Compiler Construction

Lecture 13: Semantic Analysis I (Attribute Grammars)

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Summer Semester 2012


Studieren Ohne Grenzen e.V. präsentiert die
Nacht der Professoren
15.06.
Apollo
22:00
Ab 23:00 legen eure Professoren von der RWTH für den guten Zweck auf:
Prof. Reicher-Marek | Philosophie
Prof. Reicherter | Neotektonik
Prof. Bientinesi | Informatik
Prof. Panstruga | Biologie
Prof. Blank | Biologie
Dr. Pratzer | Physik
4€, ab 23h 5€
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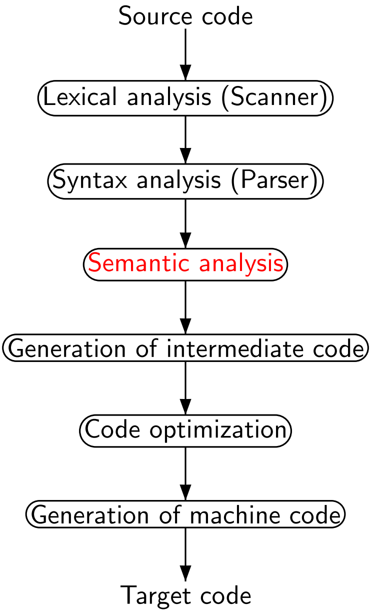
Outline

Conceptual Structure of a Compiler



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To generate (efficient) code, the compiler needs to answer many questions:

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These cannot be expressed using context-free grammars!
(e.g., $\{ww \mid w \in \Sigma^*\} \notin CFL_\Sigma$)

Static Semantics

Static semantics

Static semantics refers to properties of program constructs

- which are true for every occurrence of this construct in every program execution (static) and
- can be decided at compile time
- but are context-sensitive and thus not expressible using context-free grammars (semantics).

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Example properties

Static: type or declaredness of an identifier, number of registers required to evaluate an expression, ...

Dynamic: value of an expression, size of runtime stack, ...

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Attribute Grammars I

Goal: compute context-dependent but runtime-independent properties of a given program

Idea: enrich context-free grammar by semantic rules which annotate syntax tree with attribute values

$\Longrightarrow$ Semantic analysis = attribute evaluation

Result: attributed syntax tree

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$\Longrightarrow$ Semantic analysis = attribute evaluation

Result: attributed syntax tree

In greater detail:

- With every nonterminal a set of attributes is associated.
- Two types of attributes are distinguished:

  Synthesized: bottom-up computation (from the leaves to the root)

  Inherited: top-down computation (from the root to the leaves)
- With every production a set of semantic rules is associated.

Attribute Grammars II

Advantage: attribute grammars provide a very flexible and broadly applicable mechanism for transporting information throught the syntax tree ("syntax-directed translation")

- Attribute values: symbol tables, data types, code, error flags, ...
- Application in Compiler Construction:
  - static semantics
  - program analysis for optimization
  - code generation
  - error handling
- Automatic attribute evaluation by compiler generators (cf. `yacc`'s synthesized attributes)
- Originally designed by D. Knuth for defining the semantics of context-free languages (Math. Syst. Theory 2 (1968), pp. 127–145)

Example: Knuth's Binary Numbers I

Example 13.1 (only synthesized attributes)

Binary numbers (with fraction):

G_B : Numbers	$S \to L$	
	$S \to L\,.\,L$	
Lists	$L \to B$	
	$L \to LB$	
Bits	$B \to 0$	
Bits	$B \to 1$	

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\begin{array}{llllll}
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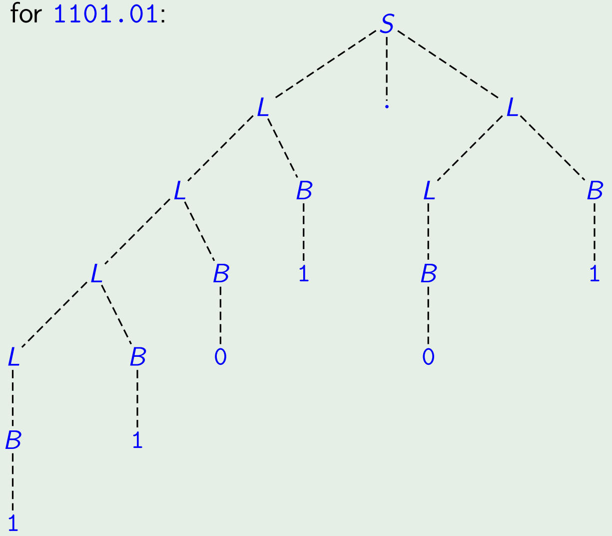
 & \text{Lists} & L \to B & d.0 & = & d.1 \\
 & & & l.0 & = & 1 \\
 & & L \to LB & d.0 & = & 2 * d.1 + d.2 \\
 & & & l.0 & = & l.1 + 1 \\
 & \text{Bits} & B \to 0 & d.0 & = & 0 \\
 & \text{Bits} & B \to 1 & d.0 & = & 1
\end{array}
$$

Synthesized attributes of S, L, B: d (decimal value; domain: $V^d := \mathbb{Q}$)
of L: l (length; domain: $V^l := \mathbb{N}$)

Semantic rules: equations with attribute variables
(index = position of symbol; 0 = left-hand side)

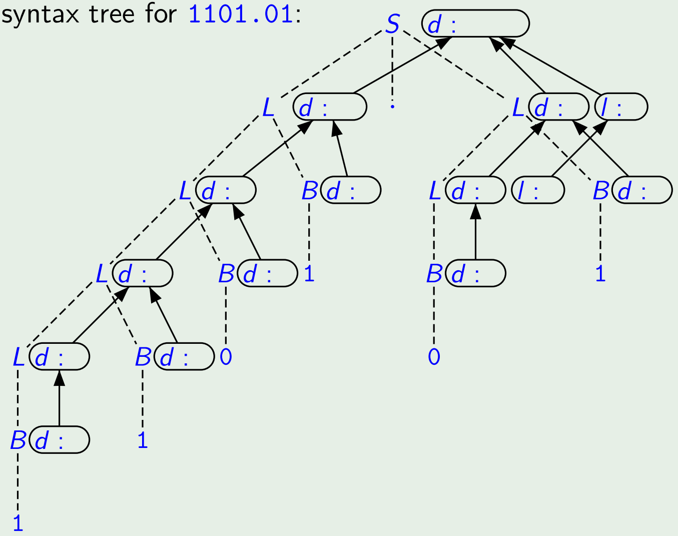
Example 13.1 (continued)

Syntax tree for 1101.01:



Example 13.1 (continued)

Attributed syntax tree for `1101.01`:



Example: Knuth's Binary Numbers II

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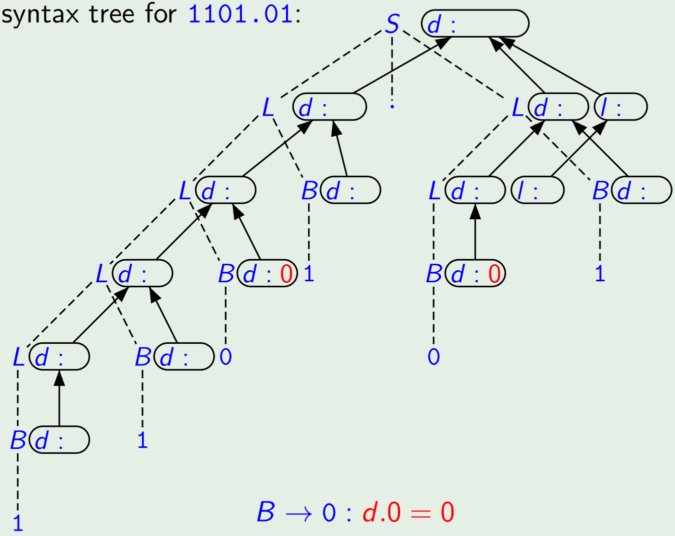

$B \rightarrow 0 : d.0 = 0$

Example: Knuth's Binary Numbers II

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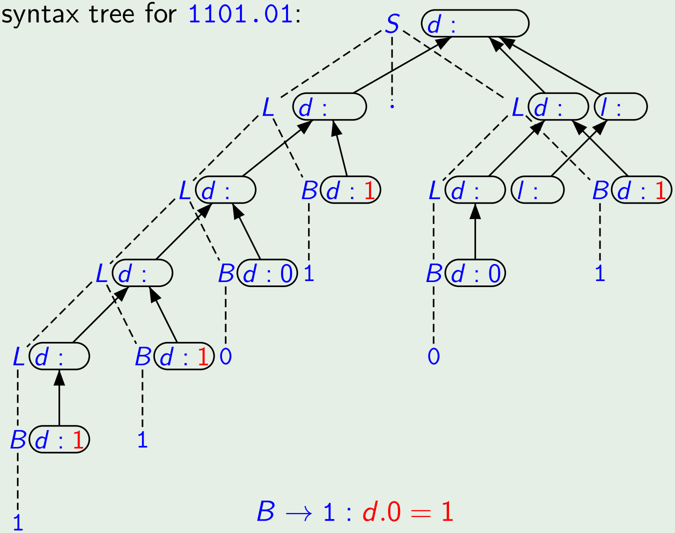

$B \rightarrow 1 : d.0 = 1$

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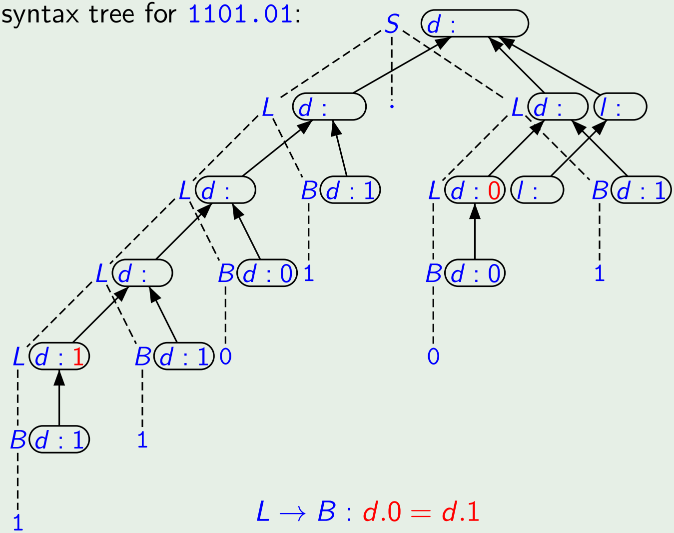

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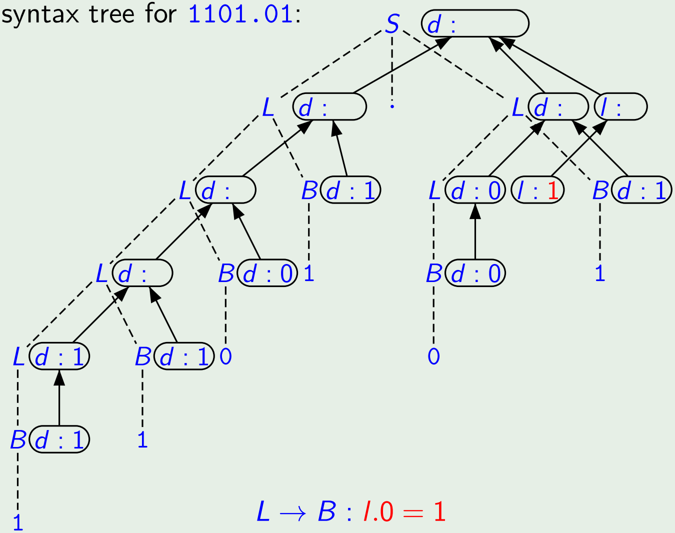

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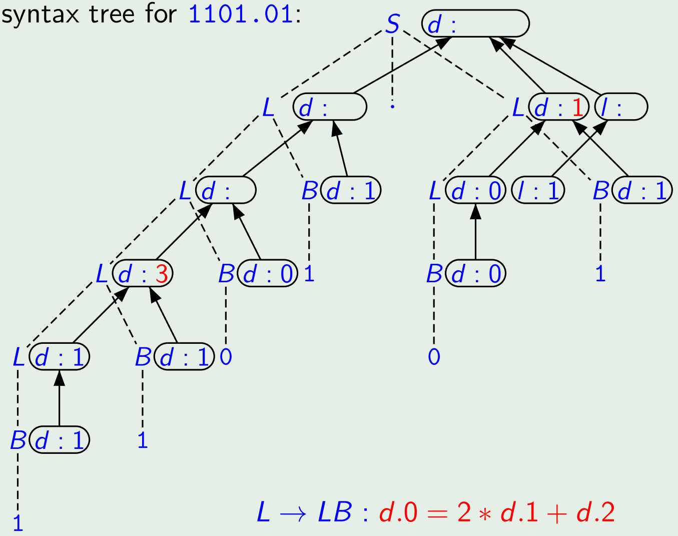

$L \to LB : d.0 = 2 * d.1 + d.2$

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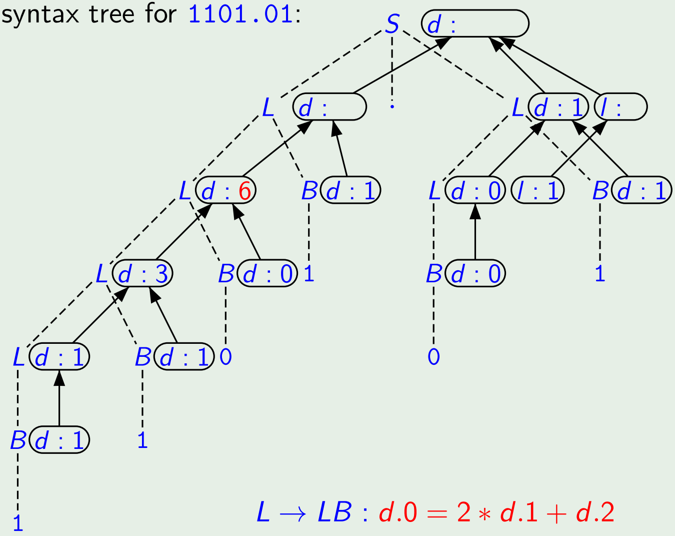

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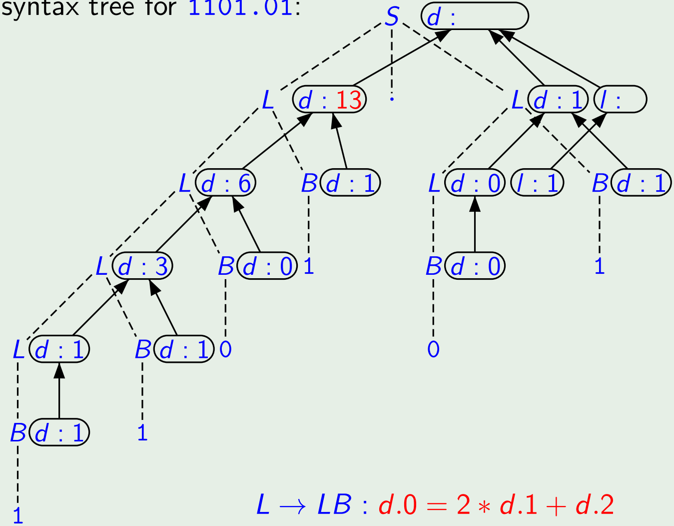

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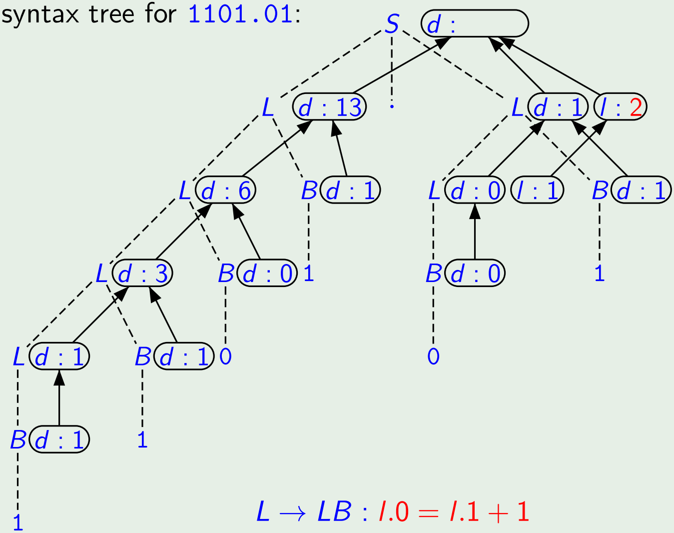

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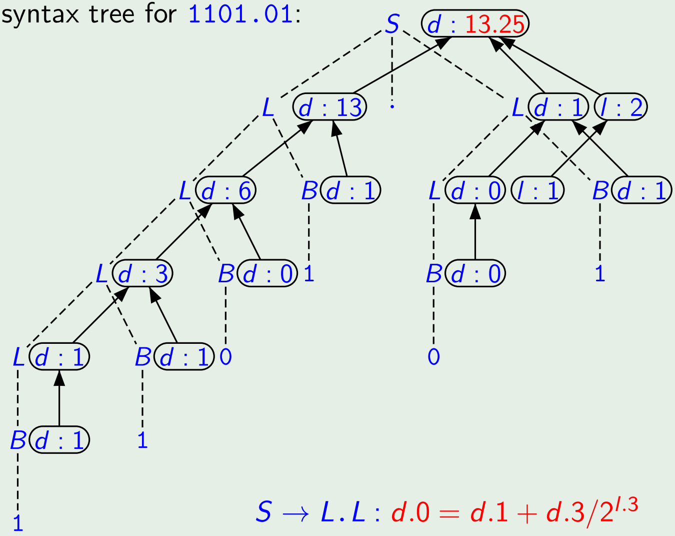

$S \rightarrow L\,.\,L : d.0 = d.1 + d.3/2^{l.3}$

Outline

Adding Inherited Attributes I

Example 13.2 (synthesized + inherited attributes)

Binary numbers (with fraction):

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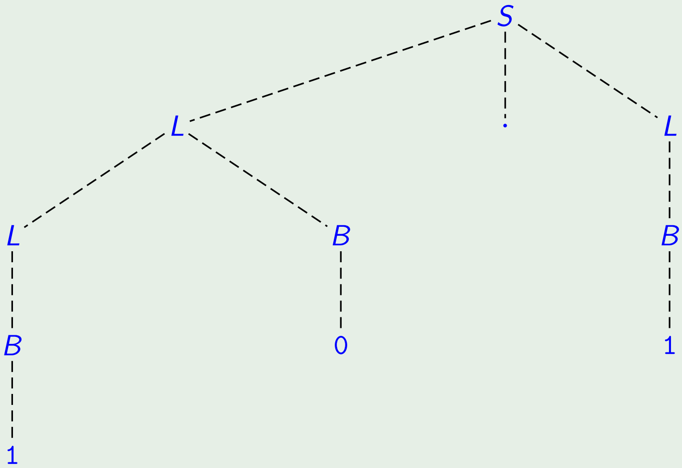

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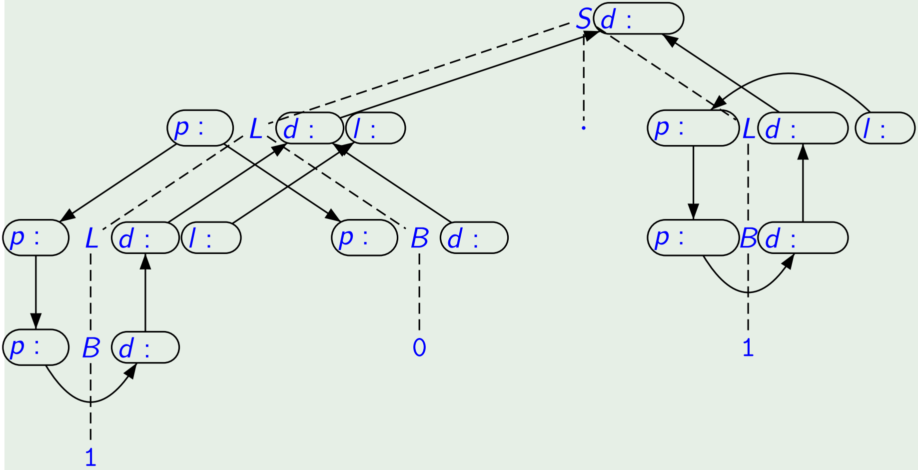

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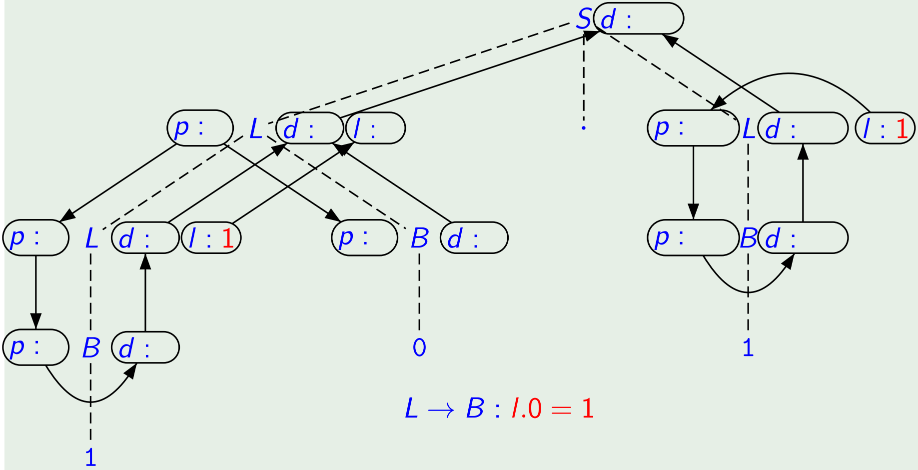

$L \to B : l.0 = 1$

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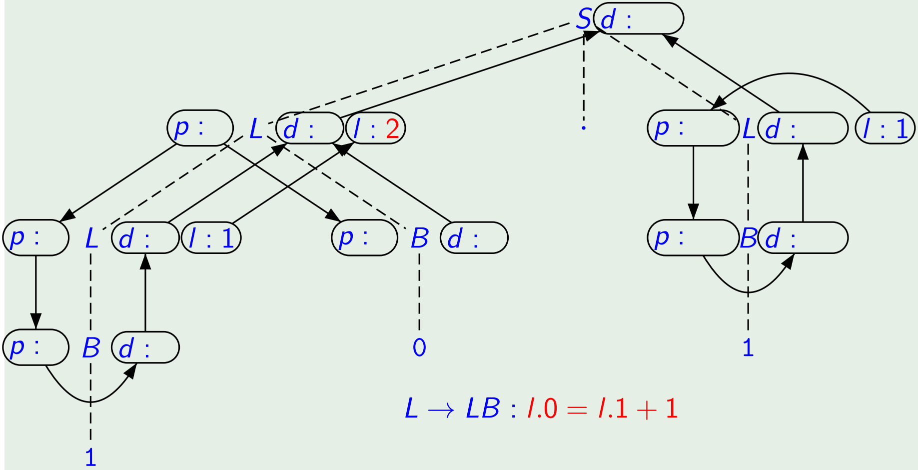

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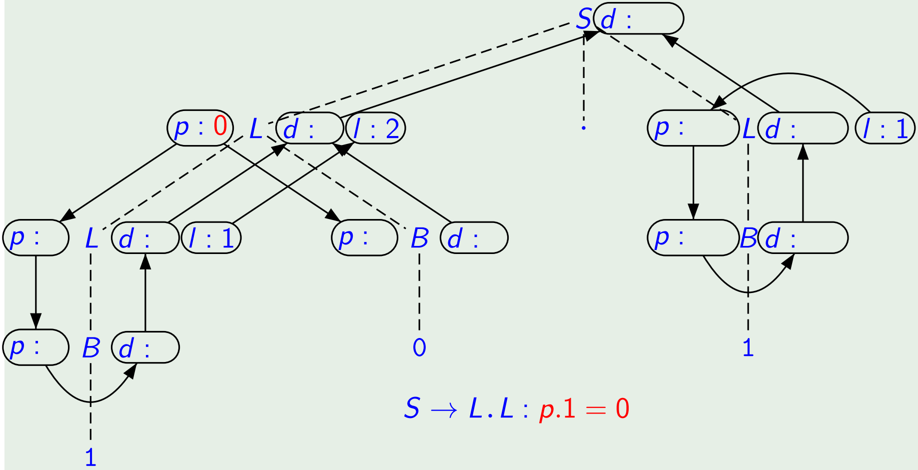

$S \to L \,.\, L : p.1 = 0$

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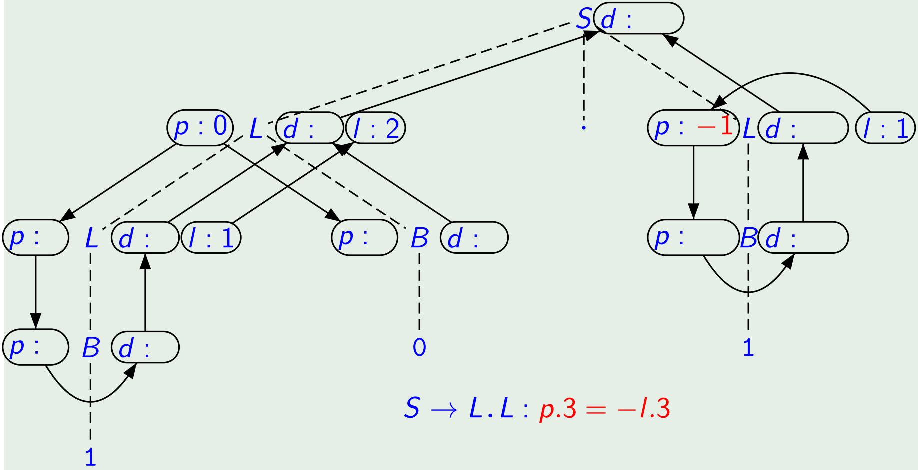

$S \rightarrow L.L : p.3 = -l.3$

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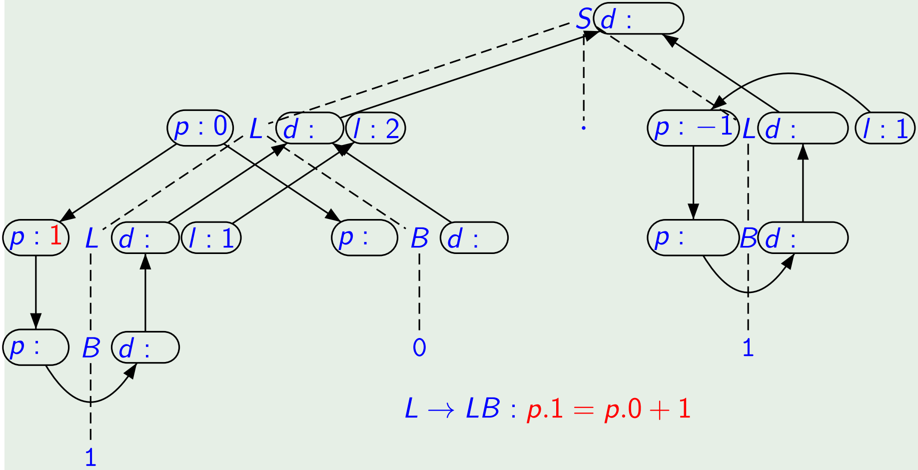

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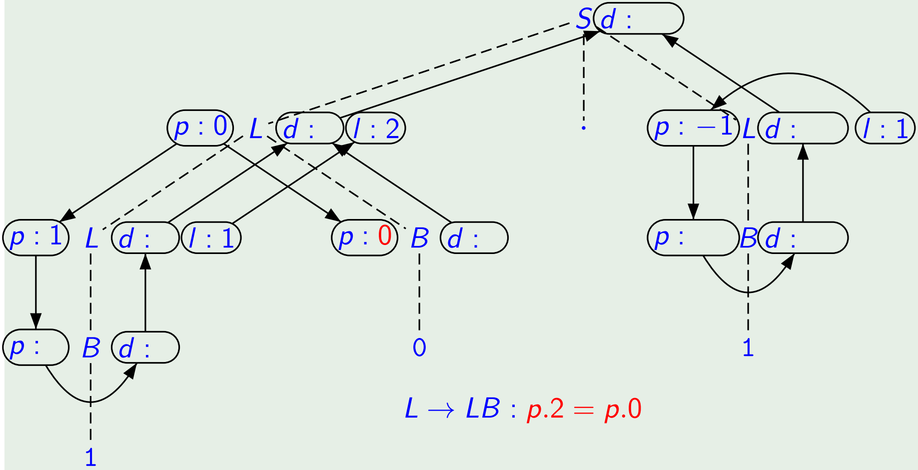

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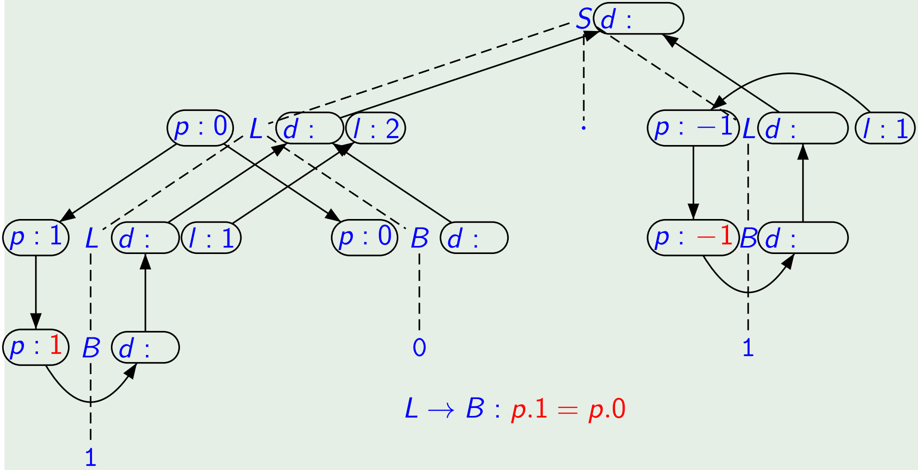

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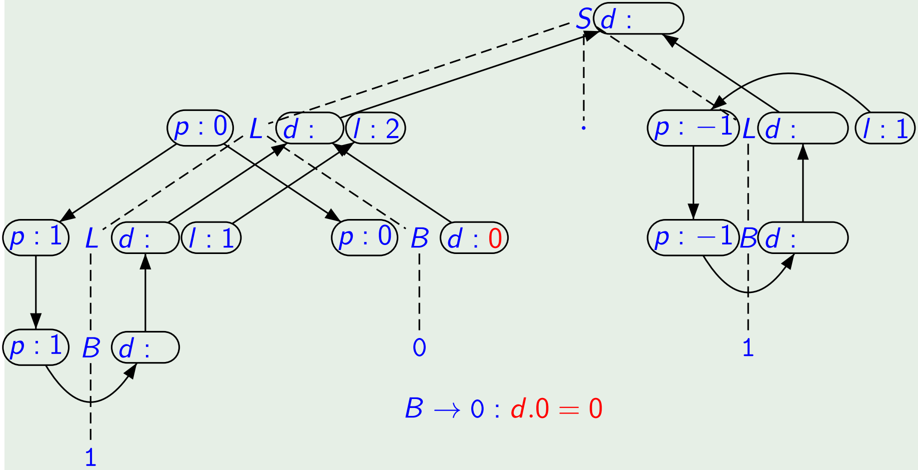

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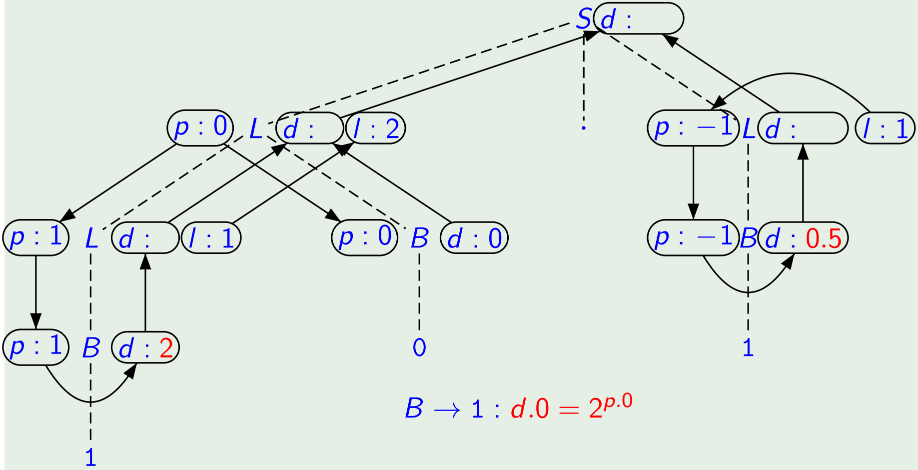

$B \to 1 : d.0 = 2^{p.0}$

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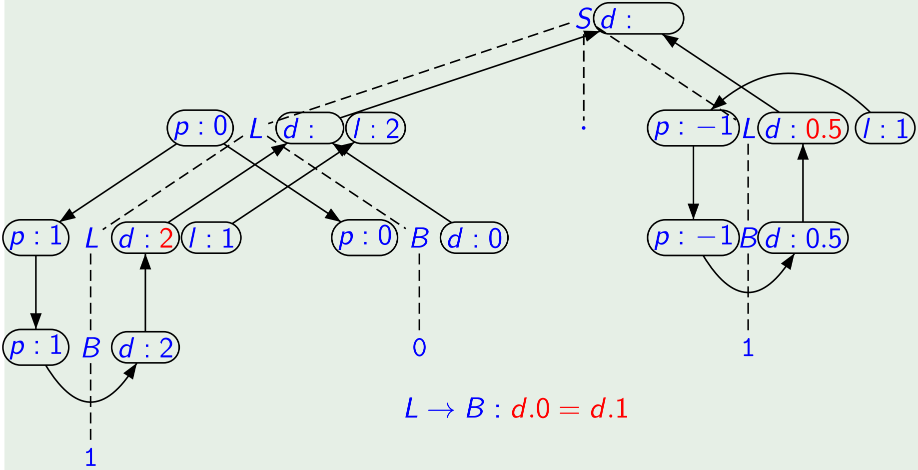

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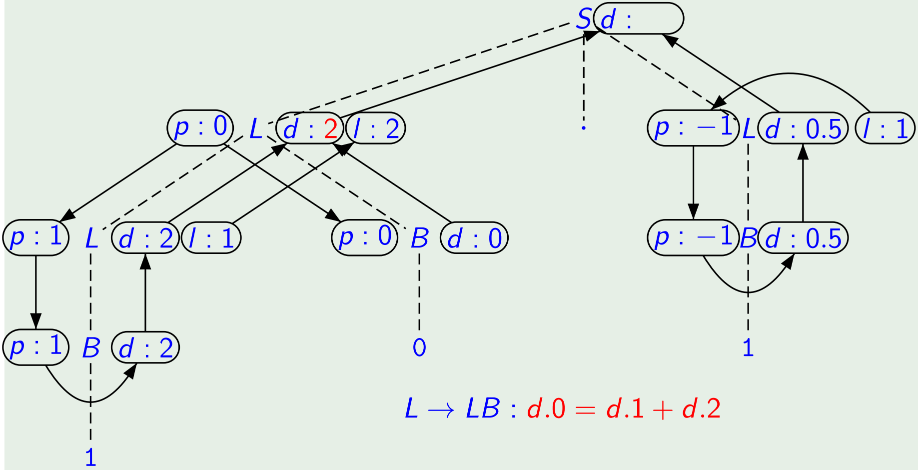

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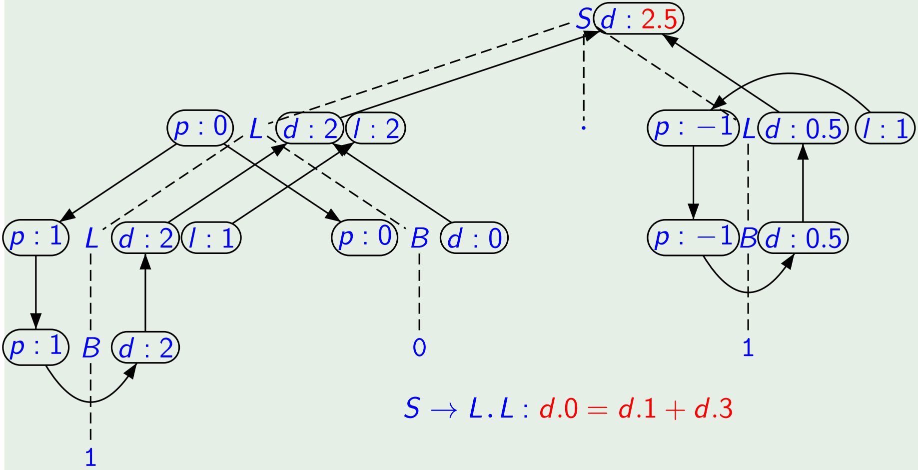

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Formal Definition of Attribute Grammars I

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- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.

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- Every production $\pi = Y_0 \to Y_1 \ldots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \ldots, r\}\}$$
of attribute variables of π with the subsets of inner and outer variables:
$$\begin{aligned} In_\pi &:= \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\} \\ Out_\pi &:= Var_\pi \setminus In_\pi \end{aligned}$$

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  $$Out_\pi := Var_\pi \setminus In_\pi$$
- A semantic rule of π is an equation of the form
  $$\alpha.i = f(\alpha_1.i_1, \ldots, \alpha_n.i_n)$$
  where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \ldots \times V^{\alpha_n} \to V^\alpha$.

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  where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \to V^\alpha$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every inner variable of π, and let $E := (E_\pi \mid \pi \in P)$.

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Definition 13.3 (Attribute grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a union of value sets.
- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.
- Every production $\pi = Y_0 \to Y_1 \dots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \dots, r\}\}$$
of attribute variables of π with the subsets of inner and outer variables:
$$\begin{aligned} In_\pi &:= \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\} \\ Out_\pi &:= Var_\pi \setminus In_\pi \end{aligned}$$
- A semantic rule of π is an equation of the form
$$\alpha.i = f(\alpha_1.i_1, \dots, \alpha_n.i_n)$$
where $n \in \mathbb{N}$, $\alpha.i \in In_\pi$, $\alpha_j.i_j \in Out_\pi$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \to V^\alpha$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every inner variable of π, and let $E := (E_\pi \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V \rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Formal Definition of Attribute Grammars II

Example 13.4 (cf. Example 13.2)

$\mathfrak{A}_B \in AG$ for binary numbers:

- Attributes: $Att = Syn \uplus Inh$ with $Syn = \{d, l\}$ and $Inh = \{p\}$

Formal Definition of Attribute Grammars II

Example 13.4 (cf. Example 13.2)

$\mathfrak{A}_B \in AG$ for binary numbers:

- Attributes: $Att = Syn \uplus Inh$ with $Syn = \{d, l\}$ and $Inh = \{p\}$
- Value sets: $V^d = \mathbb{Q}$, $V^l = \mathbb{N}$, $V^p = \mathbb{Z}$

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- Value sets: $V^d = \mathbb{Q}$, $V^l = \mathbb{N}$, $V^p = \mathbb{Z}$
- Attribute assignment:

$Y \in X$	S	L	B	0	1	.
$\text{syn}(Y)$	$\{d\}$	$\{d, l\}$	$\{d\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\text{inh}(Y)$	$\emptyset$	$\{p\}$	$\{p\}$	$\emptyset$	$\emptyset$	$\emptyset$

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$\mathrm{inh}(Y)$	$\emptyset$	$\{p\}$	$\{p\}$	$\emptyset$	$\emptyset$	$\emptyset$

- Attribute variables:

$\pi \in P$	$S \to L$	$S \to L.L$	$L \to B$
In_π	$\{d.0, p.1\}$	$\{d.0, p.1, p.3\}$	$\{d.0, l.0, p.1\}$
Out_π	$\{d.1, l.1\}$	$\{d.1, l.1, d.3, l.3\}$	$\{d.1, p.0\}$

$\pi \in P$	$L \to LB$	$B \to 0$	$B \to 1$
In_π	$\{d.0, l.0, p.1, p.2\}$	$\{d.0\}$	$\{d.0\}$
Out_π	$\{d.1, d.2, l.1, p.0\}$	$\{p.0\}$	$\{p.0\}$

Formal Definition of Attribute Grammars II

Example 13.4 (cf. Example 13.2)

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- Attribute variables:

$\pi \in P$	$S \to L$	$S \to L\,.\,L$	$L \to B$
In_π	$\{d.0, p.1\}$	$\{d.0, p.1, p.3\}$	$\{d.0, l.0, p.1\}$
Out_π	$\{d.1, l.1\}$	$\{d.1, l.1, d.3, l.3\}$	$\{d.1, p.0\}$

$\pi \in P$	$L \to LB$	$B \to 0$	$B \to 1$
In_π	$\{d.0, l.0, p.1, p.2\}$	$\{d.0\}$	$\{d.0\}$
Out_π	$\{d.1, d.2, l.1, p.0\}$	$\{p.0\}$	$\{p.0\}$

- Semantic rules: see Example 13.2
  (e.g., $E_{S \to L} = \{d.0 = d.1, p.1 = 0\}$)

Outline

Attribution of Syntax Trees I

Definition 13.5 (Attribution of syntax trees)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K.

- K determines the set of attribute variables of t:

$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \mathrm{att}(Y)\}.$$

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$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \mathrm{att}(Y)\}.$$

- Let $k_0 \in K$ be an (inner) node where production $\pi = Y_0 \to Y_1 \dots Y_r \in P$ is applied, and let $k_1, \dots, k_r \in K$ be the corresponding successor nodes. The attribute equation system E_{k_0} of k_0 is obtained from E_π by substituting every attribute index $i \in \{0, \dots, r\}$ by k_i.

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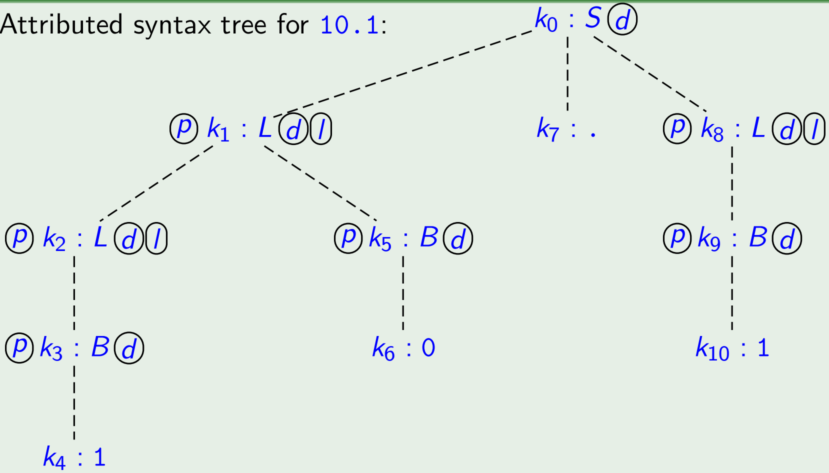
- The attribute equation system of t is given by
  $$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Example 13.6 (cf. Example 13.2)

Attributed syntax tree for `10.1`:



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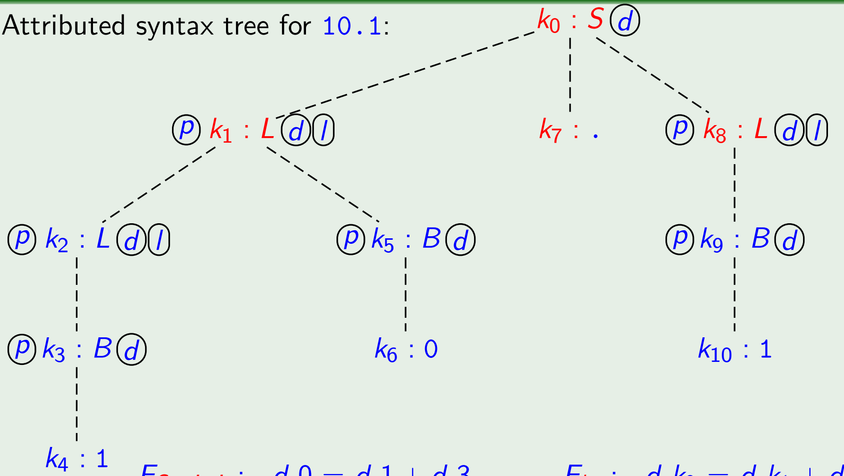

$$E_{S \to L.L}: \quad \begin{aligned} d.0 &= d.1 + d.3 \\ p.1 &= 0 \\ p.3 &= -l.3 \end{aligned} \quad \xrightarrow{\text{subst}} \quad E_{k_0}: \quad \begin{aligned} d.k_0 &= d.k_1 + d.k_8 \\ p.k_1 &= 0 \\ p.k_8 &= -l.k_8 \end{aligned}$$

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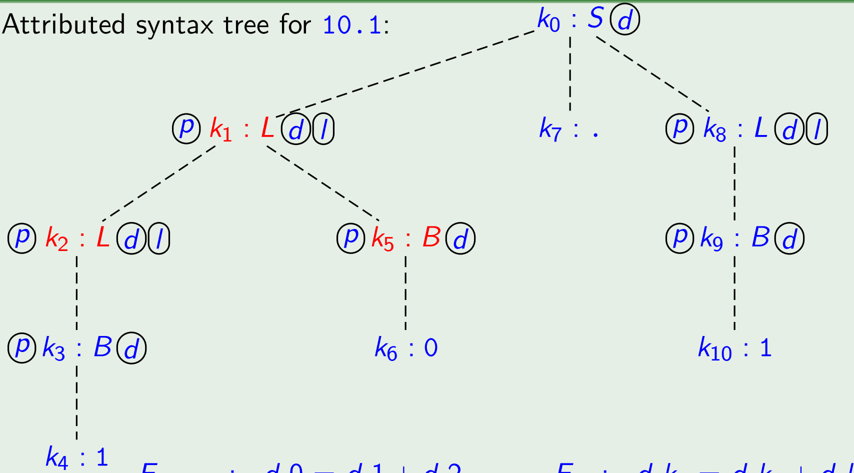

$$E_{L \to LB}: \quad \begin{aligned} d.0 &= d.1 + d.2 \\ l.0 &= l.1 + 1 \\ p.1 &= p.0 + 1 \\ p.2 &= p.0 \end{aligned} \quad \xrightarrow{\text{subst}} \quad E_{k_1}: \quad \begin{aligned} d.k_1 &= d.k_2 + d.k_5 \\ l.k_1 &= l.k_2 + 1 \\ p.k_2 &= p.k_1 + 1 \\ p.k_5 &= p.k_1 \end{aligned}$$

Attribution of Syntax Trees III

Corollary 13.7

For each $\alpha.k \in Var_t$ except the inherited attribute variables at the root and the synthesized attribute variables at the leaves of t, E_t contains exactly one equation with left-hand side $\alpha.k$.

Attribution of Syntax Trees III

Corollary 13.7

For each $\alpha.k \in Var_t$ except the inherited attribute variables at the root and the synthesized attribute variables at the leaves of t, E_t contains exactly one equation with left-hand side $\alpha.k$.

Assumptions:

- The start symbol does not have inherited attributes: $\mathrm{inh}(S) = \emptyset$.
- Synthesized attributes of terminal symbols are provided by the scanner.