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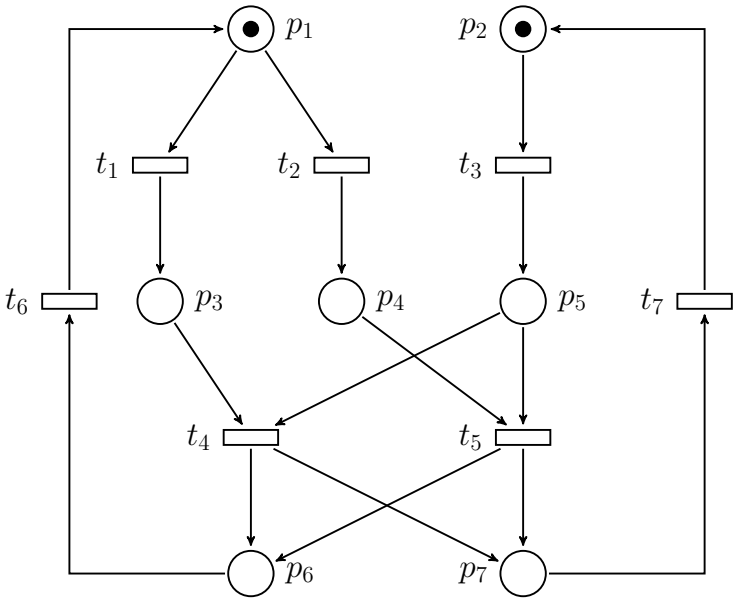
Concurrency Theory WS 2013/2014

— Series 10 —

Hand in until January 21st before the exercise class.

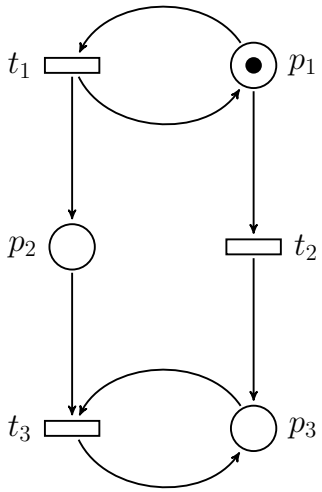
Exercise 1 (Marking Graph and Distributed Runs) (3 Points)



1) Give the marking graph of the Petri net shown above.

2) Give at least three distributed runs which cover at least four transition of this Petri net. If the Petri net admits an infinite distributed run, please provide it.

Exercise 2 (Infinite Marking Graph) (3 Points)



1) Give the (partial) marking graph of the Petri net shown above and show that it is infinite.

2) Since the marking graphs of a Petri net can be infinite, *coverability tree* (or coverability graph) is defined for Petri net, which is always finite. In order to represent *infinite* tokens in a place, a special symbol ω is introduced, and for all natural number n it has the following properties: $\omega > n$, $\omega \pm n = \omega$ and $\omega \geq \omega$.

The coverability tree for a Petri net (P, T, F, M_0) can then be computed by following algorithm.

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s.1) Label M_0 as the root and tag it "new".
s.2) While "new" markings exist, do the following:
  s.2.1) Select a new marking M.
  s.2.2) If M is identical to a marking on the path from the root
         to M, tag M "old" and to another new marking.
  s.2.3) If no transitions as are enabled at M, tag M as "dead-end".
  s.2.4) While there exist enabled transitions at M, do the
         following for each enabled transition t at M:
    s.2.4.1) Obtain the marking M' that results from firing t at M.
    s.2.4.2) On the path from the root to M, if there exist a marking
             M'' such that M'(p) ≥ M''(p) for each place p and
             M' ≠ M'', replace M'(p) by ω for each p
             such that M'(p) > M''(p).
    s.2.4.4) Introduce M' as a node, draw an arc with label t
             from M to M', and tag M' "new".
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Compute the coverability tree of the above Petri net.

3) The *synchronic distance* between two transitions t and t' in an elementary system net (P, T, F, M_0) is defined by

$$d_{t,t'} = \max_{\sigma} \, | \#_\sigma(t) - \#_\sigma(t') |$$

where σ is a sequential run starting at any reachable marking M of M_0 and $\#_\sigma(t)$ is the number of times t fires in σ. Please give d_{t_1,t_2}, d_{t_1,t_3} and d_{t_2,t_3} for the above net.

Exercise 3 (Modelling with Petri nets) (4 Points)

1) Model the traffic light system at a cross road in *Germany* by means of elementary Petri nets. If you don't know how it works, you should do some on-the-spot investigation.

2) In Exercise Series 2, we have introduced Hyman's algorithm for mutual exclusion. Please model this algorithm by means of elementary Petri nets.