Prof. Dr. Ir. Joost-Pieter Katoen
apl. Prof. Dr. Thomas Noll
Chakraborty, Kaminski, Wu

Concurrency Theory WS 2013/2014

— Series 9 —

Hand in until January 7 before the exercise class.

Exercise 1 (Complexity of Checking Trace Equivalence) (1 Point)

Checking for trace equivalence is PSPACE complete for finite LTS. As bisimilarity implies trace equivalence, we can simply check for bisimilarity in order to check for trace equivalence. Checking bisimilarity for finite LTS is in P. We have thereby reduced a PSPACE complete problem to a problem in P. Contradiction!

Find the mistake in the above proof!

Exercise 2 (Monotonicity of $\mathcal{F}$) (1 Point)

Recall the definition of $\mathcal{F}\colon : \mathcal{P}(\mathit{Prc} \times \mathit{Prc}) \to \mathcal{P}(\mathit{Prc} \times \mathit{Prc})$, $\mathcal{R} \mapsto \mathcal{R}'$, where $P \mathcal{R}' Q$ if and only if

1. if $P \xrightarrow{\alpha} P'$ then there exists $Q' \in \mathit{Prc}$, such that $Q \xrightarrow{\alpha} Q'$ and $P' \mathcal{R} Q'$, and
2. if $Q \xrightarrow{\alpha} Q'$ then there exists $P' \in \mathit{Prc}$, such that $P \xrightarrow{\alpha} P'$ and $P' \mathcal{R} Q'$.

Prove that $\mathcal{F}$ is monotonic on $(\mathcal{P}(\mathit{Prc} \times \mathit{Prc}), \supseteq)$!

Exercise 3 (Strong Bisimilarity and Finite Automata) (1 + 1 Points)

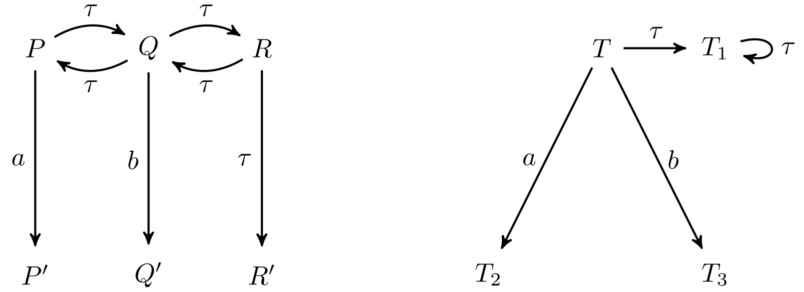
Let $\mathfrak{A}$ be a finite automaton with $\mathrm{L}(\mathfrak{A}) = L$, let $\mathfrak{A}_d$ be a deterministic finite automaton with $\mathrm{L}(\mathfrak{A}_d) = L$, and let $\mathfrak{A}_L$ be the minimal deterministic finite automaton which recognizes L. Prove or disprove:

(a) The initial states of $\mathfrak{A}$ and $\mathfrak{A}_L$ are strongly bisimilar.

(b) The initial states of $\mathfrak{A}_d$ and $\mathfrak{A}_L$ are strongly bisimilar.

Exercise 4 (Weak Bisimilarity) (1 Point)

Consider the following LTS:



Prove or disprove: $P \approx T$.

Exercise 5 (Branching Bisimulation) (2 + 2 Points)

A branching bisimulation is a relation $\mathcal{R}$, such that $\mathcal{R}$ is symmetric and if $P \mathrel{\mathcal{R}} P'$ and $P \xrightarrow{\alpha} Q$, then either

- $\alpha = \tau$ and $P' \mathrel{\mathcal{R}} Q$, or
- $\alpha \neq \tau$ and there is a sequence of $k \geq 0$ transitions

$$Q = Q_0 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_k \xrightarrow{\alpha} Q',$$

such that $\forall i \in \{0, \ldots, k\} \colon P \mathrel{\mathcal{R}} Q_i$ and $P' \mathrel{\mathcal{R}} Q'$.

Two processes P and Q are called branching bisimilar, denoted $P \approx_{BB} Q$, iff there exists a branching bisimulation $\mathcal{R}$ such that $P \mathrel{\mathcal{R}} Q$. Prove or disprove:

(a) $P \approx_{BB} Q \implies P \approx Q$

(b) $P \approx Q \implies P \approx_{BB} Q$

Exercise 6 (Fixed–Point Iteration) (1 Points)

Let $(M, \sqsubseteq)$ be a complete lattice with least element $\perp$, let $f \colon M \to M$ be monotonic, and let $p = \sup \left\{ f^i(\perp) \mid i \in \mathbb{N} \right\}$ be a fixed–point of f. Prove that p is the least fixed–point of f!