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Concurrency Theory WS 2013/2014

— Series 1 —

Hand in until October 29th before the exercise class.

Exercise 1 (2 Points)

Consider the following process definition:

$$B = a.\overline{a}.B + b.\overline{b}.B$$

(a) Draw LTS(B)! Also write down all necessary derivation trees for drawing LTS(B)!

(b) Is Tr(B) regular? Justify your answer!

Exercise 2 (3 Points)

Consider the following situation: A user can insert a coin into a vending machine. The user then selects whether he wants the vending machine to vend coffee or tea. The vending machine then vends the selected beverage to the user.

From the point of view of the vending machine, the situation can be described as follows: The vending machine can accept coins. If a coin is inserted, the vending machine lets the user select coffee or tea. If coffee is selected by the user, the vending machine vends coffee; if tea is selected, it vends tea. Note that the vending machine is able to handle an arbitrary number of users consecutively.

Now consider the following process definition:

$$\begin{aligned} C_{\|} &= (C_{\text{user}} \parallel C_{\text{machine}}) \setminus \{\text{coin, tea, coffee}\} \\ C_{\text{user}} &= \text{uIC}.\overline{\text{coin}}.\big(\text{uST}.\overline{\text{tea}}.\text{nil} + \text{uSC}.\overline{\text{coffee}}.\text{nil}\big) \\ C_{\text{machine}} &= P_{\text{machine}}, \end{aligned}$$

where A is the set of action names given by $A = \{\text{coin, tea, coffee, uIC, uST, uSC, mVT, mVC}\}$, where uIC indicates “user inserts coin”, uST indicates “user selects tea”, uSC indicates “user selects coffee”, mVT indicates “machine vends tea” and mVC indicates “machine vends coffee”.

(a) Complete the above specification that formalizes the described situation by providing P_{machine}!
Hint: The trace language $\text{Tr}(C_{\text{machine}})$ of C_{machine} should be given by:

$$\begin{aligned} &\big(\text{coin} \cdot (\text{tea} \cdot \text{mVT} + \text{coffee} \cdot \text{mVC})\big)^* \\ &\cdot (\varepsilon + \text{coin} + \text{coin} \cdot \text{tea} + \text{coin} \cdot \text{tea} \cdot \text{mVT} + \text{coin} \cdot \text{coffee} + \text{coin} \cdot \text{coffee} \cdot \text{mVC}) \end{aligned}$$

(b) Draw the corresponding labeled transition system LTS $\left(C_{\|}\right)$!

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Exercise 3 (2 Points)

For any word w we write $v \preceq w$, if v is a prefix of w. The prefix set $\text{pref}(L)$ of a language $L \subseteq \Sigma^*$ is defined as

$$\text{pref}(L) \;=\; \{v \in \Sigma^* \mid v \preceq w,\; w \in L\}\,.$$

A language L is called prefix–closed if $L = \text{pref}(L)$.

Prove that every regular language can be prefix–completed while preserving regularity, i.e. prove that for every regular language L the language $\text{pref}(L)$ is again regular!

Exercise 4 (4 Points)

A language L is called CCS–recognizable, if there exists a recursive process definition

$$(C_i = P_i \mid 1 \leq i \leq k)\,,$$

such that $\text{Tr}(C_1) = L$.

(a) Show that the language $a^* \cdot (c + b + \varepsilon)$ is CCS–recognizable by providing an according process definition and drawing the corresponding labeled transition system!

(b) Prove that every prefix–closed regular language is CCS–recognizable!