# Concurrency Theory WS 2013/2014
## — Series 3 —

Hand in until November 12th before the exercise class.

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### Exercise 1 (Complementation of HML Formulae)          (2+2 Points)

**(a)** Suppose it holds for some arbitrary but fixed HML formula $F$ that $\llbracket F^{\mathrm{c}} \rrbracket = S \setminus \llbracket F \rrbracket$ for every LTS $(S, Act, \rightarrow)$.

Prove that $\llbracket \left( [\alpha]F \right)^{\mathrm{c}} \rrbracket = S \setminus \llbracket [\alpha]F \rrbracket$!

**(b)** Prove that $(F^{\mathrm{c}})^{\mathrm{c}} = F$ holds for every HMF formula $F$!

### Exercise 2 (Modal Depth of HML Formulae)          (1 Point)

Give a formal definition for the modal depth $\mathrm{mdepth}(F) \colon HMF \to \mathbb{N}$ of an HML formulae $F$!

### Exercise 3 (Semantics of HML Formulae)          (1+1 Points)

Let Clock = tick.Clock.

**(a)** Show that Clock satisfies the HML formula $[\mathrm{tick}](\langle \mathrm{tick} \rangle tt \wedge [\mathrm{tock}]\mathit{ff})$!

**(b)** Show that Clock satisfies the following HML formula $F_n$ for every $n \in \mathbb{N}$:

$$F_n \;=\; \underbrace{\langle \mathrm{tick} \rangle \cdots \langle \mathrm{tick} \rangle}_{n \text{ times}} tt$$

### Exercise 4 (Construction of HML Formulae)          (1+1+1 Points)

Let $A = a.b.d.\mathrm{nil} + a.c.d.\mathrm{nil}$ and $B = a.(b.d.\mathrm{nil} + c.d.\mathrm{nil})$.

**(a)** Construct an HML formula which is satisfied by both $A$ and $B$!

**(b)** Construct an HML formula which is satisfied by $B$ but not by $A$!

**(c)** Construct an HML formula which is satisfied by $A$ but not by $B$!