Example 11.7

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

SLR(1) Conflicts

Problem: not all conflicts can be resolved using *fo* sets

Example 11.7

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(0)(G_{LR})$:

$I_0 := LR(0)(\varepsilon) :$	$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot L=R]$	$[S \rightarrow \cdot R]$
	$[L \rightarrow \cdot *R]$	$[L \rightarrow \cdot a]$	$[R \rightarrow \cdot L]$
$I_1 := LR(0)(S) :$	$[S' \rightarrow S \cdot]$		
$I_2 := LR(0)(L) :$	$[S \rightarrow L \cdot =R]$	$[R \rightarrow L \cdot]$	
$I_3 := LR(0)(R) :$	$[S \rightarrow R \cdot]$		
$I_4 := LR(0)(*) :$	$[L \rightarrow * \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$
$I_5 := LR(0)(a) :$	$[L \rightarrow a \cdot]$		
$I_6 := LR(0)(L=) :$	$[S \rightarrow L= \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$
$I_7 := LR(0)(*R) :$	$[L \rightarrow *R \cdot]$		
$I_8 := LR(0)(*L) :$	$[R \rightarrow L \cdot]$		
$I_9 := LR(0)(L=R) :$	$[S \rightarrow L=R \cdot]$		

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$LR(0)(G_{LR})$:

$I_0 := LR(0)(\varepsilon) :$	$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot L=R]$	$[S \rightarrow \cdot R]$
	$[L \rightarrow \cdot *R]$	$[L \rightarrow \cdot a]$	$[R \rightarrow \cdot L]$
$I_1 := LR(0)(S) :$	$[S' \rightarrow S \cdot]$		
$I_2 := LR(0)(L) :$	$[S \rightarrow L \cdot =R]$	$[R \rightarrow L \cdot]$	
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$I_6 := LR(0)(L=) :$	$[S \rightarrow L= \cdot R]$	$[R \rightarrow \cdot L]$	$[L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$
$I_7 := LR(0)(*R) :$	$[L \rightarrow *R \cdot]$		
$I_8 := LR(0)(*L) :$	$[R \rightarrow L \cdot]$		
$I_9 := LR(0)(L=R) :$	$[S \rightarrow L=R \cdot]$		

But: conflict in I_2 not $SLR(1)$ -solvable since $= \in \text{fo}(R)$

Observation: not every element of $\text{fo}(A)$ can follow every occurrence of A
 $\Rightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

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Definition 11.8 ($LR(1)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$ be start separated by $S' \rightarrow S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha \beta_1 \beta_2 a w$, then $[A \rightarrow \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha \beta_1$.

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$, then $[A \rightarrow \beta_1 \cdot \beta_2, \epsilon]$ is called an $LR(1)$ item for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ , called the $LR(1)$ set (or: $LR(1)$ information) of γ .

Observation: not every element of $\text{fo}(A)$ can follow every occurrence of A
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Let $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$ be start separated by $S' \rightarrow S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha \beta_1 \beta_2 a w$, then $[A \rightarrow \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha \beta_1$.
- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$, then $[A \rightarrow \beta_1 \cdot \beta_2, \epsilon]$ is called an $LR(1)$ item for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ , called the $LR(1)$ set (or: $LR(1)$ information) of γ .
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$.

Corollary 11.9

① For every $\gamma \in X^*$, $LR(1)(\gamma)$ is finite.

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- 1 For every $\gamma \in X^*$, $LR(1)(\gamma)$ is finite.
- 2 $LR(1)(G)$ is finite.
- 3 For every $\gamma \in X^*$, $LR(1)(\gamma)$ “contains” $LR(0)(\gamma)$, i.e.,
$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

Corollary 11.9

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- ③ For every $\gamma \in X^*$, $LR(1)(\gamma)$ “contains” $LR(0)(\gamma)$, i.e.,
$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$
- ④ $[A \rightarrow \beta_1 \cdot \beta_2, x] \in I \in LR(1)(G) \implies x \in \text{fo}(A)$

Definition 11.10 (LR(1) conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$ and $I \in \text{LR}(1)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ and $x \in \Sigma_\epsilon$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2, x], [B \rightarrow \beta \cdot, a] \in I.$$

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- I has a **reduce/reduce conflict** if there exist $x \in \Sigma_\epsilon$ and $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

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Definition 11.10 (LR(1) conflicts)

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- I has a **reduce/reduce conflict** if there exist $x \in \Sigma_\epsilon$ and $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot, x], [B \rightarrow \beta \cdot, x] \in I.$$

Lemma 11.11

$G \in \text{LR}(1)$ iff no $I \in \text{LR}(1)(G)$ contains conflicting items.

The computation of $LR(0)$ sets (cf. Theorem 9.11) can be extended to cover right contexts:

Theorem 11.12 (Computing $LR(1)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

① $LR(1)(\varepsilon)$ is the least set such that

- $[S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$ and
- if $[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon)$, $B \rightarrow \beta \in P$, and $y \in \text{fi}(\gamma x)$, then $[B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$.

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② $LR(1)(\alpha Y)$ ($\alpha \in X^*$, $Y \in X$) is the least set such that

- if $[A \rightarrow \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$, then $[A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$ and
- if $[A \rightarrow \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y)$, $B \rightarrow \beta \in P$, and $y \in \text{fi}(\gamma_2 x)$, then $[B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$.

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

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$LR(1)(G_{LR}) : [S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : \quad [S' \rightarrow \cdot S, \varepsilon]$

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

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$I'_0 := LR(1)(\varepsilon) : \quad [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \quad [S \rightarrow \cdot R, \varepsilon]$

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 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot a, =]$			

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 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$		

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$I'_0 := LR(1)(\varepsilon) :$

$[S' \rightarrow \cdot S, \varepsilon]$	$[S \rightarrow \cdot L=R, \varepsilon]$	$[S \rightarrow \cdot R, \varepsilon]$	$[L \rightarrow \cdot *R, =]$
$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \varepsilon]$	$[L \rightarrow \cdot *R, \varepsilon]$	$[L \rightarrow \cdot a, \varepsilon]$

Computing $LR(1)$ Sets II

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$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) : \quad [S' \rightarrow \cdot S, \epsilon] \quad [S \rightarrow \cdot L=R, \epsilon] \quad [S \rightarrow \cdot R, \epsilon] \quad [L \rightarrow \cdot *R, =]$
 $\quad \quad \quad [L \rightarrow \cdot a, =] \quad [R \rightarrow \cdot L, \epsilon] \quad [L \rightarrow \cdot *R, \epsilon] \quad [L \rightarrow \cdot a, \epsilon]$

$I'_1 := LR(1)(S) : \quad [S' \rightarrow S \cdot, \epsilon]$

Computing $LR(1)$ Sets II

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 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$ $[S' \rightarrow \cdot S, \epsilon]$ $[S \rightarrow \cdot L=R, \epsilon]$ $[S \rightarrow \cdot R, \epsilon]$ $[L \rightarrow \cdot *R, =]$
 $[L \rightarrow \cdot a, =]$ $[R \rightarrow \cdot L, \epsilon]$ $[L \rightarrow \cdot *R, \epsilon]$ $[L \rightarrow \cdot a, \epsilon]$

$I'_1 := LR(1)(S) :$ $[S' \rightarrow S \cdot, \epsilon]$

$I'_2 := LR(1)(L) :$ $[S \rightarrow L \cdot =R, \epsilon]$

Computing $LR(1)$ Sets II

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 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		

Computing $LR(1)$ Sets II

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$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			

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 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \epsilon]$		

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

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$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \epsilon]$

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$ $[L \rightarrow \cdot a, =]$	$[S \rightarrow \cdot L=R, \epsilon]$ $[R \rightarrow \cdot L, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$ $[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot *R, =]$ $[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$ $[L \rightarrow \cdot *R, =]$	$[L \rightarrow * \cdot R, \epsilon]$ $[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, =]$ $[L \rightarrow \cdot *R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$ $[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \epsilon]$		

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
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$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$		

Computing $LR(1)$ Sets II

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$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$

Computing $LR(1)$ Sets II

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$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, =]$
	$[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, =]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, =]$	$[L \rightarrow \cdot a, =]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : \quad [A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$		

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) :$ $[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x)$
 $\implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L = \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_{12} := LR(1)(L=a) :$	$[L \rightarrow a \cdot, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR}) : [A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha)$
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_{12} := LR(1)(L=a) :$	$[L \rightarrow a \cdot, \epsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \epsilon]$			

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$ $[L \rightarrow \cdot a, =]$	$[S \rightarrow \cdot L=R, \epsilon]$ $[R \rightarrow \cdot L, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$ $[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot *R, =]$ $[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, =]$ $[L \rightarrow \cdot *R, =]$	$[L \rightarrow * \cdot R, \epsilon]$ $[L \rightarrow \cdot a, =]$	$[R \rightarrow \cdot L, =]$ $[L \rightarrow \cdot *R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$ $[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, =]$ $[L \rightarrow \cdot a, \epsilon]$			
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, =]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, =]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_{12} := LR(1)(L=a) :$	$[L \rightarrow a \cdot, \epsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \epsilon]$			
$I'_{14} := \emptyset$				

Computing $LR(1)$ Sets II

Example 11.13 (cf. Example 11.7)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\epsilon) :$	$[S' \rightarrow \cdot S, \epsilon]$	$[S \rightarrow \cdot L=R, \epsilon]$	$[S \rightarrow \cdot R, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$
	$[L \rightarrow \cdot a, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_1 := LR(1)(S) :$	$[S' \rightarrow S \cdot, \epsilon]$			
$I'_2 := LR(1)(L) :$	$[S \rightarrow L \cdot =R, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_3 := LR(1)(R) :$	$[S \rightarrow R \cdot, \epsilon]$			
$I'_4 := LR(1)(*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$
	$[L \rightarrow * \cdot R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_5 := LR(1)(a) :$	$[L \rightarrow a \cdot, \epsilon]$	$[L \rightarrow a \cdot, \epsilon]$		
$I'_6 := LR(1)(L=) :$	$[S \rightarrow L= \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_7 := LR(1)(*R) :$	$[L \rightarrow *R \cdot, \epsilon]$	$[L \rightarrow *R \cdot, \epsilon]$		
$I'_8 := LR(1)(*L) :$	$[R \rightarrow L \cdot, \epsilon]$	$[R \rightarrow L \cdot, \epsilon]$		
$I'_9 := LR(1)(L=R) :$	$[S \rightarrow L=R \cdot, \epsilon]$			
$I'_{10} := LR(1)(L=L) :$	$[R \rightarrow L \cdot, \epsilon]$			
$I'_{11} := LR(1)(L=*) :$	$[L \rightarrow * \cdot R, \epsilon]$	$[R \rightarrow \cdot L, \epsilon]$	$[L \rightarrow \cdot *R, \epsilon]$	$[L \rightarrow \cdot a, \epsilon]$
$I'_{12} := LR(1)(L=a) :$	$[L \rightarrow a \cdot, \epsilon]$			
$I'_{13} := LR(1)(L=*R) :$	$[L \rightarrow *R \cdot, \epsilon]$			
$I'_{14} := \emptyset$				

In I'_2 : shift on $=$ /reduce on $\epsilon \implies G_{LR} \in LR(1)$

The $LR(1)$ Action Function

Definition 11.14 ($LR(1)$ action function)

The $LR(1)$ action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $LR(1)$ Action Function

Definition 11.14 ($LR(1)$ action function)

The $LR(1)$ action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Corollary 11.15

For every $G \in CFG_\Sigma$, $G \in LR(1)$ iff its $LR(1)$ action function is well defined.

The $LR(1)$ goto Function

The `goto` function is defined in analogy to the $LR(0)$ case (Definition 10.1).

Definition 11.16 ($LR(1)$ goto function)

The function $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

The $LR(1)$ goto Function

The **goto** function is defined in analogy to the $LR(0)$ case (Definition 10.1).

Definition 11.16 ($LR(1)$ goto function)

The function $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

Again, **act** and **goto** form the $LR(1)$ parsing table of G .

The LR(1) Parsing Table

Example 11.17 (cf. Example 11.13)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5			I'_8	I'_7
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

The $LR(1)$ Parsing Automaton I

Definition 11.18 ($LR(1)$ parsing automaton)

The $LR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 10.6), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I', z)$ if $\text{act}(I, a) = \text{shift}$ and $\text{goto}(I, a) = I'$

reduce _{a} : $(aw, \alpha I_1 \dots I_n, z) \vdash (aw, \alpha I', zi)$ if $\text{act}(I_n, a) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

reduce _{ε} : $(\varepsilon, \alpha I_1 \dots I_n, z) \vdash (\varepsilon, \alpha I', zi)$ if $\text{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(I, \varepsilon) = \text{accept}$

error _{a} : $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, a) = \text{error}$

error _{ε} : $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(I, \varepsilon) = \text{error}$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S \quad (0) \quad S \rightarrow L=R \mid R \quad (1,2) \quad L \rightarrow *R \mid a \quad (3,4) \quad R \rightarrow L \quad (5)$

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ε	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5			I'_8	I'_7
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

($a=*a$, I'_0 , ϵ)

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S \ (0) \quad S \rightarrow L=R \mid R \ (1,2) \quad L \rightarrow *R \mid a \ (3,4) \quad R \rightarrow L \ (5)$

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

$(a=*a, I'_0, \epsilon)$
 $\vdash (=*a, I'_0 I'_5, \epsilon)$
 $\vdash (=*a, I'_0 I'_2, 4)$
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

```

(a=*a,  $I'_0$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_5$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_2$ , 4)
⊢ ( *a,  $I'_0 I'_2 I'_6$ , 4)
⊢ ( a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)
    
```

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R \mid R$ (1,2) $L \rightarrow *R \mid a$ (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

```

(a=*a,  $I'_0$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_5$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_2$ , 4)
⊢ ( *a,  $I'_0 I'_2 I'_6$ , 4)
⊢ ( a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)
    
```

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S$ (0) $S \rightarrow L=R$ | R (1,2) $L \rightarrow *R$ | a (3,4) $R \rightarrow L$ (5)

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ε	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

```

(a=*a,  $I'_0$ ,  $\varepsilon$ )
⊢ ( =*a,  $I'_0 I'_5$ ,  $\varepsilon$ )
⊢ ( =*a,  $I'_0 I'_2$ , 4)
⊢ ( *a,  $I'_0 I'_2 I'_6$ , 4)
⊢ ( a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)
⊢ (  $\varepsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)
⊢ (  $\varepsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)
⊢ (  $\varepsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)
⊢ (  $\varepsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)
⊢ (  $\varepsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535)
    
```

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S \quad (0) \quad S \rightarrow L=R \mid R \quad (1,2) \quad L \rightarrow *R \mid a \quad (3,4) \quad R \rightarrow L \quad (5)$

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

```

(a=*a,  $I'_0$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_5$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_2$ , 4)
⊢ ( *a,  $I'_0 I'_2 I'_6$ , 4)
⊢ ( a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535)
⊢ (  $\epsilon$ ,  $I'_0 I'_1$ , 445351)
    
```

The LR(1) Parsing Automaton II

Example 11.19 (cf. Example 11.13)

$G_{LR} : S' \rightarrow S \quad (0) \quad S \rightarrow L=R \mid R \quad (1,2) \quad L \rightarrow *R \mid a \quad (3,4) \quad R \rightarrow L \quad (5)$

LR(1)(G_{LR})	act/goto Σ				goto N		
	*	=	a	ϵ	S	L	R
I'_0	shift/ I'_4		shift/ I'_5		I'_1	I'_2	I'_3
I'_1				accept			
I'_2		shift/ I'_6		red 5			
I'_3				red 2			
I'_4	shift/ I'_4		shift/ I'_5		I'_8	I'_7	
I'_5		red 4		red 4			
I'_6	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_9	
I'_7		red 3		red 3			
I'_8		red 5					
I'_9				red 1			
I'_{10}				red 5			
I'_{11}	shift/ I'_{11}		shift/ I'_{12}		I'_{10}	I'_{13}	
I'_{12}				red 4			
I'_{13}				red 3			

(empty = error/ $\emptyset$)

LR(1) parsing of $a=*a$:

```

(a=*a,  $I'_0$ ,  $\epsilon$ )
⊢ ( =*a,  $I'_0 I'_5$ ,  $\epsilon$  )
⊢ ( =*a,  $I'_0 I'_2$ , 4 )
⊢ ( *a,  $I'_0 I'_2 I'_6$ , 4 )
⊢ ( a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4 )
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4 )
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44 )
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445 )
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453 )
⊢ (  $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535 )
⊢ (  $\epsilon$ ,  $I'_0 I'_1$ , 445351 )
⊢ (  $\epsilon$ ,  $\epsilon$ , 4453510 )
    
```