Prof. Dr. Ir. Joost-Pieter Katoen
apl. Prof. Dr. Thomas Noll
B. Kaminski, H. Wu, S. Chakraborty

Concurrency Theory WS 2013/2014

— Series 4 —

Hand in until November 19th before the exercise class.

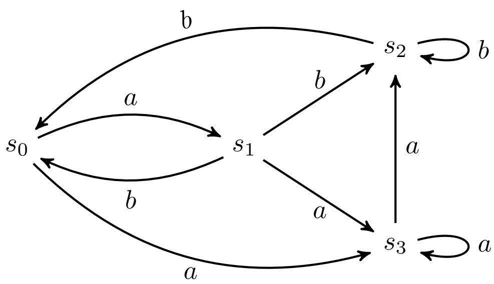
Exercise 1 (Semantics of HML with recursion) (5 Points)

Let $(S, Act, \rightarrow)$ be an LTS, the the semantics of a HML formula $F \in HMF_X$ is defined in lecture 5 as a function

$$[\![F]\!] : 2^S \rightarrow 2^S.$$

1) Show that $[\![F]\!]$ is monotonic over the complete lattice $(2^S, \subseteq)$. How about if we allow negations in F ?

2) An LTS $(S, Act, \rightarrow)$ is given as follows:



Consider following questions:

- compute $[\![\langle b\rangle[a]tt \wedge \langle b\rangle[b]X]\!](\{s_0, s_2\})$ iteratively.
- compute the set of processes satisfying following property

$$X \overset{\min}{=} \langle b\rangle\langle a\rangle tt \vee \langle b\rangle[b]X$$

- compute the sets of processes satisfying following equational systems

$$\begin{aligned} A &\overset{\max}{=} [a]B \\ B &\overset{\max}{=} \langle a\rangle C \wedge [b]B \\ C &\overset{\max}{=} [b]B \end{aligned}$$

Exercise 2 (Strong bisimulation as fixed point) (5 Points)

Let $(S, Act, \rightarrow)$ be an LTS. A binary relation $\mathcal{R}$ over S is an element of set $2^{(S\times S)}$. Now we define the set $\mathcal{F}(\mathcal{R})$ as follows. A pair $(p, q) \in \mathcal{F}(\mathcal{R})$ for all $p, q \in S$ and for every $\alpha \in Act$ iff

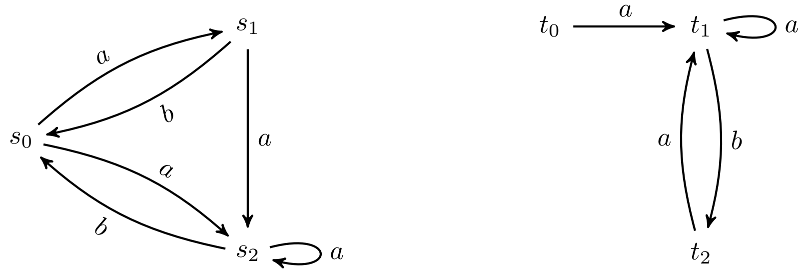
1. $p \xrightarrow{\alpha} p'$ implies $q \xrightarrow{\alpha} q'$ for some q' such that $(p', q') \in \mathcal{R}$;
2. $q \xrightarrow{\alpha} q'$ implies $p \xrightarrow{\alpha} p'$ for some p' such that $(p', q') \in \mathcal{R}$.

1) Show that the function $\mathcal{F}$ is monotonic over the complete lattice $(2^{(S\times S)}, \subseteq)$, and

$$\sim\, = \bigcup \{\mathcal{R} \in 2^{(S\times S)} \mid \mathcal{R} \subseteq \mathcal{F}(\mathcal{R})\}$$

is the largest fixed point of $\mathcal{F}$.

2) Compute $\sim$ for following LTS $(S, Act, \rightarrow)$ iteratively:


s_0
s_1
s_2
a
b
a
a
b
a
t_0
t_1
t_2
a
a
a
b