Concurrency Theory

Lecture 2: Calculus of Communicating Systems (CCS)

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(Software Modeling and Verification)

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Outline

The Calculus of Communicating Systems

History:

- Robin Milner: *A Calculus of Communicating Systems*
  LNCS 92, Springer, 1980
- Robin Milner: *Communication and Concurrency*
  Prentice-Hall, 1989
- Robin Milner: *Communicating and Mobile Systems: the π-calculus*
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Approach: describing parallelism on a simple and abstract level, using only a few basic primitives

- no explicit storage (variables)
- no explicit representation of values (numbers, Booleans, ...)

$\Longrightarrow$ parallel system reduced to communication potential

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Definition 2.1 (Syntax of CCS)

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- $Act := A \cup \overline{A} \cup \{\tau\}$ is the set of actions where τ denotes the silent (or: unobservable) action.
- Let Pid be a set of process identifiers.
- The set Prc of process expressions is defined by the following syntax:

$$
\begin{array}{rll}
P ::= & \mathsf{nil} & \text{(inaction)} \\
\mid & \alpha.P & \text{(prefixing)} \\
\mid & P_1 + P_2 & \text{(choice)} \\
\mid & P_1 \parallel P_2 & \text{(parallel composition)} \\
\mid & P \setminus L & \text{(restriction)} \\
\mid & P[f] & \text{(relabelling)} \\
\mid & C & \text{(process call)}
\end{array}
$$

where $\alpha \in Act$, $L \subseteq A$, $C \in Pid$, and $f : Act \to Act$ such that $f(\tau) = \tau$ and $f(\overline{a}) = \overline{f(a)}$ for each $a \in A$.

Syntax of CCS II

Definition 2.1 (continued)

- A (recursive) process definition is an equation system of the form

$$(C_i = P_i \mid 1 \leq i \leq k)$$

where $k \geq 1$, $C_i \in \mathit{Pid}$ (pairwise distinct), and $P_i \in \mathit{Prc}$ (with process identifiers from $\{C_1, \ldots, C_k\}$).

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Notational Conventions:

- $\overline{\overline{a}}$ means a
- $\sum_{i=1}^{n} P_i$ ($n \in \mathbb{N}$) means $P_1 + \ldots + P_n$ (where $\sum_{i=1}^{0} P_i := \mathsf{nil}$)
- $P \setminus a$ abbreviates $P \setminus \{a\}$
- $[a_1 \mapsto b_1, \ldots, a_n \mapsto b_n]$ stands for $f : Act \to Act$ with $f(a_i) = b_i$ ($i \in [n]$) and $f(\alpha) = \alpha$ otherwise
- restriction and relabelling bind stronger than prefixing, prefixing stronger than composition, composition stronger than choice:

$$P \setminus a + b.Q \parallel R \quad \text{means} \quad (P \setminus a) + ((b.Q) \parallel R)$$

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- The relabelling $P[f]$ allows to adapt the naming of actions.
- The behaviour of a process call C is given by the right-hand side of the corresponding equation.

CCS Examples

Example 2.2

1. One-place buffer
2. Two-place buffer
3. Parallel specification of two-place buffer

(on the board)

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Labelled Transition Systems

Goal: represent behaviour of system by (infinite) graph

- nodes = system states
- edges = transitions between states

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Definition 2.3 (Labelled transition system)

A (Act-)labelled transition system (LTS) is a triple $(S, Act, \longrightarrow)$ consisting of

- a set S of states
- a set Act of (action) labels
- a transition relation $\longrightarrow \subseteq S \times Act \times S$

For $(s, \alpha, s') \in \longrightarrow$ we write $s \xrightarrow{\alpha} s'$. An LTS is called finite if S is so.

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Remarks:

- sometimes an initial state $s_0 \in S$ is distinguished ("$LTS(s_0)$")
- (finite) LTSs correspond to (finite) automata without final states

Semantics of CCS I

We define the assignment

$$\begin{array}{rcl} \text{syntax} & \rightarrow & \text{semantics} \\ \text{process definition} & \mapsto & \text{LTS} \end{array}$$

by induction over the syntactic structure of process expressions. Here we employ derivation rules of the form

$$\text{rule name}\,\frac{\text{premise(s)}}{\text{conclusion}}$$

which can be composed to complete derivation trees.

Definition 2.4 (Semantics of CCS)

A process definition $(C_i = P_i \mid 1 \leq i \leq k)$ determines the LTS $(Prc, Act, \longrightarrow)$ whose transitions can be inferred from the following rules ($P, P', Q, Q' \in Prc$, $\alpha \in Act$, $\lambda \in A \cup \overline{A}$, $a \in A$):

$$(\text{Act})\ \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad (\text{Sum}_1)\ \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad (\text{Sum}_2)\ \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

$$(\text{Par}_1)\ \frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q} \qquad (\text{Par}_2)\ \frac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'}$$

$$(\text{Com})\ \frac{P \xrightarrow{\lambda} P' \quad Q \xrightarrow{\overline{\lambda}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'} \qquad (\text{Res})\ \frac{P \xrightarrow{\alpha} P' \quad (\alpha, \overline{\alpha} \notin L)}{P \setminus L \xrightarrow{\alpha} P' \setminus L}$$

$$(\text{Rel})\ \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]} \qquad (\text{Call})\ \frac{P \xrightarrow{\alpha} P' \quad (C = P)}{C \xrightarrow{\alpha} P'}$$

Semantics of CCS III

Example 2.5

1. One-place buffer:

$$B = in.\overline{out}.B$$

2. Sequential two-place buffer:

$$\begin{aligned} B_0 &= in.B_1 \\ B_1 &= \overline{out}.B_0 + in.B_2 \\ B_2 &= \overline{out}.B_1 \end{aligned}$$

3. Parallel two-place buffer:

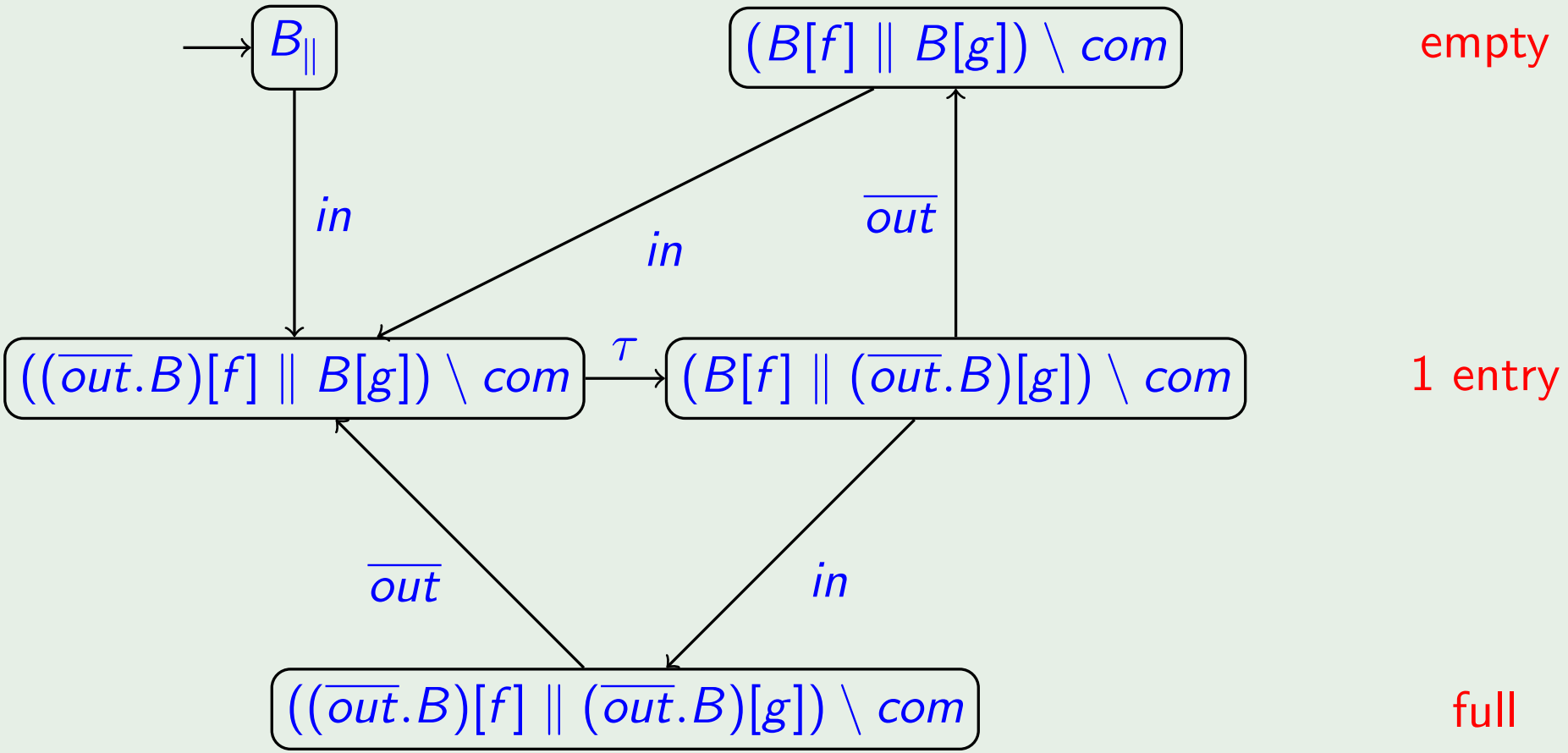
$$\begin{aligned} B_{\|} &= (B[f] \,\|\, B[g]) \setminus com \\ B &= in.\overline{out}.B \end{aligned}$$

where $f := [out \mapsto com]$ and $g := [in \mapsto com]$

(on the board)

Example 2.5 (continued)

Complete LTS of parallel two-place buffer:



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Process Traces

Goal: reduce processes to the action sequences they can perform

Definition 2.6 (Trace language)

For every $P \in Prc$, let

$$Tr(P) := \{w \in Act^* \mid \text{ex. } P' \in Prc \text{ such that } P \xrightarrow{w} P'\}$$

be the trace language of P
(where $\xrightarrow{w} := \xrightarrow{a_1} \circ \ldots \circ \xrightarrow{a_n}$ for $w = a_1 \ldots a_n$).

$P, Q \in Prc$ are called trace equivalent if $Tr(P) = Tr(Q)$.

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$P, Q \in Prc$ are called trace equivalent if $Tr(P) = Tr(Q)$.

Example 2.7 (One-place buffer)

$B = in.\overline{out}.B$

$\Longrightarrow \; Tr(B) = (in \cdot \overline{out})^* \cdot (in + \varepsilon)$

Trace Equivalence II

Remarks:

- The trace language of $P \in Prc$ is accepted by the LTS of P, interpreted as a (finite or infinite) automaton with initial state P and where every state is final.

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- The trace language of $P \in Prc$ is accepted by the LTS of P, interpreted as a (finite or infinite) automaton with initial state P and where every state is final.
- Trace equivalence is obviously an equivalence relation (i.e., reflexive, symmetric, and transitive).
- Trace equivalence identifies processes with identical LTSs: the trace language of a process consists of the (finite) paths in the LTS. Thus:

$$LTS(P) = LTS(Q) \implies Tr(P) = Tr(Q)$$