

Compiler Construction

Lecture 15: Semantic Analysis III (Attribute Evaluation)

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Summer Semester 2012

- 1 Repetition: Circularity of Attribute Grammars
- 2 The Circularity Check
- 3 Correctness and Complexity of the Circularity Check
- 4 Attribute Evaluation
- 5 Attribute Evaluation by Topological Sorting
- 6 L-Attributed Grammars

Goal: unique solvability of equation system
⇒ avoid cyclic dependencies

Definition (Circularity)

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is called **circular** if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called **noncircular**.

Remark: because of the division of Var_π into In_π and Out_π , cyclic dependencies cannot occur at production level.

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a “cover” production

$\pi = A_0 \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the “upper end” of the cycle and
- for at least one $i \in [r]$, some attributes in $\text{syn}(A_i)$ depend on attributes in $\text{inh}(A_i)$.

Example

on the board

To identify such “critical” situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\text{syn}(A_i)$ can depend on attributes in $\text{inh}(A_i)$.

Definition (Attribute dependence)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k , $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \xrightarrow{A} \alpha$).
- For every syntax tree t with root label $A \in N$,
$$is(A, t) := \{(\beta, \alpha) \in \text{inh}(A) \times \text{syn}(A) \mid \beta \xrightarrow{A} \alpha \text{ in } t\}.$$
- For every $A \in N$,
$$IS(A) := \{is(A, t) \mid t \text{ syntax tree with root label } A\} \subseteq 2^{Inh \times Syn}.$$

Remark: it is important that $IS(A)$ is a **system** of attribute dependence sets, not a **union** (otherwise: **strong noncircularity**—see exercises).

Example

on the board

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In the circularity check, the dependency systems $IS(A)$ are iteratively computed. The following notation is employed:

Definition 15.1

Given $\pi = A \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ and $is_i \subseteq \text{inh}(A_i) \times \text{syn}(A_i)$ for every $i \in [r]$, let

$$is[\pi; is_1, \dots, is_r] \subseteq \text{inh}(A) \times \text{syn}(A)$$

be given by

$$is[\pi; is_1, \dots, is_r] :=$$

$$\left\{ (\beta, \alpha) \mid (\beta.0, \alpha.0) \in (\rightarrow_\pi \cup \bigcup_{i=1}^r \{(\beta'.p_i, \alpha'.p_i) \mid (\beta', \alpha') \in is_i\})^+ \right\}$$

where $p_i := \sum_{j=1}^i |w_{j-1}| + i$.

The Circularity Check I

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Example 15.2

on the board

Algorithm 15.3 (Circularity check for attribute grammars)

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Procedure: ① for every $A \in N$, *iteratively construct $IS(A)$ as follows:*

- ① if $\pi = A \rightarrow w \in P$, then $is[\pi] \in IS(A)$
- ② if $\pi = A \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ and $is_i \in IS(A_i)$ for every $i \in [r]$, then $is[\pi; is_1, \dots, is_r] \in IS(A)$

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② test whether \mathfrak{A} is circular by checking if there exist $\pi = A \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ and $is_i \in IS(A_i)$ for every $i \in [r]$ such that the following relation is cyclic:

$$\rightarrow_\pi \cup \bigcup_{i=1}^r \{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in is_i\}$$

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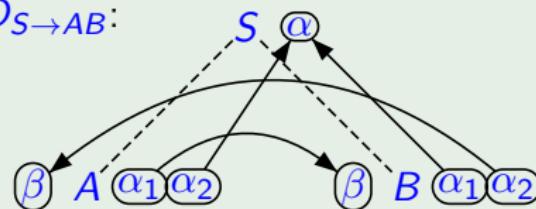
(where $p_i := \sum_{j=1}^i |w_{j-1}| + i$)

Output: "yes" or "no"

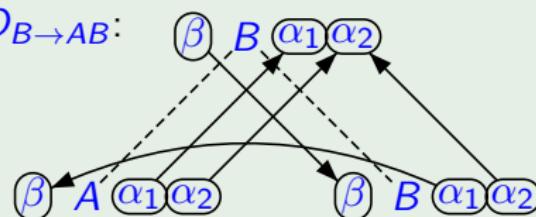
The Circularity Check III

Example 15.4

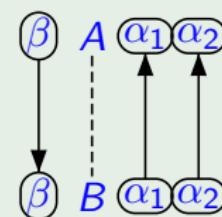
$D_{S \rightarrow AB}$:



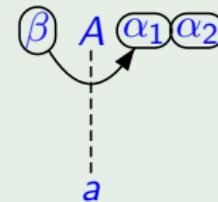
$D_{B \rightarrow AB}$:



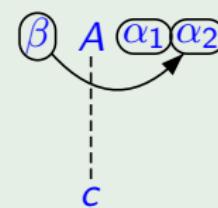
$D_{A \rightarrow B}$:



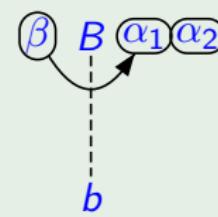
$D_{A \rightarrow a}$:



$D_{A \rightarrow c}$:



$D_{B \rightarrow b}$:



Application of Algorithm 15.3: on the board

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Theorem 15.5 (Correctness of circularity check)

An attribute grammar is circular iff Algorithm 15.3 yields the answer “yes”.

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Proof.

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The time complexity of the circularity check is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

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□

Lemma 15.6

The time complexity of the circularity check is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

Proof.

by reduction of the word problem of alternating Turing machines (see
M. Jazayeri: *A Simpler Construction for Showing the Intrinsically Exponential Complexity of the Circularity Problem for Attribute Grammars*, Comm. of the ACM 28(4), 1981, pp. 715–720)

□

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Attribute Evaluation Methods

Given:

- noncircular attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$
- syntax tree t of G
- valuation $v : Syn_{\Sigma} \rightarrow V$ where
 $Syn_{\Sigma} := \{\alpha.k \mid k \text{ labelled by } a \in \Sigma, \alpha \in \text{syn}(a)\} \subseteq Var_t$

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- ① start with variables which depend at most on Syn_{Σ}
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- ① for every $A \in N$ and $\alpha \in \text{syn}(A)$, define evaluation function $g_{A,\alpha}$ with the following parameters:
 - the node of t where α has to be evaluated and
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- ➍ **S-attributed grammars** (i.e., $Inh = \emptyset$): yacc

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Procedure:

- ① *let* $Var := Var_t \setminus Syn_{\Sigma}$ (* attributes to be evaluated *)
- ② *while* $Var \neq \emptyset$ *do*
 - ① *let* $x \in Var$ *such that* $\{y \in Var \mid y \rightarrow_t x\} = \emptyset$
 - ② *let* $x = f(x_1, \dots, x_n) \in E_t$
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Example 15.8

see Examples 13.1 and 13.2 (Knuth's binary numbers)

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Remark: note that no restrictions are imposed for $\beta \in Syn$ (for $i = 0$) or $\alpha \in Inh$ (for $j = 0$). Thus, in an L-attributed grammar,

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L-Attributed Grammars I

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Corollary 15.2

Every $\mathfrak{A} \in LAG$ is noncircular.

Example 15.3

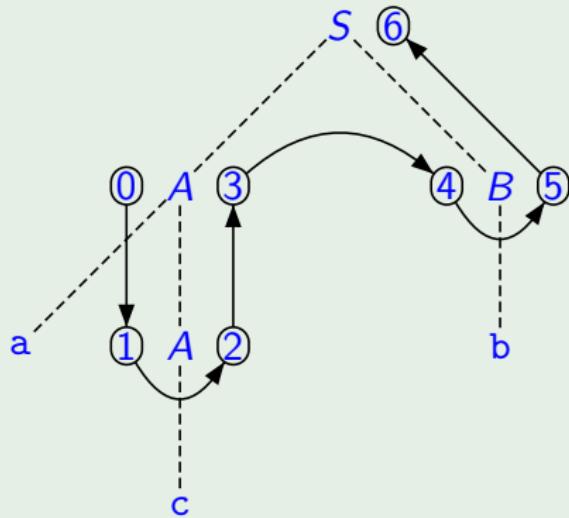
L-attributed grammar:

$$\begin{array}{ll} S \rightarrow AB & i.1 = 0 \\ & i.2 = s.1 + 1 \\ & s.0 = s.2 + 1 \\ A \rightarrow aA & i.2 = i.0 + 1 \\ & s.0 = s.2 + 1 \\ A \rightarrow c & s.0 = i.0 + 1 \\ B \rightarrow b & s.0 = i.0 + 1 \end{array}$$

Example 15.3

L-attributed grammar:

$S \rightarrow AB$ $i.1 = 0$
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Observation 1: the syntax tree of an L-attributed grammar can be attributed by a **depth-first, left-to-right tree traversal** with **two visits** to each node

- ① **top-down**: evaluation of **inherited** attributes
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Observation 2: visit sequence fits nicely with **parsing**

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Observation 2: visit sequence fits nicely with **parsing**

- ① **top-down**: expansion steps
- ② **bottom-up**: reduction steps

Idea: extend LL parsing to support reduction steps, and integrate attribute evaluation \Rightarrow

- use **recursive-descent parser**
- add variables and operations for **attribute evaluation**

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- function `next()` for invoking the scanner
- procedure `print(i)` for displaying the leftmost analysis (or errors)

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which

- tests `token` with regard to the lookahead sets of the A -productions,
- prints the corresponding rule number and
- evaluates the corresponding right-hand side as follows:
 - for $a \in \Sigma$: check `token`; call `next()`
 - for $A \in N$: call A

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Method: to every $A \in N$ we assign a procedure

$$A(\text{in: } \text{inh}(A), \text{out: } \text{syn}(A))$$

which

- declares local variables for synthesized attributes on right-hand sides,
- tests `token` with regard to the lookahead sets of the A -productions,
- prints the corresponding rule number and
- evaluates the corresponding right-hand side as follows:
 - for $a \in \Sigma$: check `token`; call `next()`
 - for $A \in N$: call A with appropriate parameters

Example 15.4 (cf. Example 15.3)

```
proc main();
    token := next(); S()
proc S();    (* S → A B *)
    if token in {'a', 'c'} then
        print(1); A(); B()
    else print(error); stop fi
proc A();    (* A → a A | c *)
    if token = 'a' then
        print(2); token := next(); A()
    elseif token = 'c' then
        print(3); token := next()
    else print(error); stop fi
proc B();    (* B → b *)
    if token = 'b' then
        print(4); token := next()
    else print(error); stop fi
```

Example 15.5 (cf. Example 15.3)

```
proc main(); var s;
  token := next(); S(s); print(s)
proc S(out s0); var s1,s2;  (* S → A B *)
  if token in {'a','c'} then
    print(1); A(0,s1); B(s1 + 1,s2); s0 := s2 + 1
  else print(error); stop fi
proc A(in i0,out s0); var s2;  (* A → a A | c *)
  if token = 'a' then
    print(2); token := next(); A(i0 + 1,s2); s0 := s2 + 1
  elsif token = 'c' then
    print(3); token := next(); s0 := i0 + 1
  else print(error); stop fi
proc B(in i0,out s0);  (* B → b *)
  if token = 'b' then
    print(4); token := next(); s0 := i0 + 1
  else print(error); stop fi
```