Compiler Construction

Lecture 16: Code Generation I (Intermediate Code)

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Summer Semester 2012

Online Registration for Seminars and Practical Courses (Praktika) in Winter Term 2012/13

Who?

Students of:
- Master Courses
- Bachelor Informatik (~~Pro~~Seminar!)

Where?

web-info8.informatik.rwth-aachen.de/apse

When?

18.06.2012 - 01.07.2012

Outline

Conceptual Structure of a Compiler

$$\begin{tikzcd}
\text{Source code} \arrow[d] \\
\boxed{\text{Lexical analysis (Scanner)}} \arrow[d] \\
\boxed{\text{Syntax analysis (Parser)}} \arrow[d] \\
\boxed{\text{Semantic analysis}} \arrow[d] \\
\boxed{\color{red}\text{Generation of intermediate code}} \arrow[d] \\
\boxed{\text{Code optimization}} \arrow[d] \\
\boxed{\text{Generation of machine code}} \arrow[d] \\
\text{Target code}
\end{tikzcd}$$

Modularization of Code Generation I

Splitting of code generation for programming language PL:

$$\mathsf{PL} \xrightarrow{\text{trans}} \mathsf{IC} \xrightarrow{\text{code}} \mathsf{MC}$$

Frontend:	trans generates machine-independent intermediate code (IC) for abstract (stack) machine
Backend:	code generates actual machine code (MC)

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Advantages: IC machine independent $\Longrightarrow$

Portability: much easier to write IC compiler/interpreter for a new machine (as opposed to rewriting the whole compiler)

Fast compiler implementation: generating IC much easier than generating MC

Code size: IC programs usually smaller than corresponding MC programs

Code optimization: division into machine-independent and machine-dependent parts

Modularization of Code Generation II

Example 16.1

1. UNiversal Computer-Oriented Language (UNCOL; $\approx$ 1960; http://en.wikipedia.org/wiki/UNCOL):
universal intermediate language for compilers (never fully specified or implemented; too ambitious)

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\mathrm{PL}_1 \arrow[dr] & & \mathrm{MC}_1 \\
\vdots & \mathit{UNCOL} \arrow[ur] \arrow[dr] & \vdots \\
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only $n + m$ translations (in place of $n \cdot m$)

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5. Common Intermediate Language (CIL; Microsoft .NET; ≈ 2002; http://en.wikipedia.org/wiki/Common_Intermediate_Language)

Language Structures I

Structures in high-level programming languages:

- Basic data types and basic operations
- Static and dynamic data structures
- Expressions and assignments
- Control structures (sequences, branching statements, loops, ...)
- Procedures and functions
- Modularity: blocks, modules, and classes

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Use of procedures and blocks:

- FORTRAN: non-recursive and non-nested procedures
  $\Longrightarrow$ static memory management (requirements determined at compile time)
- C: recursive and non-nested procedures
  $\Longrightarrow$ dynamic memory management using runtime stack (requirements only known at runtime), no static links
- Algol-like languages (Pascal, Modula): recursive and nested procedures
  $\Longrightarrow$ dynamic memory management using runtime stack with static links
- Object-oriented languages (C++, Java): object creation and removal
  $\Longrightarrow$ dynamic memory management using heap

Language Structures II

Structures in machine code: (von Neumann/SISD)

Memory hierarchy: accumulators, registers, cache, main memory, background storage

Instruction types: arithmetic/Boolean/... operation, test/jump instruction, transfer instruction, I/O instruction, ...

Addressing modes: direct/indirect, absolute/relative, ...

Architectures: RISC (few [fast but simple] instructions, many registers), CISC (many [complex but slow] instructions, few registers)

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Structures in intermediate code:

- Data types and operations like PL
- Data stack with basic operations
- Jumping instructions for control structures
- Runtime stack for blocks, procedures, and static data structures
- Heap for dynamic data structures

Outline

The Example Programming Language EPL

Structures of EPL:

- Only integer and Boolean values
- Arithmetic and Boolean expressions with strict and non-strict semantics
- Control structures: sequence, branching, iteration
- Nested blocks and recursive procedures with local and global variables ($\Longrightarrow$ dynamic memory management using runtime stack with static links)
- (not considered: procedure parameters and [dynamic] data structures)

Syntax of EPL

Definition 16.2 (Syntax of EPL)

The syntax of EPL is defined as follows:

$\mathbb{Z}$:	z	(* z is an integer *)
Ide :	I	(* I is an identifier *)
$AExp$:	$A ::= z \mid I \mid A_1 + A_2 \mid \ldots$	
$BExp$:	$B ::= A_1 < A_2 \mid \texttt{not } B \mid B_1 \texttt{ and } B_2 \mid B_1 \texttt{ or } B_2$	
Cmd :	$C ::= I \texttt{ := } A \mid C_1 ; C_2 \mid \texttt{if } B \texttt{ then } C_1 \texttt{ else } C_2 \mid \texttt{while } B \texttt{ do } C \mid I()$	
Dcl :	$D ::= D_C\ D_V\ D_P$	
	$D_C ::= \varepsilon \mid \texttt{const } I_1 \texttt{ := } z_1, \ldots, I_n \texttt{ := } z_n;$	
	$D_V ::= \varepsilon \mid \texttt{var } I_1, \ldots, I_n;$	
	$D_P ::= \varepsilon \mid \texttt{proc } I_1; K_1; \ldots; \texttt{proc } I_n; K_n;$	
Blk :	$K ::= D\ C$	
Pgm :	$P ::= \texttt{in/out } I_1, \ldots, I_n; K.$	

EPL Example: Factorial Function

Example 16.3 (Factorial function)

```
in/out x;
  var y;
  proc F;
    if x > 1 then
      y := y * x;
      x := x - 1;
      F()
  y := 1;
  F();
  x := y.
```

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- Multiple declarations of an identifier in different blocks are possible. Each usage in a command C refers to the "innermost" declaration.
- Static scoping: the usage of an identifier in the body of a called procedure refers to its declaration environment (and not to its calling environment).

Static Semantics of EPL II

Example 16.4

```
in/out x;
  const c = 10;
  var y;
  proc P;
    var y, z;
    proc Q;
      var x, z;
      [... z := 1; P() ...]
    [... P() ... R() ...]
  proc R;
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- “Innermost” principle

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- “Innermost” principle
- Static scoping: body of P can refer to x, y, z

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- "Innermost" principle
- Static scoping: body of `P` can refer to `x, y, z`
- Later declaration: call of `R` in `P` followed by declaration (in Pascal: `forward` declarations for one-pass compilation)

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(omitting the details)

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- Effect of statement = modification of state
  - assignment I `:=` A: update of I by value of A
  - composition C_1 `;` C_2: sequential execution
  - branching `if` B `then` C_1 `else` C_2: test of B, followed by jump to respective branch
  - iteration `while` B `do` C: execution of C as long as B is true
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- Consequently, an EPL program $P = \texttt{in/out}\ I_1, \ldots, I_n; K. \in Pgm$ has as semantics a function

$$\mathfrak{M}[\![P]\!] : \mathbb{Z}^n \dashrightarrow \mathbb{Z}^n$$

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Example 16.5 (Factorial function; cf. Example 16.3)

here $n = 1$ and $\mathfrak{M}[\![P]\!](x) = x!$ (where $x! := 1$ for $x \leq 1$)

Outline

The Abstract Machine AM

Definition 16.6 (Abstract machine for EPL)

The abstract machine for EPL (AM) is defined by the state space

$$S := PC \times DS \times PS$$

with

- the program counter $PC := \mathbb{N}$,
- the data stack $DS := \mathbb{Z}^*$ (top of stack to the right), and
- the procedure stack (or: runtime stack) $PS := \mathbb{Z}^*$ (top of stack to the left).

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- the procedure stack (or: runtime stack) $PS := \mathbb{Z}^*$ (top of stack to the left).

Thus a state $s = (l, d, p) \in S$ is given by

- a program label $l \in PC$,
- a data stack $d = d.r : \ldots : d.1 \in DS$, and
- a procedure stack $p = p.1 : \ldots : p.t \in PS$.

AM Instructions

Definition 16.7 (AM instructions)

The set of AM instructions is divided into

arithmetic instructions: `ADD`, `MULT`, ...

Boolean instructions: `NOT`, `AND`, `OR`, `LT`, ...

jumping instructions: `JMP`(ca), `JFALSE`(ca) ($ca \in PC$)

procedure instructions: `CALL`(ca, dif, loc) ($ca \in PC$, $dif, loc \in \mathbb{N}$), `RET`

transfer instructions: `LOAD`(dif, off), `STORE`(dif, off) ($dif, off \in \mathbb{N}$), `LIT`(z) ($z \in \mathbb{Z}$)

Semantics of Instructions

Definition 16.8 (Semantics of AM instructions (1st part))

The semantics of an AM instruction O

$$[\![O]\!] : S \dashrightarrow S$$

is defined as follows:

$$\begin{aligned}
[\![\texttt{ADD}]\!](l, d : z_1 : z_2, p) &:= (l+1, d : z_1 + z_2, p) \\
[\![\texttt{NOT}]\!](l, d : b, p) &:= (l+1, d : \neg b, p) && \text{if } b \in \{0,1\} \\
[\![\texttt{AND}]\!](l, d : b_1 : b_2, p) &:= (l+1, d : b_1 \wedge b_2, p) && \text{if } b_1, b_2 \in \{0,1\} \\
[\![\texttt{OR}]\!](l, d : b_1 : b_2, p) &:= (l+1, d : b_1 \vee b_2, p) && \text{if } b_1, b_2 \in \{0,1\} \\
[\![\texttt{LT}]\!](l, d : z_1 : z_2, p) &:= \begin{cases} (l+1, d : 1, p) & \text{if } z_1 < z_2 \\ (l+1, d : 0, p) & \text{if } z_1 \geq z_2 \end{cases} \\
[\![\texttt{JMP}(ca)]\!](l, d, p) &:= (ca, d, p) \\
[\![\texttt{JFALSE}(ca)]\!](l, d : b, p) &:= \begin{cases} (ca, d, p) & \text{if } b = 0 \\ (l+1, d, p) & \text{if } b = 1 \end{cases}
\end{aligned}$$

Outline

Structure of Procedure Stack I

The semantics of procedure and transfer instructions requires a particular structure of the procedure stack $p \in PS$: it must be composed of frames (or: activation records) of the form

$$sl : dl : ra : v_1 : \ldots : v_k$$

where

static link sl: points to frame of surrounding declaration environment
$\Longrightarrow$ used to access non-local variables

dynamic link dl: points to previous frame (i.e., of calling procedure)
$\Longrightarrow$ used to remove topmost frame after termination of procedure call

return address ra: program label after termination of procedure call
$\Longrightarrow$ used to continue program execution after termination of procedure call

local variables v_i: values of locally declared variables

Structure of Procedure Stack II

- Frames are created whenever a procedure call is performed
- Two special frames:

I/O frame:	for keeping values of `in/out` variables ($sl = dl = ra = 0$)
MAIN frame:	for keeping values of top-level block ($sl = dl =$ I/O frame)

Structure of Procedure Stack III

Example 16.9 (cf. Example 16.4)

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in/out x;
  const c = 10;
  var y;
  proc P;
    var y, z;
    proc Q;
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Structure of Procedure Stack III

Example 16.9 (cf. Example 16.4)

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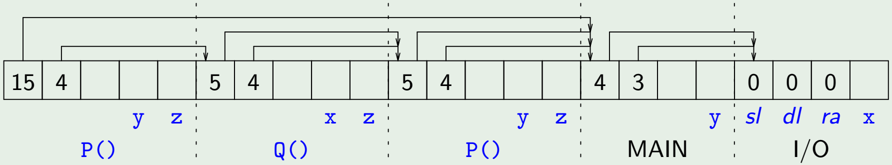
Procedure stack after second call of P:



Structure of Procedure Stack IV

Observation:

- The usage of a variable in a procedure body refers to its innermost declaration.
- If the level difference between the usage and the declaration is *dif* , then a chain of *dif* static links has to be followed to access the corresponding frame.

Structure of Procedure Stack IV

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Example 16.10 (cf. Example 16.9)

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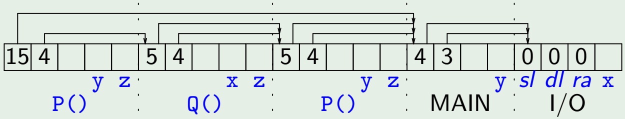

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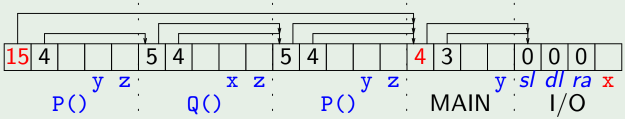

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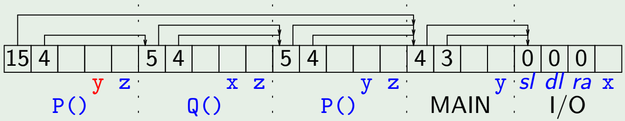

P uses y $\Longrightarrow$ $\mathit{dif} = 0$

The base Function

Upon procedure call, the static link information is computed by the following auxiliary function which, given a procedure stack and a level difference, determines the begin of the corresponding frame.

Definition 16.11 (base function)

The function

$$\mathrm{base} : PS \times \mathbb{N} \dashrightarrow \mathbb{N}$$

is given by

$$\begin{aligned} \mathrm{base}(p, 0) &:= 1 \\ \mathrm{base}(p, \mathit{dif} + 1) &:= \mathrm{base}(p, \mathit{dif}) + p.\mathrm{base}(p, \mathit{dif}) \end{aligned}$$

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In the second call of `P` (from `Q`): $dif = 2$

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Definition 16.11 (base function)

The function

$$\mathrm{base} : PS \times \mathbb{N} \dashrightarrow \mathbb{N}$$

is given by

$$\begin{aligned} \mathrm{base}(p, 0) &:= 1 \\ \mathrm{base}(p, dif + 1) &:= \mathrm{base}(p, dif) + p.\mathrm{base}(p, dif) \end{aligned}$$

Example 16.12 (cf. Example 16.10)

In the second call of `P` (from `Q`): $dif = 2$

$$\begin{aligned} & \mathrm{base}(p, 0) = 1 \\ \Longrightarrow\ & \mathrm{base}(p, 1) = 1 + p.1 = 6 \\ \Longrightarrow\ & \mathrm{base}(p, 2) = 6 + p.6 = 11 \end{aligned}$$

$$\Longrightarrow\ sl = \mathrm{base}(p, 2) + \underbrace{2}_{\texttt{y,z}} + \underbrace{2}_{ra,dl} = 15$$