

Compiler Construction

Lecture 7: Syntax Analysis III ($LL(1)$ Parsing)

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Summer Semester 2012

- 1 Repetition: $LL(k)$ Grammars
- 2 Characterization of $LL(1)$
- 3 Computing Lookahead Sets
- 4 Deterministic Top-Down Parsing

$LL(k)$: reading of input from Left to right with k -lookahead, computing a Leftmost analysis

Definition ($LL(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $k \in \mathbb{N}$. Then G has the $LL(k)$ property (notation: $G \in LL(k)$) if for all leftmost derivations of the form

$$S \Rightarrow_I^* wA\alpha \begin{cases} \Rightarrow_I w\beta\alpha \Rightarrow_I^* wx \\ \Rightarrow_I w\gamma\alpha \Rightarrow_I^* wy \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\text{first}_k(x) \neq \text{first}_k(y)$ (i.e., different productions must not yield the same lookahead).

Motivation:

- $k = 1$ sufficient to resolve nondeterminism in “most” practical applications
- Implementation of $LL(k)$ parsers for $k > 1$ rather involved (cf. **ANTLR** [ANother Tool for Language Recognition; formerly PCCTS] at <http://wwwantlr.org/>)

Abbreviations: $fi := first_1$, $fo := follow_1$, $\Sigma_\epsilon := \Sigma \cup \{\epsilon\}$

Corollary

- 1 For every $\alpha \in X^*$,
$$fi(\alpha) = \{a \in \Sigma \mid \text{ex. } w \in \Sigma^* : \alpha \Rightarrow^* aw\} \cup \{\epsilon \mid \alpha \Rightarrow^* \epsilon\} \subseteq \Sigma_\epsilon$$
- 2 For every $A \in N$,
$$fo(A) = \{x \in fi(\alpha) \mid \text{ex. } w \in \Sigma^*, \alpha \in X^* : S \Rightarrow_j^* wA\alpha\} \subseteq \Sigma_\epsilon.$$

Definition (Lookahead set)

Given $\pi = A \rightarrow \beta \in P$,

$$\text{la}(\pi) := \text{fi}(\beta \cdot \text{fo}(A)) \subseteq \Sigma_\varepsilon$$

is called the **lookahead set** of π (where $\text{fi}(\Gamma) := \bigcup_{\gamma \in \Gamma} \text{fi}(\gamma)$).

Corollary

① For all $a \in \Sigma$,

$$a \in \text{la}(A \rightarrow \beta) \text{ iff } a \in \text{fi}(\beta) \text{ or } (\beta \Rightarrow^* \varepsilon \text{ and } a \in \text{fo}(A))$$

② $\varepsilon \in \text{la}(A \rightarrow \beta)$ iff $\beta \Rightarrow^* \varepsilon$ and $\varepsilon \in \text{fo}(A)$

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Theorem 7.1 (Characterization of $LL(1)$)

$G \in LL(1)$ iff for all pairs of rules $A \rightarrow \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\text{la}(A \rightarrow \beta) \cap \text{la}(A \rightarrow \gamma) = \emptyset.$$

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Proof.

on the board



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Proof.

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Remark: the above theorem generally does not hold if $k > 1$
(cf. exercises)

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(see Waite/Goos: *Compiler Construction*, p. 164f)

Lemma 7.2 (Computation of fi/fo)

The sets $\text{fi}(\alpha) \subseteq \Sigma_\varepsilon$ (for $\alpha \in X^*$) and $\text{fo}(A) \subseteq \Sigma_\varepsilon$ (for $A \in N$) are the least sets such that:

① $\text{fi}(Y)$ for $Y \in X$:

- $Y \in \Sigma \implies \text{fi}(Y) = \{Y\}$
- $Y \rightarrow A_1 \dots A_k Z \alpha \in P, k \in \mathbb{N}, Z \in X, \varepsilon \in \text{fi}(A_1) \cap \dots \cap \text{fi}(A_k), a \in \text{fi}(Z) \implies a \in \text{fi}(Y)$
- $Y \rightarrow A_1 \dots A_k \in P, k \in \mathbb{N}, \varepsilon \in \text{fi}(A_1) \cap \dots \cap \text{fi}(A_k) \implies \varepsilon \in \text{fi}(Y)$

Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

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② $\text{fi}(Y_1 \dots Y_n)$ for $n \in \mathbb{N}, Y_i \in X$:

- $\varepsilon \in \text{fi}(Y_1 \dots Y_{k-1}), a \in \text{fi}(Y_k), k \in [n] \implies a \in \text{fi}(Y_1 \dots Y_n)$
- $\varepsilon \in \text{fi}(Y_1) \cap \dots \cap \text{fi}(Y_n) \implies \varepsilon \in \text{fi}(Y_1 \dots Y_n)$

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 - $\varepsilon \in \text{fi}(Y_1) \cap \dots \cap \text{fi}(Y_n) \implies \varepsilon \in \text{fi}(Y_1 \dots Y_n)$
- ③ $\text{fo}(A)$ for $A \in N$:
 - $\varepsilon \in \text{fo}(S)$
 - $A \rightarrow \alpha B \beta \in P, a \in \text{fi}(\beta) \implies a \in \text{fo}(B)$
 - $A \rightarrow \alpha B \beta \in P, \varepsilon \in \text{fi}(\beta), x \in \text{fo}(A) \implies x \in \text{fo}(B)$

Corollary 7.3

- ① $A \rightarrow a\alpha \in P \implies a \in \text{fi}(A)$
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Example 7.4

Grammar for
arithmetic expressions
(cf. Example 5.10):

$$\begin{array}{lcl} G_{AE} : & E \rightarrow E+T & | \ T \\ & T \rightarrow T * F & | \ F \\ & F \rightarrow (E) & | \ a \ | \ b \end{array}$$

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(cf. Example 5.10):

$$\bullet F \rightarrow a \in P \implies a \in \text{fi}(F)$$

$$\begin{array}{l} G_{AE} : \quad E \rightarrow E+T \mid T \\ \quad \quad T \rightarrow T*F \mid F \\ \quad \quad F \rightarrow (E) \mid a \mid b \end{array}$$

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- $F \rightarrow a \in P \implies a \in \text{fi}(F)$
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$$\begin{array}{lcl} G_{AE} : & E \rightarrow E+T & | \ T \\ & T \rightarrow T*F & | \ F \\ & F \rightarrow (E) & | \ a \ | \ b \end{array}$$

Computing Lookahead Sets II

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$$G_{AE} : \begin{array}{l|l} E \rightarrow E+T & T \\ T \rightarrow T * F & F \\ F \rightarrow (E) & a \mid b \end{array}$$

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- $\implies a \in \text{la}(T \rightarrow T*F) \cap \text{la}(T \rightarrow F) \neq \emptyset$
- $\implies G_{AE} \notin LL(1)$

Fixing the Problem

(general methods later)

Example 7.5 (continuing Example 7.4)

Restructuring (such that $L(G'_{AE}) = L(G_{AE})$):

$$\begin{aligned} G'_{AE} : \quad & E \rightarrow TE' \\ & E' \rightarrow +TE' \mid \varepsilon \\ & T \rightarrow FT' \\ & T' \rightarrow *FT' \mid \varepsilon \\ & F \rightarrow (E) \mid a \mid b \end{aligned}$$

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$A \in N$	$\text{fi}(A)$
E	$\{ (, a, b \}$
E'	$\{ +, \varepsilon \}$
T	$\{ (, a, b \}$
T'	$\{ *, \varepsilon \}$
F	$\{ (, a, b \}$

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$T \rightarrow FT'$	$\{ (, a, b \}$
$T' \rightarrow *FT'$	$\{ * \}$
$T' \rightarrow \varepsilon$	$\{ +, \varepsilon,) \}$
$F \rightarrow (E)$	$\{ (\}$
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$$\Rightarrow G'_{AE} \in LL(1)$$

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Deterministic Top-Down Parsing

Approach: given $G \in CFG_{\Sigma}$,

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Deterministic Top-Down Parsing

Approach: given $G \in CFG_{\Sigma}$,

- ① Verify that $G \in LL(1)$ by computing the lookahead sets and checking alternatives for disjointness
 - ② Start with nondeterministic top-down parsing automaton $NTA(G)$
 - ③ Use **1-symbol lookahead** to control the choice of expanding productions:
 - $(aw, A\alpha, z) \vdash (aw, \beta\alpha, zi)$
if $\pi_i = A \rightarrow \beta$ and $a \in la(\pi_i)$
 - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$
if $\pi_i = A \rightarrow \beta$ and $\varepsilon \in la(\pi_i)$
 - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$]
- $\Rightarrow$ **deterministic top-down parsing automaton** $DTA(G)$

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 - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$
if $\pi_i = A \rightarrow \beta$ and $\varepsilon \in la(\pi_i)$
 - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$] $\Rightarrow$ **deterministic top-down parsing automaton** $DTA(G)$

Remarks:

- $DTA(G)$ is actually **not a pushdown automaton** (a is read but not consumed). But: can be simulated using the finite control.
- Advantage of using lookahead is **twofold**:
 - Removal of nondeterminism
 - Earlier detection of syntax errors
(in configurations $(aw, A\alpha, z)$ where $a \notin \bigcup_{A \rightarrow \beta \in P} la(A \rightarrow \beta)$)

The Deterministic Top-Down Automaton I

Definition 7.6 (Deterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LL(1)$. The **deterministic top-down parsing automaton** of G , $DTA(G)$, is defined by the following components.

- **Input alphabet** Σ , **pushdown alphabet** X , **output alphabet** $[p]$
- **Configurations** $\Sigma^* \times X^* \times [p]^*$, **initial configuration** (w, S, ε) , **final configurations** $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$ (as $NTA(G)$)
- **Action function**
$$\text{act} : \Sigma_\varepsilon \times X_\varepsilon \rightarrow \{(\alpha, i) \mid \pi_i = A \rightarrow \alpha\} \cup \{\text{pop}, \text{accept}, \text{error}\}$$

with $\text{act}(x, A) := (\alpha, i)$ if $\pi_i = A \rightarrow \alpha$ and $x \in \text{la}(\pi_i)$
$$\begin{aligned}\text{act}(a, a) &:= \text{pop} \\ \text{act}(\varepsilon, \varepsilon) &:= \text{accept} \\ \text{act}(x, y) &:= \text{error} \quad \text{otherwise}\end{aligned}$$
- **Transitions** for $x \in \Sigma_\varepsilon$, $w \in \Sigma^*$, $Y \in X$, $\beta \in X^*$, and $z \in [p]^*$:
$$(xw, Y\beta, z) \vdash \begin{cases} (xw, \alpha\beta, zi) & \text{if } \text{act}(x, Y) = (\alpha, i) \\ (w, \beta, z) & \text{if } \text{act}(x, Y) = \text{pop} \end{cases}$$

The Deterministic Top-Down Automaton II

Example 7.7 (cf. Example 7.5)

$$\begin{array}{lll} G'_{AE} : & E \rightarrow TE' & (1) \\ & E' \rightarrow +TE' \mid \varepsilon & (2, 3) \\ & T \rightarrow FT' & (4) \\ & T' \rightarrow *FT' \mid \varepsilon & (5, 6) \\ & F \rightarrow (E) \mid a \mid b & (7, 8, 9) \end{array}$$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta)$
$E \rightarrow TE'$	$\{ (, a, b \}$
$E' \rightarrow +TE'$	$\{ + \}$
$E' \rightarrow \varepsilon$	$\{ \varepsilon,) \}$
$T \rightarrow FT'$	$\{ (, a, b \}$
$T' \rightarrow *FT'$	$\{ * \}$
$T' \rightarrow \varepsilon$	$\{ +, \varepsilon,) \}$
$F \rightarrow (E)$	$\{ (\}$
$F \rightarrow a$	$\{ a \}$
$F \rightarrow b$	$\{ b \}$

The Deterministic Top-Down Automaton II

Example 7.7 (cf. Example 7.5)

$$\begin{aligned}
 G'_{AE} : \quad & E \rightarrow TE' & (1) \\
 & E' \rightarrow +TE' \mid \varepsilon & (2, 3) \\
 & T \rightarrow FT' & (4) \\
 & T' \rightarrow *FT' \mid \varepsilon & (5, 6) \\
 & F \rightarrow (E) \mid a \mid b & (7, 8, 9)
 \end{aligned}$$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta)$
$E \rightarrow TE'$	$\{ (, a, b \}$
$E' \rightarrow +TE'$	$\{ + \}$
$E' \rightarrow \varepsilon$	$\{ \varepsilon,) \}$
$T \rightarrow FT'$	$\{ (, a, b \}$
$T' \rightarrow *FT'$	$\{ * \}$
$T' \rightarrow \varepsilon$	$\{ +, \varepsilon,) \}$
$F \rightarrow (E)$	$\{ (\}$
$F \rightarrow a$	$\{ a \}$
$F \rightarrow b$	$\{ b \}$

$act : \Sigma_\varepsilon \times X_\varepsilon \rightarrow \{(\alpha, i) \mid \pi_i = A \rightarrow \alpha\} \cup \{\text{pop, accept, error}\}$ (empty = error)

act	E	E'	T	T'	F	a	b	$($	$)$	$*$	$+$	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
$($	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
$)$		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
$*$				$(*FT', 5)$						pop		
$+$		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$$\begin{array}{l}
 ((a)*b, E, \varepsilon) \\
 \vdash ((a)*b, TE', 1)
 \end{array}$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT'E', 14)$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

	$(a)*b, E$	$, \varepsilon$	)
$\vdash$	$(a)*b, TE'$	$, 1$	)
$\vdash$	$(a)*b, FT'E'$	$, 14$	)
$\vdash$	$(a)*b, (E)T'E'$	$, 147$	)

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

	$(a)*b, E$	$, \varepsilon$	)
$\vdash$	$(a)*b, TE'$	$, 1$	)
$\vdash$	$(a)*b, FT'E'$	$, 14$	)
$\vdash$	$(a)*b, (E)T'E'$	$, 147$	)
$\vdash$	$(a)*b, E)T'E'$	$, 147$	)

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

	$(a)*b, E$	$, \varepsilon$	)
$\vdash$	$(a)*b, TE'$	$, 1$	)
$\vdash$	$(a)*b, FT'E'$	$, 14$	)
$\vdash$	$(a)*b, (E)T'E'$	$, 147$	)
$\vdash$	$(a)*b, E)T'E'$	$, 147$	)
$\vdash$	$(a)*b, TE')T'E'$	$, 1471$	)

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	(TE' , 1)		(FT' , 4)		(a, 8)	pop						
b	(TE' , 1)		(FT' , 4)		(b, 9)		pop					
(	(TE' , 1)		(FT' , 4)		((E), 7)			pop				
)		(ε , 3)		(ε , 6)					pop			
*				(* FT' , 5)						pop		
+		(+ TE' , 2)		(ε , 6)							pop	
ε		(ε , 3)		(ε , 6)								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT'E', 14)$
 $\vdash ((a)*b, (E)T'E', 147)$
 $\vdash ((a)*b, E)T'E', 147)$
 $\vdash ((a)*b, TE')T'E', 1471)$
 $\vdash ((a)*b, FT'E')T'E', 14714)$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	(TE' , 1)		(FT' , 4)		(a, 8)	pop						
b	(TE' , 1)		(FT' , 4)		(b, 9)		pop					
(	(TE' , 1)		(FT' , 4)		((E), 7)			pop				
)		(ε , 3)		(ε , 6)					pop			
*				(* FT' , 5)						pop		
+		(+ TE' , 2)		(ε , 6)							pop	
ε		(ε , 3)		(ε , 6)								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT'E', 14)$
 $\vdash ((a)*b, (E)T'E', 147)$
 $\vdash ((a)*b, E)T'E', 147)$
 $\vdash ((a)*b, TE')T'E', 1471)$
 $\vdash ((a)*b, FT'E')T'E', 14714)$
 $\vdash ((a)*b, aT'E')T'E', 147148)$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

```

     $((a)*b, E, \varepsilon)$ 
 $\vdash ((a)*b, TE', 1)$ 
 $\vdash ((a)*b, FT'E', 14)$ 
 $\vdash ((a)*b, (E)T'E', 147)$ 
 $\vdash (a)*b, E)T'E', 147)$ 
 $\vdash (a)*b, TE')T'E', 1471)$ 
 $\vdash (a)*b, FT'E')T'E', 14714)$ 
 $\vdash (a)*b, aT'E')T'E', 147148)$ 
 $\vdash ( )*b, T'E')T'E', 147148)$ 
    
```

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon) \vdash (()*b, E') T' E', 1471486$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT' E', 14)$
 $\vdash ((a)*b, (E) T' E', 147)$
 $\vdash (a)*b, E) T' E', 147)$
 $\vdash (a)*b, TE') T' E', 1471)$
 $\vdash (a)*b, FT' E') T' E', 14714)$
 $\vdash (a)*b, a T' E') T' E', 147148)$
 $\vdash () * b, T' E') T' E', 147148)$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	
$\vdash ((a)*b, (E) T' E', 147)$	
$\vdash ((a)*b, E) T' E', 147)$	
$\vdash ((a)*b, TE') T' E', 1471)$	
$\vdash ((a)*b, FT' E') T' E', 14714)$	
$\vdash ((a)*b, a T' E') T' E', 147148)$	
$\vdash (() * b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	
$\vdash (a)*b, E) T' E', 147)$	
$\vdash (a)*b, TE') T' E', 1471)$	
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, aT' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	(TE' , 1)		(FT' , 4)		(a, 8)	pop						
b	(TE' , 1)		(FT' , 4)		(b, 9)		pop					
(	(TE' , 1)		(FT' , 4)		((E), 7)			pop				
)		(ε , 3)		(ε , 6)					pop			
*				($*FT'$, 5)						pop		
+		($+TE'$, 2)		(ε , 6)							pop	
ε		(ε , 3)		(ε , 6)								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash (()*b, E')T'E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash (()*b,)T'E', 14714863)$
$\vdash ((a)*b, FT'E', 14)$	$\vdash (*b, T'E', 14714863)$
$\vdash ((a)*b, (E)T'E', 147)$	$\vdash (*b, *FT'E', 147148635)$
$\vdash (a)*b, E)T'E', 147)$	
$\vdash (a)*b, TE')T'E', 1471)$	
$\vdash (a)*b, FT'E')T'E', 14714)$	
$\vdash (a)*b, aT'E')T'E', 147148)$	
$\vdash ()*b, T'E')T'E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, a T' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, b T' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, a T' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, b T' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\varepsilon, T' E', 1471486359)$
$\vdash (a)*b, a T' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, b T' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\varepsilon, T' E', 1471486359)$
$\vdash (a)*b, a T' E') T' E', 147148)$	$\vdash (\varepsilon, E', 14714863596)$
$\vdash () *b, T' E') T' E', 147148)$	

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, b T' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\varepsilon, T' E', 1471486359)$
$\vdash (a)*b, a T' E') T' E', 147148)$	$\vdash (\varepsilon, E', 14714863596)$
$\vdash () *b, T' E') T' E', 147148)$	$\vdash (\varepsilon, \varepsilon, 147148635963)$