NL*—Angluin-style learning of NFA

Benedikt Bollig[1] Peter Habermehl[2]
Carsten Kern[3] Martin Leucker[4]

[1]LSV, ENS Cachan

[2]LIAFA Paris Diderot (Paris 7)

[3]RWTH Aachen University

[4] Technical University Munich

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Learning

Here, learning means:

Given exemplifying behavior of a system

Learn a *model* conforming to the given behavior

Learning

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Given exemplifying behavior of a system
in terms of words

Learn a *model* conforming to the given behavior
in terms of a regular language (deterministic finite automaton, DFA)

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- The learner is given positive and negative examples
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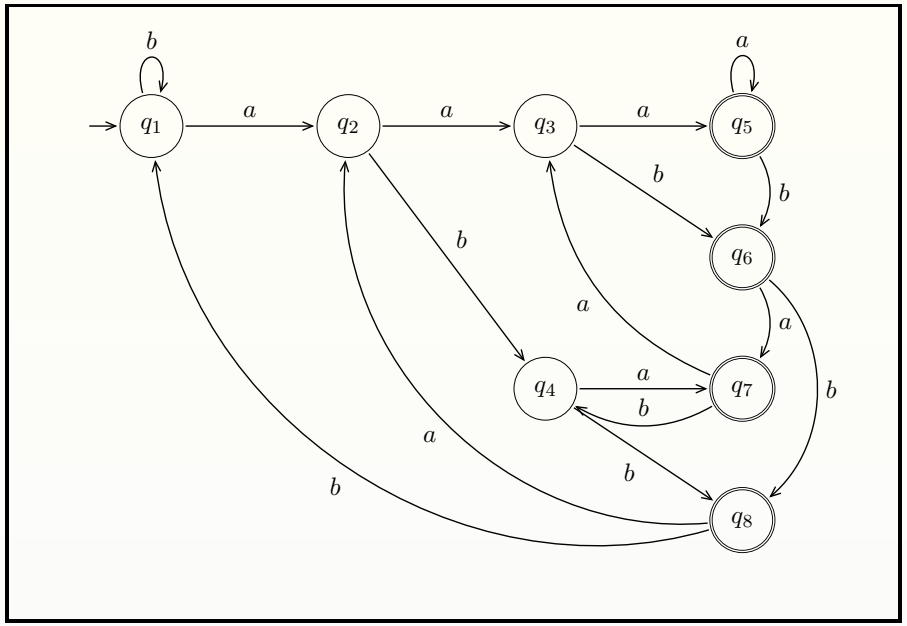
Occam's razor:

"In case of different explanations, choose the *simplest* one."
$\Rightarrow$ Learn the *minimal* DFA conforming to given examples

But there is a problem . . .



minimal DFA can be huge!

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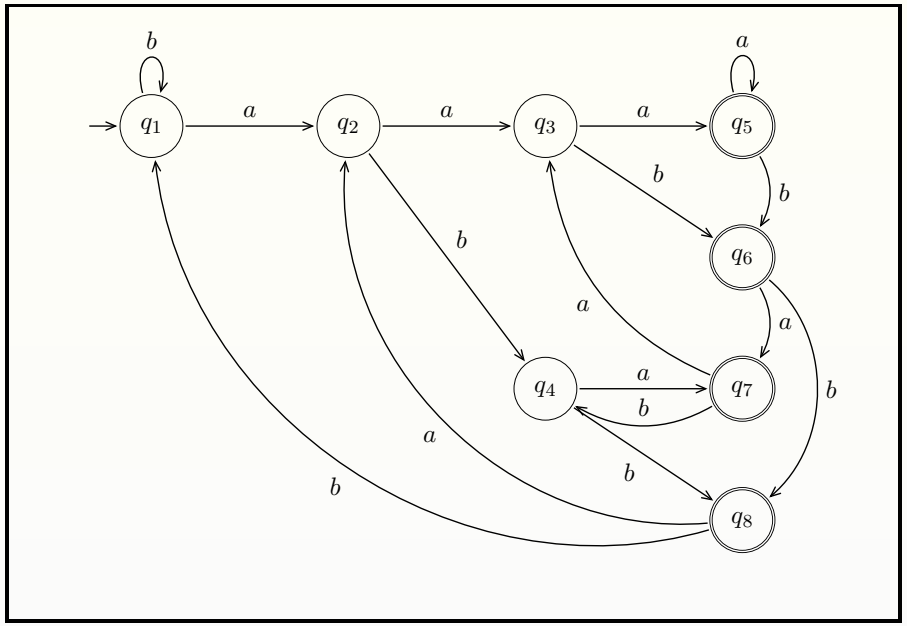

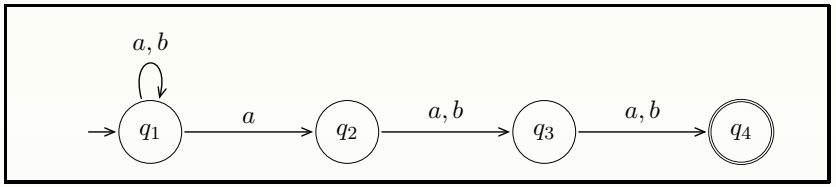

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Can we learn (a certain subclass of) NFA?

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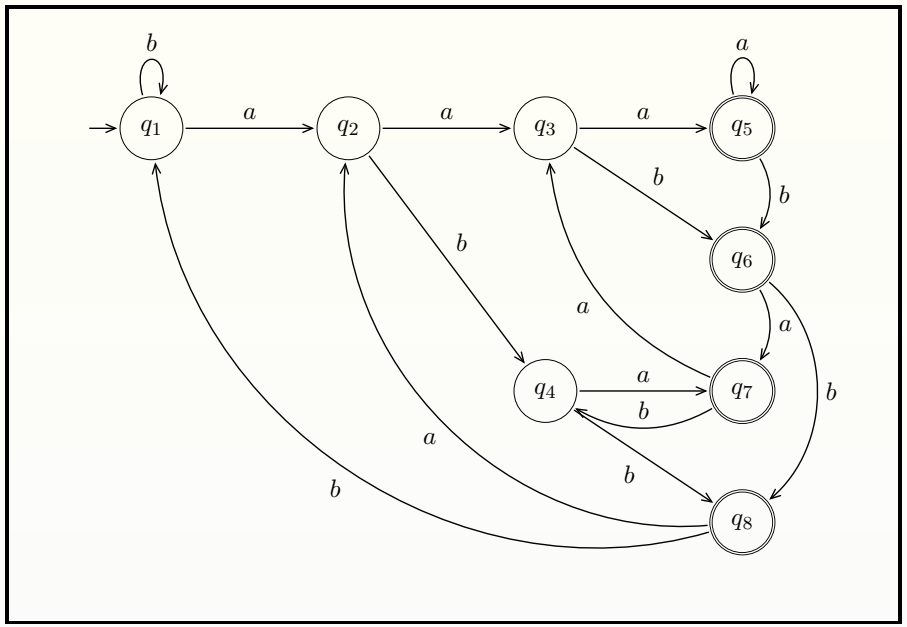

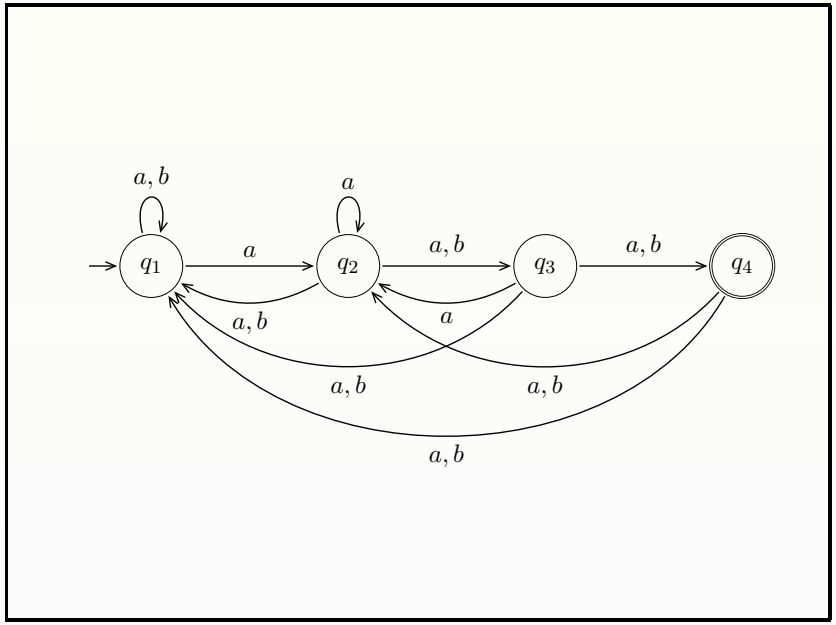

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What about NFA?

Can we learn (a certain subclass of) NFA?

Yes, we can!

Outline

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Presentation outline

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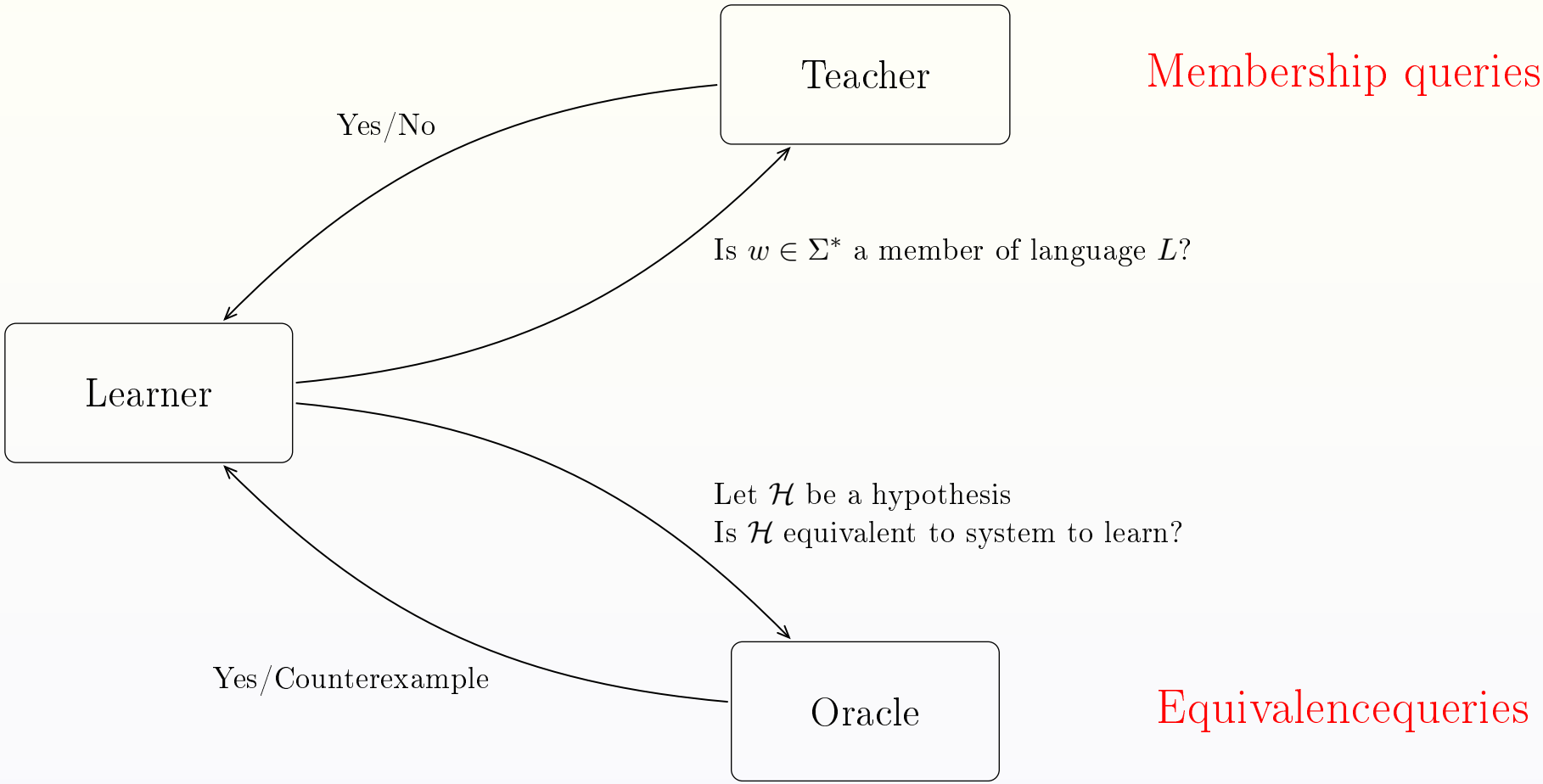
Algorithm - Overview



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- Counterexample: $w \in (L(\mathcal{H}) \setminus L) \cup (L \setminus L(\mathcal{H}))$

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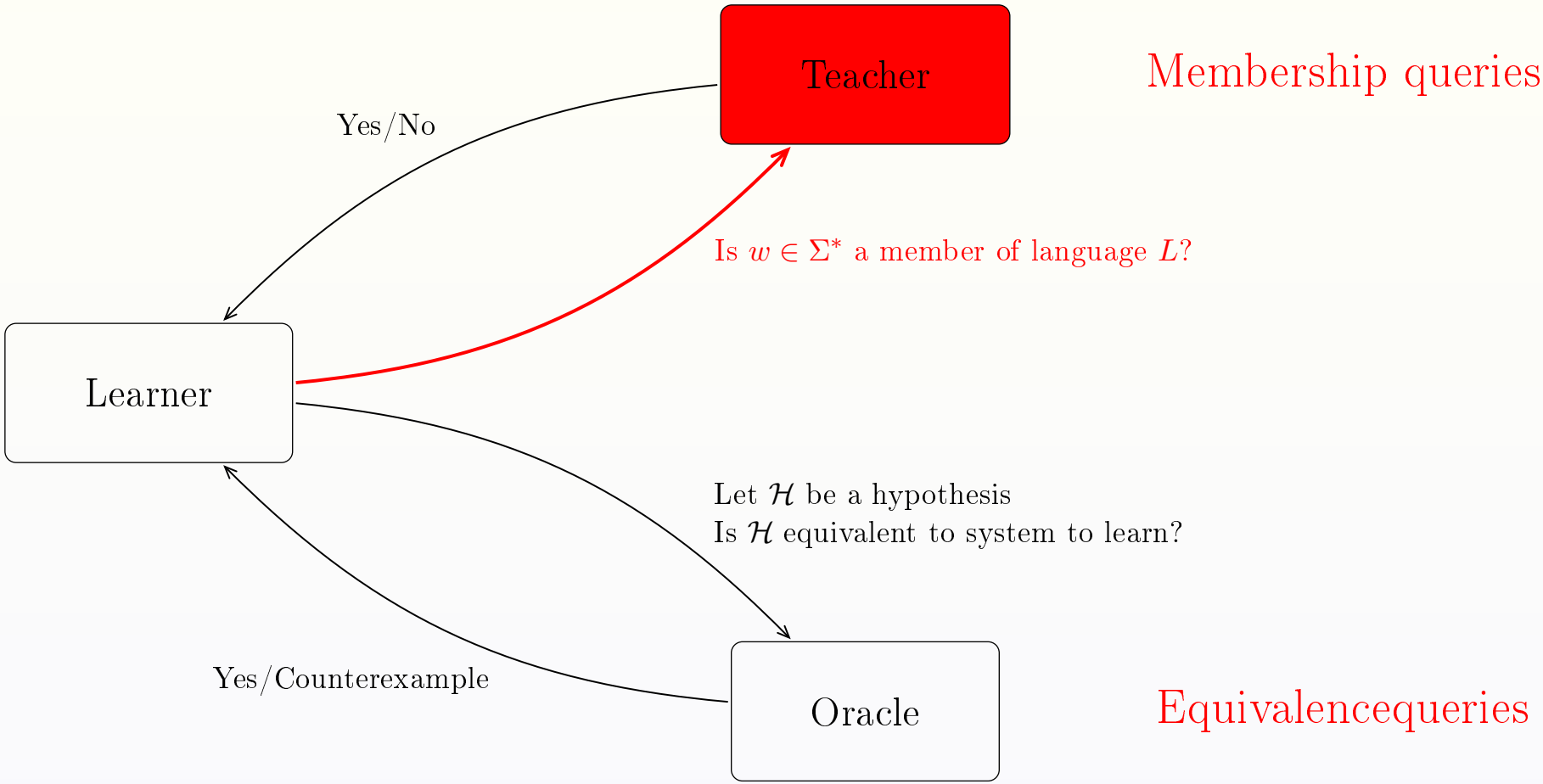

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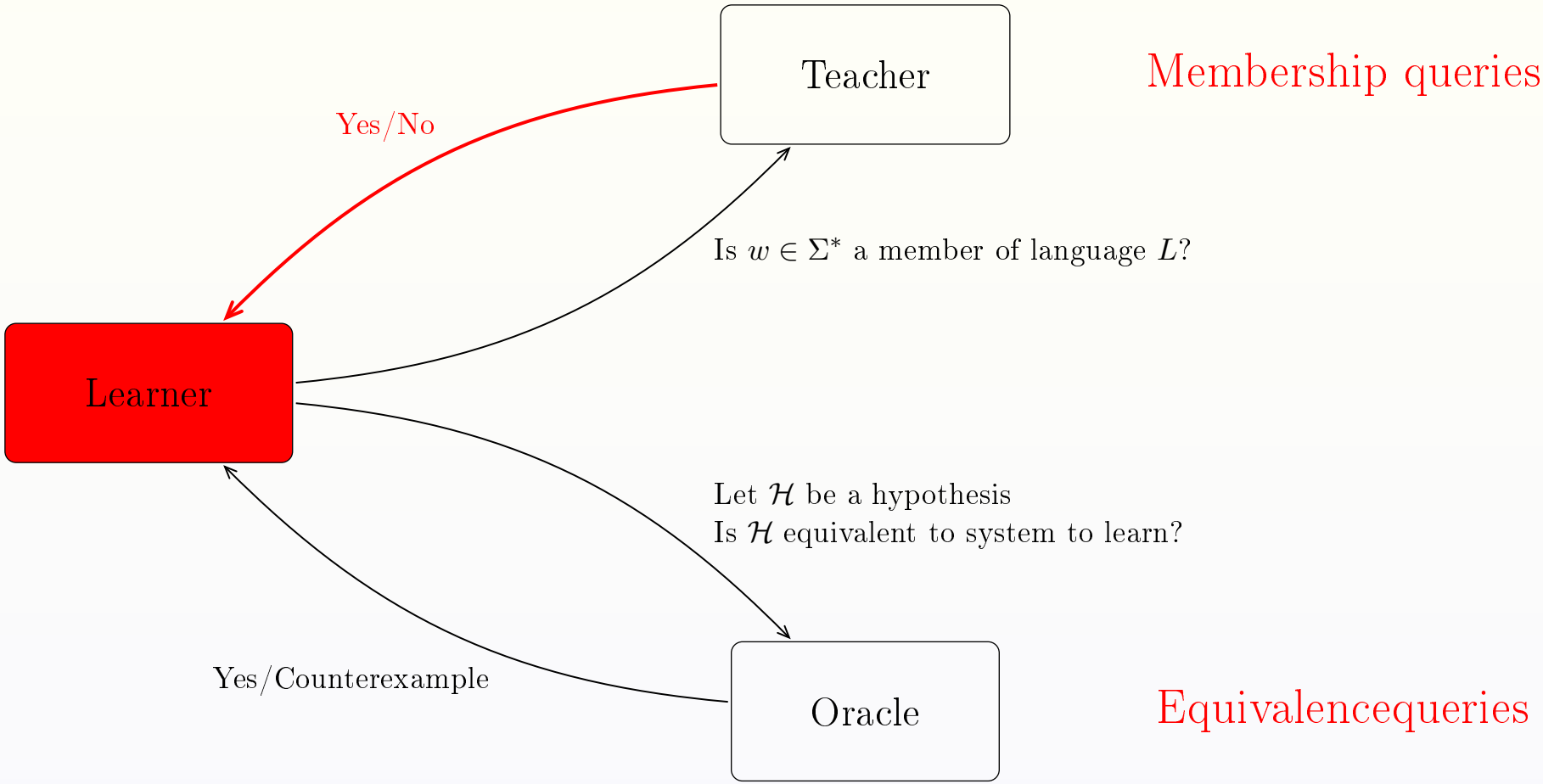

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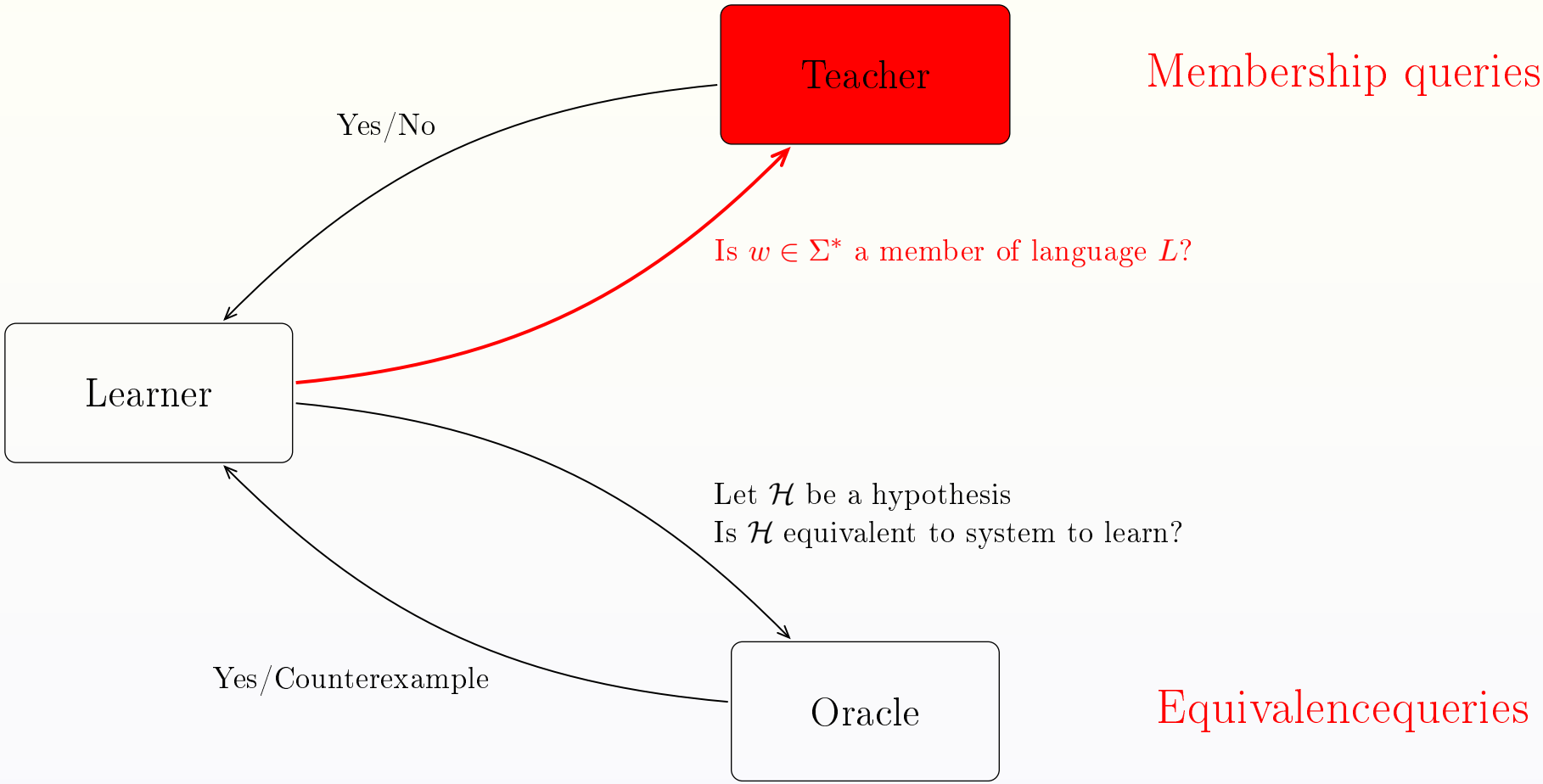

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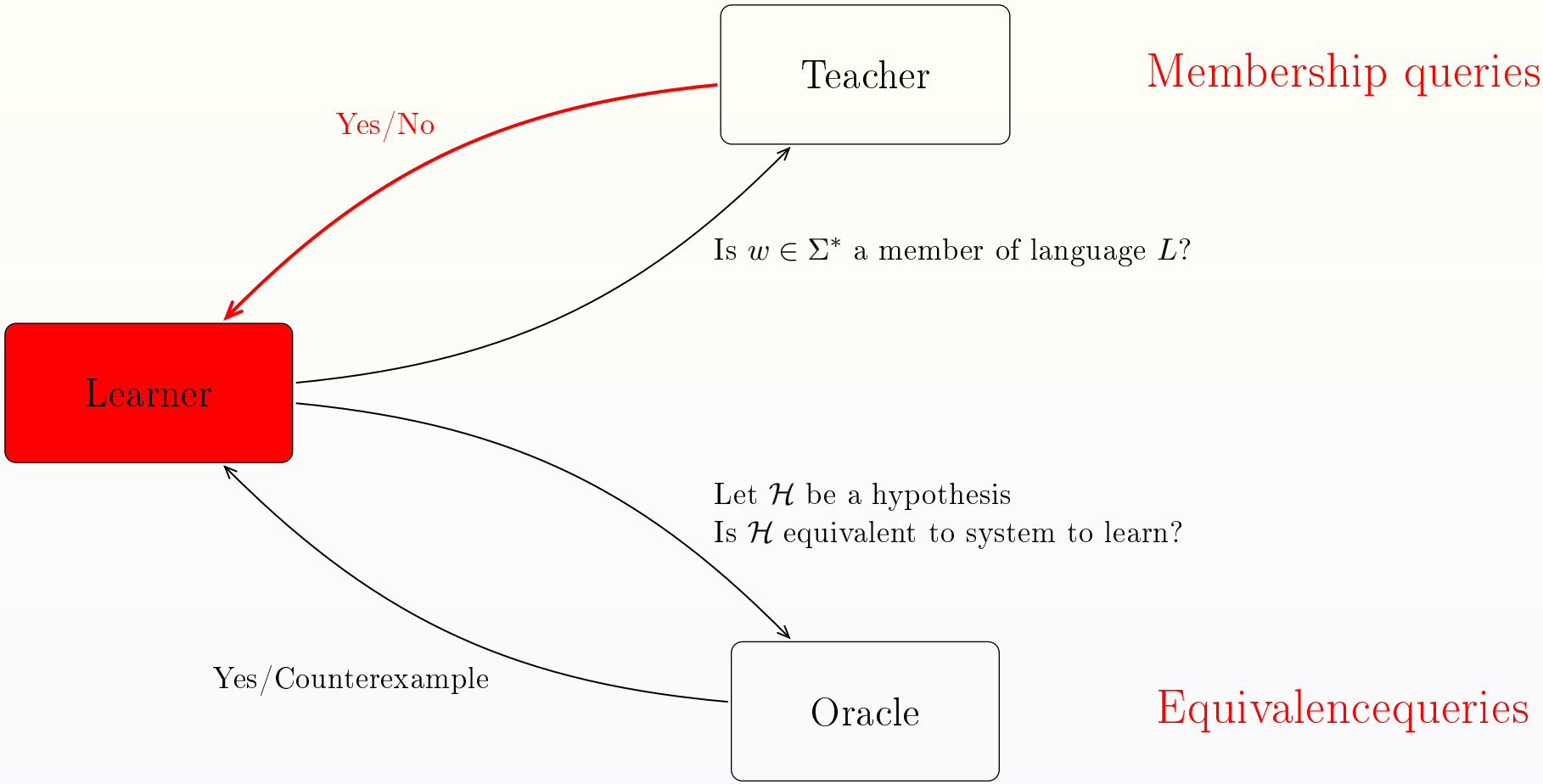

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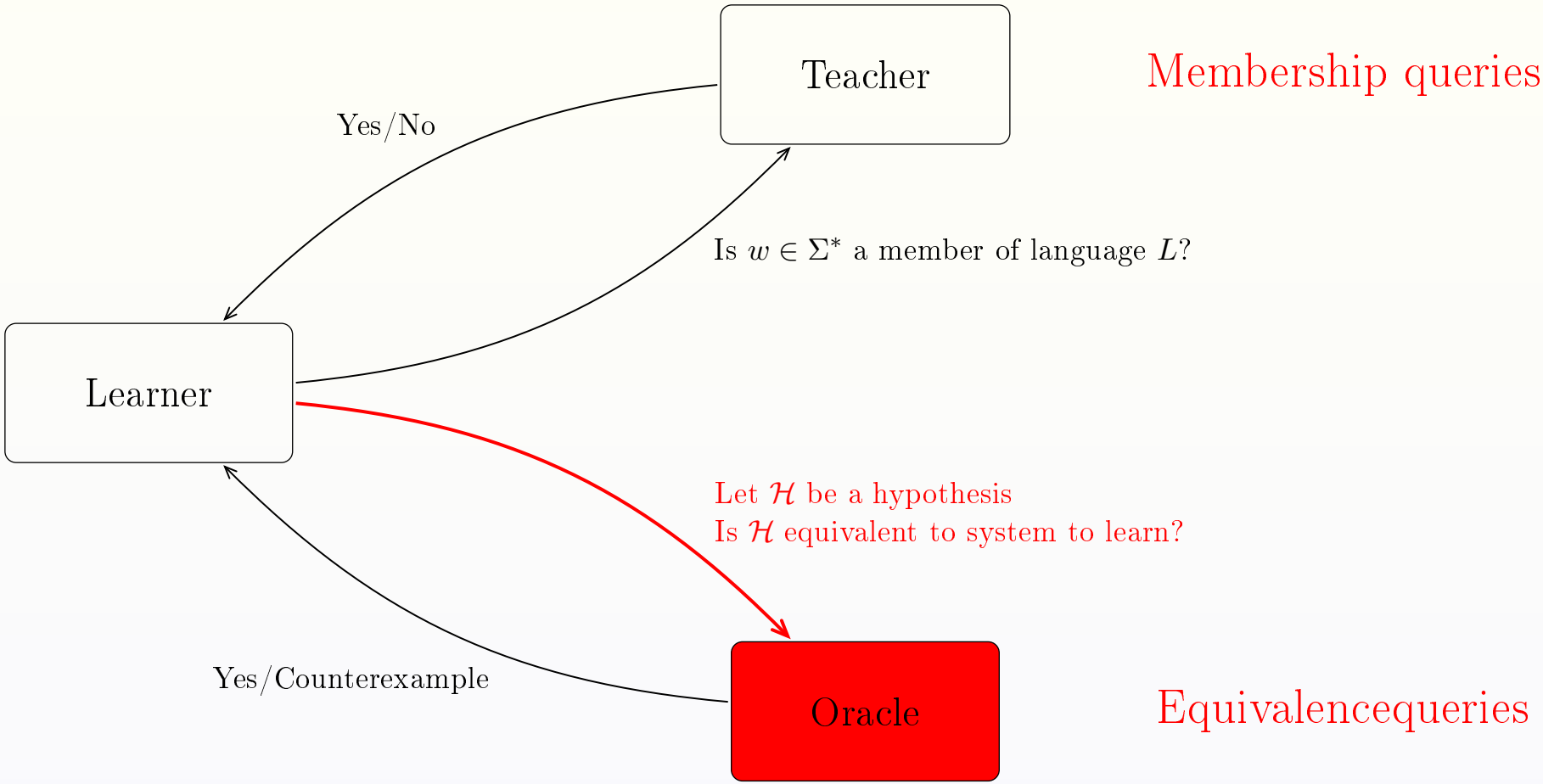

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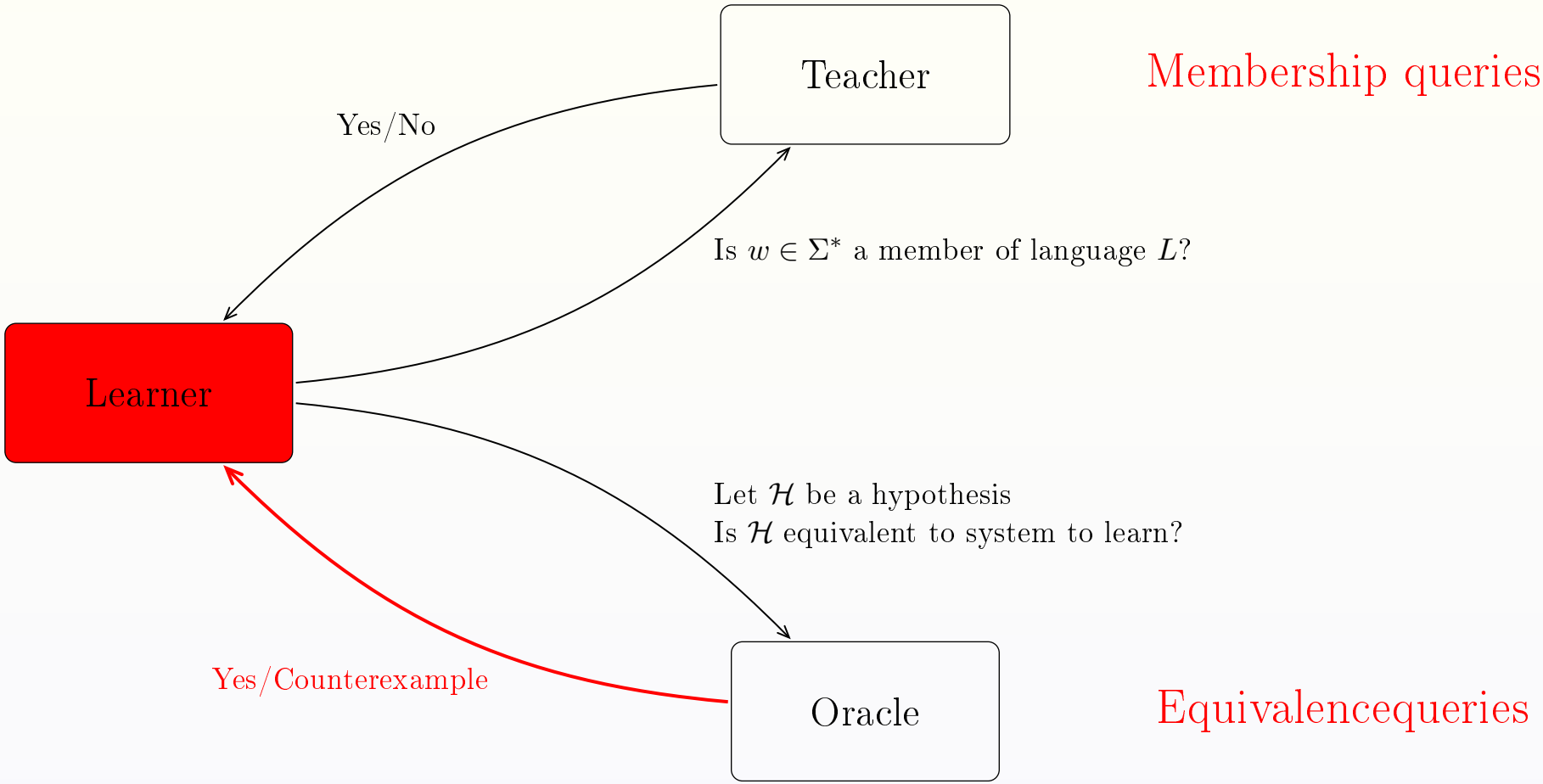

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Table-based learning

$\mathcal{T}$	ε
ε	
a	
b	
aa	
ab	

$\varepsilon \in L?$

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	
b	
aa	
ab	

$a \in L?$

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	$-$
b	
aa	
ab	

$b, aa, ab \in L?$

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	$-$
b	$-$
aa	$-$
ab	$+$

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	$-$
b	$-$
aa	$-$
ab	$+$

To derive an automaton:

- $\mathcal{T}$ must be closed, i.e., all states are derivable from $\mathcal{T}$

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	$-$
b	$-$
aa	$-$
ab	$+$

To derive an automaton:

- $\mathcal{T}$ must be closed, i.e., all states are derivable from $\mathcal{T}$
- $\mathcal{T}$ must be consistent, i.e., there are no contradicting transitions

Table-based learning

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ε	$+$
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b	$-$
aa	$-$
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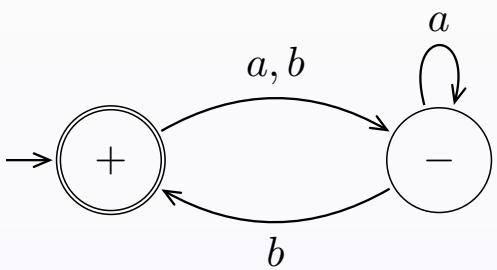

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To this end:

- upper rows serve to derive states
- lower rows serve to derive transitions

Table-based learning

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b	$-$
aa	$-$
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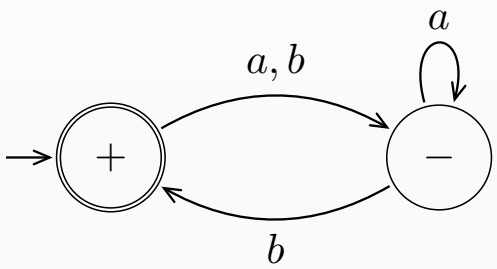

$bb \notin L$!

Table-based learning

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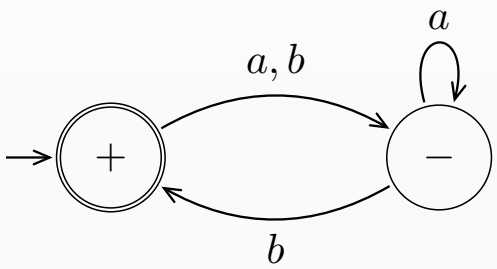

$bb \notin L$!

Counterexample can be added to:

Table-based learning

$\mathcal{T}$	ε
ε	$+$
a	$-$
b	$-$
bb	$-$
aa	$-$
ab	$+$
ba	$-$
bba	$-$
bbb	$-$

Counterexample can be added to:

- the rows (L^*)

Table-based learning

$\mathcal{T}$	ε	bb	b
ε	$+$	$-$	$-$
a	$-$	$-$	$+$
b	$-$	$-$	$-$
aa	$-$	$-$	$-$
ab	$+$	$-$	$-$

Counterexample can be added to:

- the rows (L^*)
- the columns ($\mathrm{L}^*_{\mathrm{col}}$)

Angluin's Algorithm L*

Theorem (Complexity of L*)

Let:

- n: number of states of the minimal DFA $\mathcal{A}_L$ for regular language L,
- m: length of the biggest counterexample

Then, L* returns after at most:

the minimal DFA $\mathcal{A}$.

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- $O(m|\Sigma|n^2)$ membership queries

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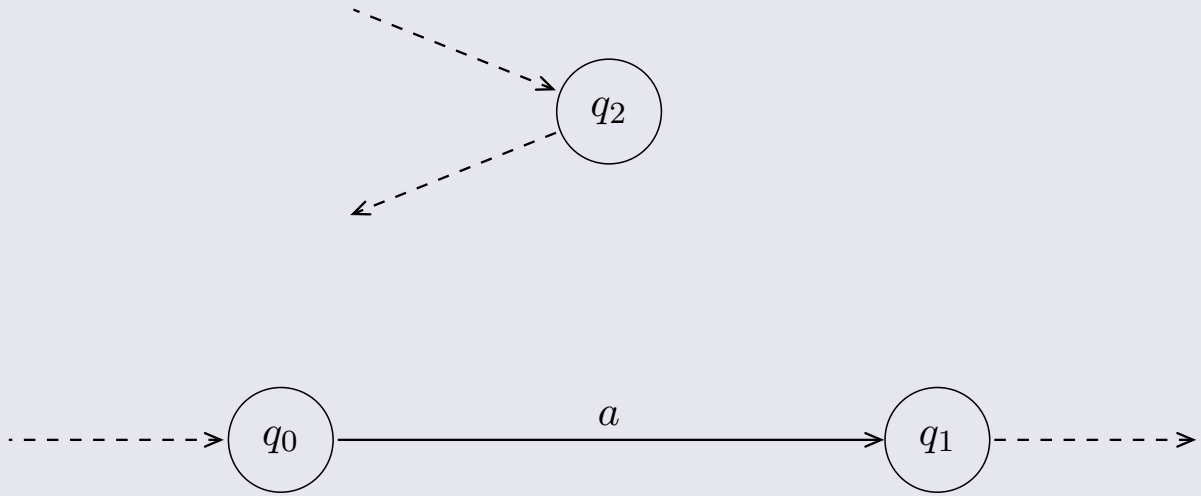
Residual Finite-State Automata [Denis et al.]



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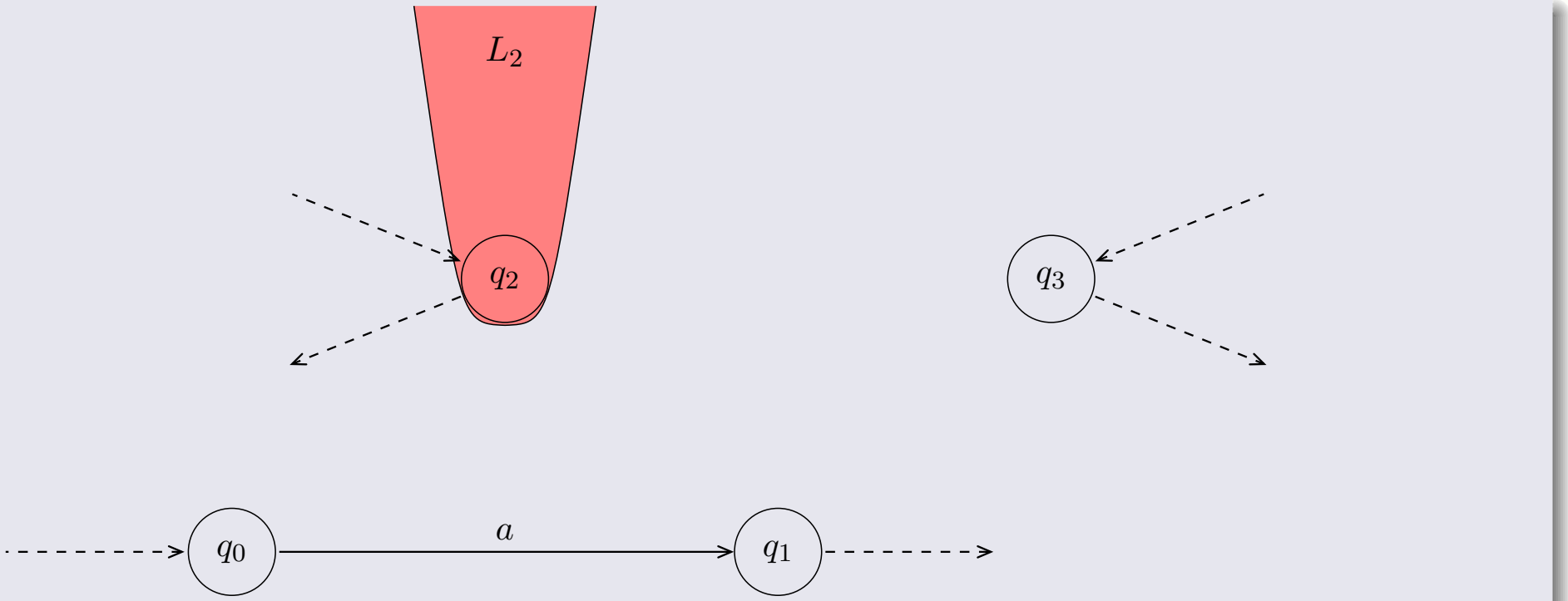

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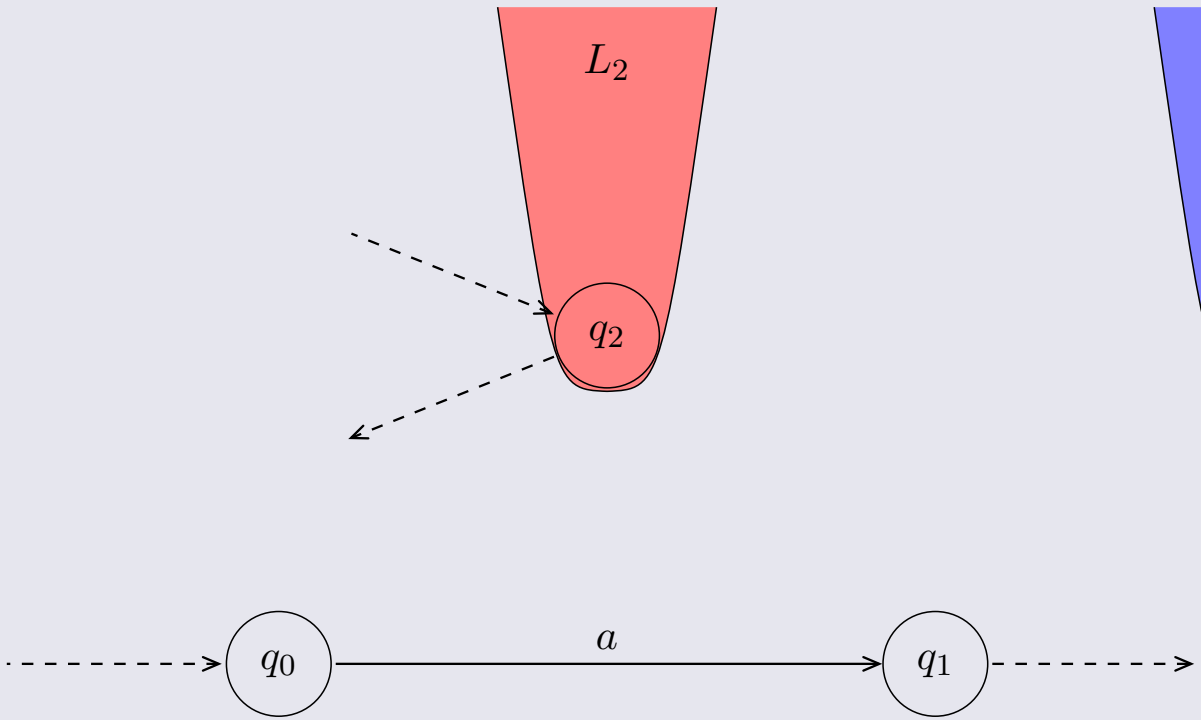

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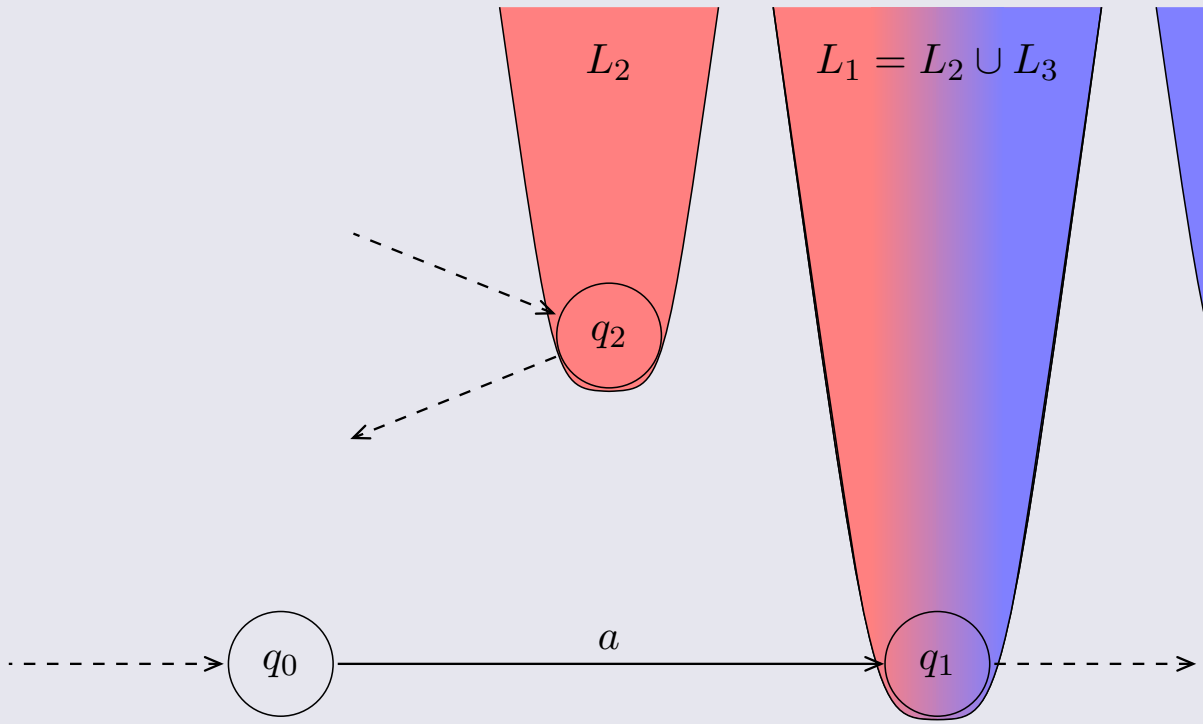

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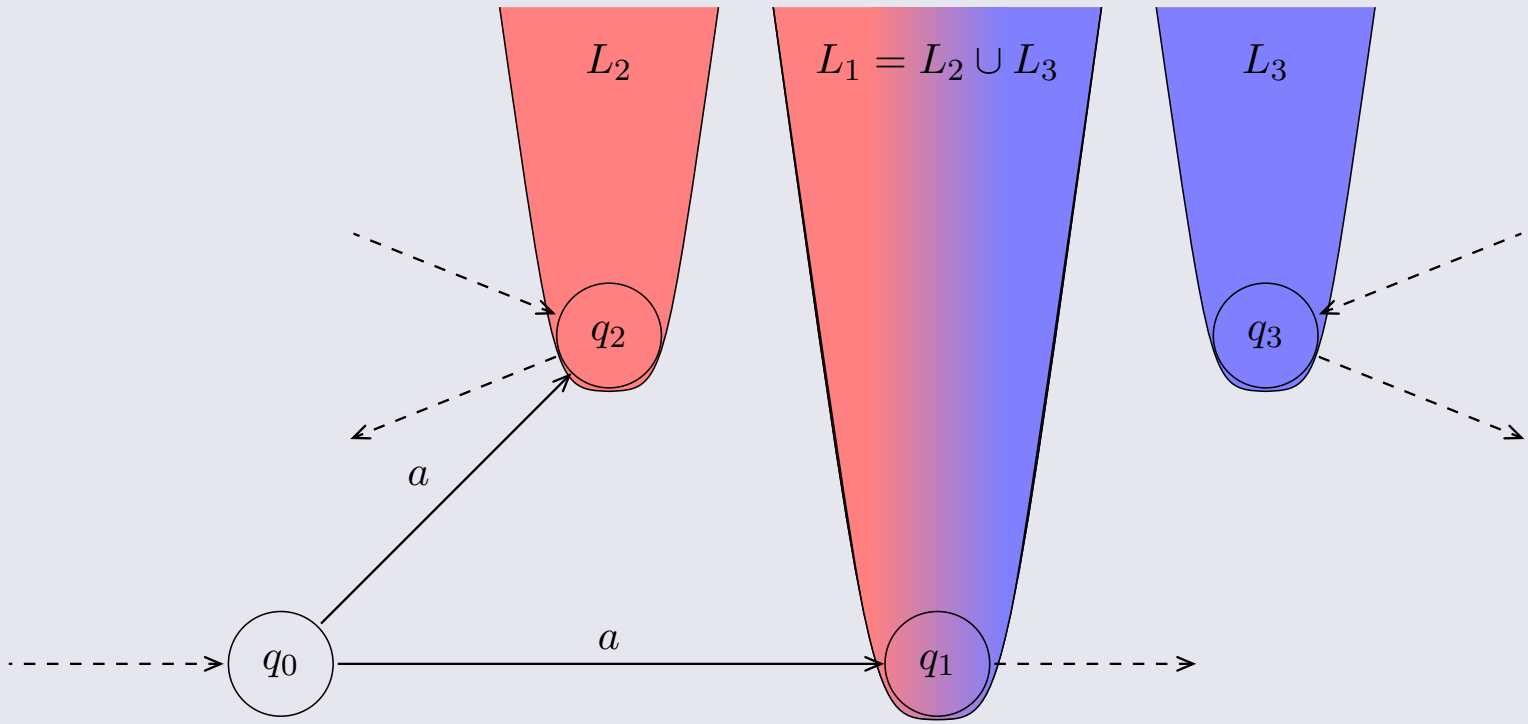

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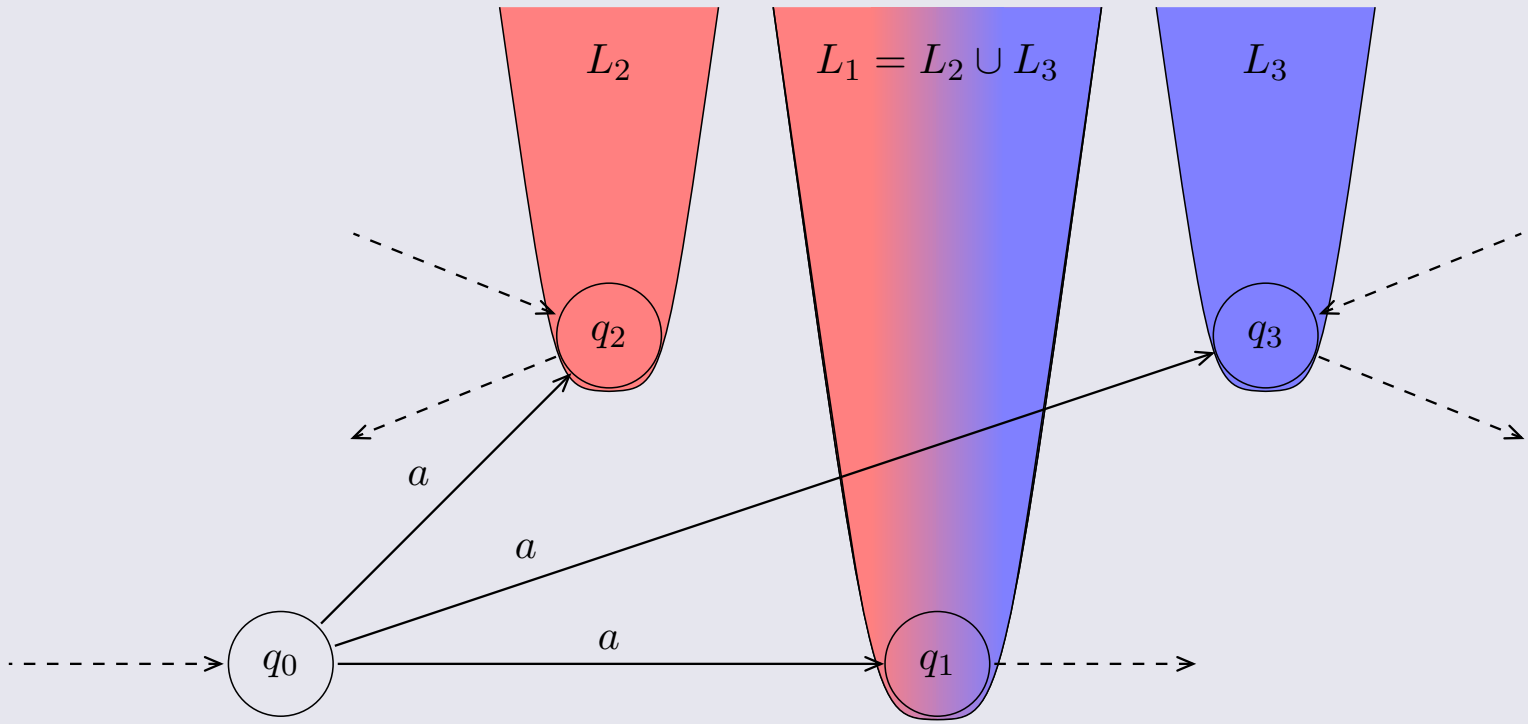

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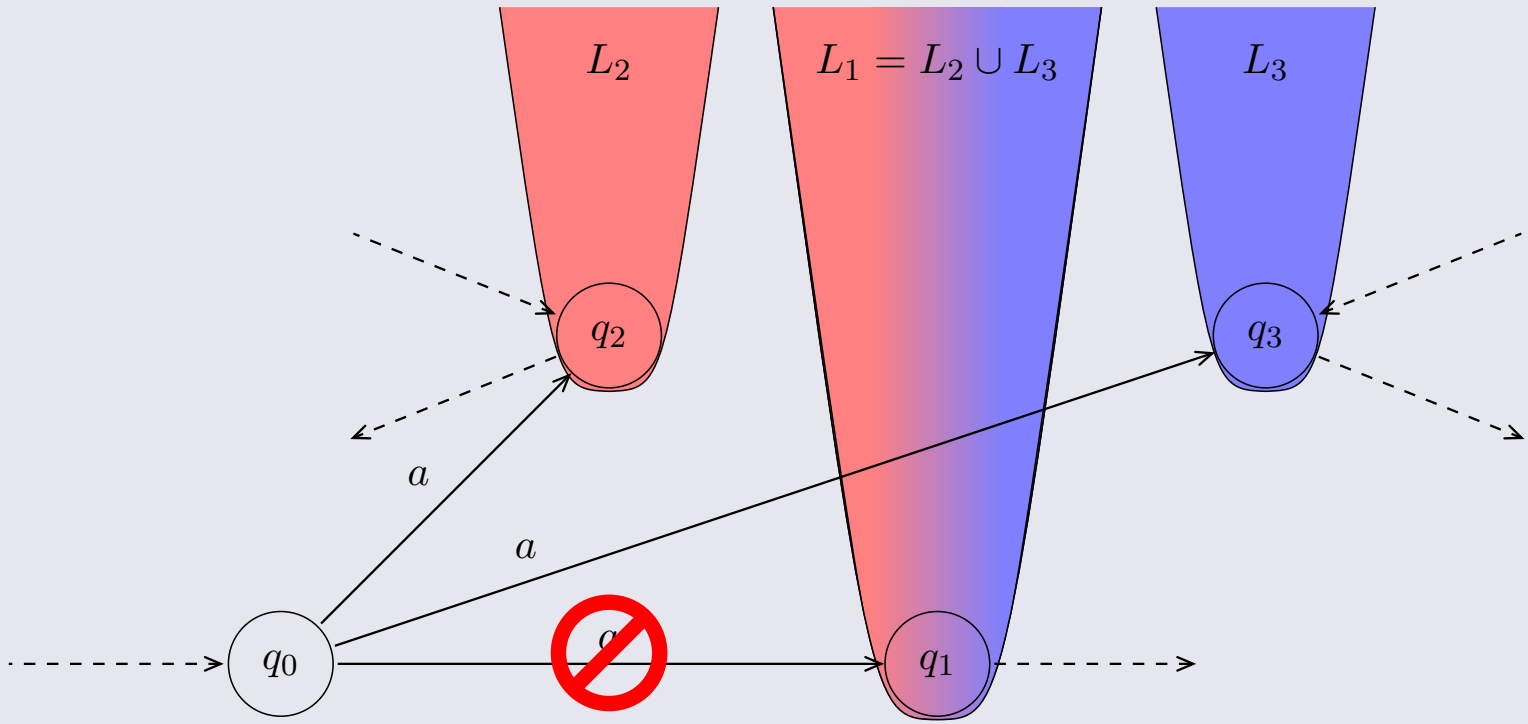

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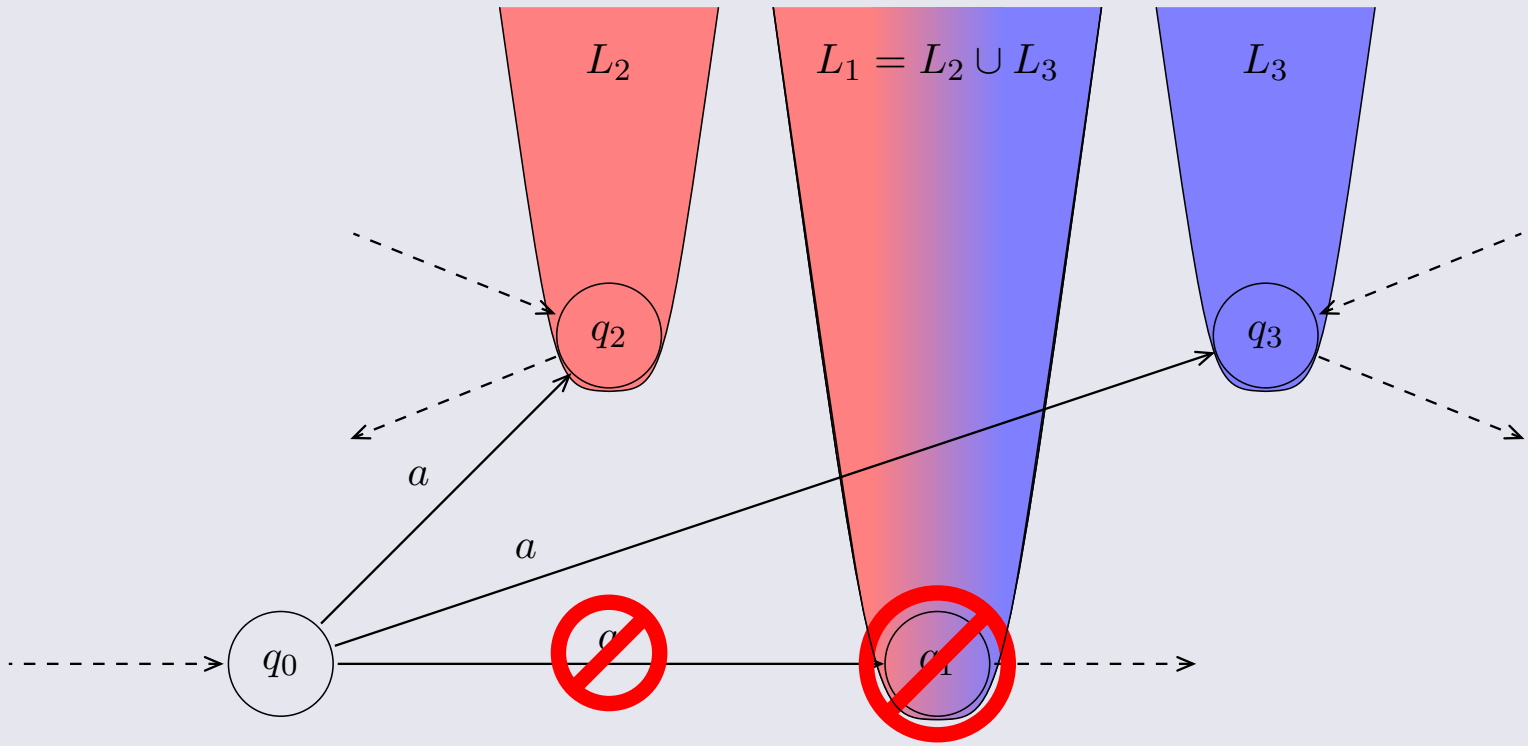

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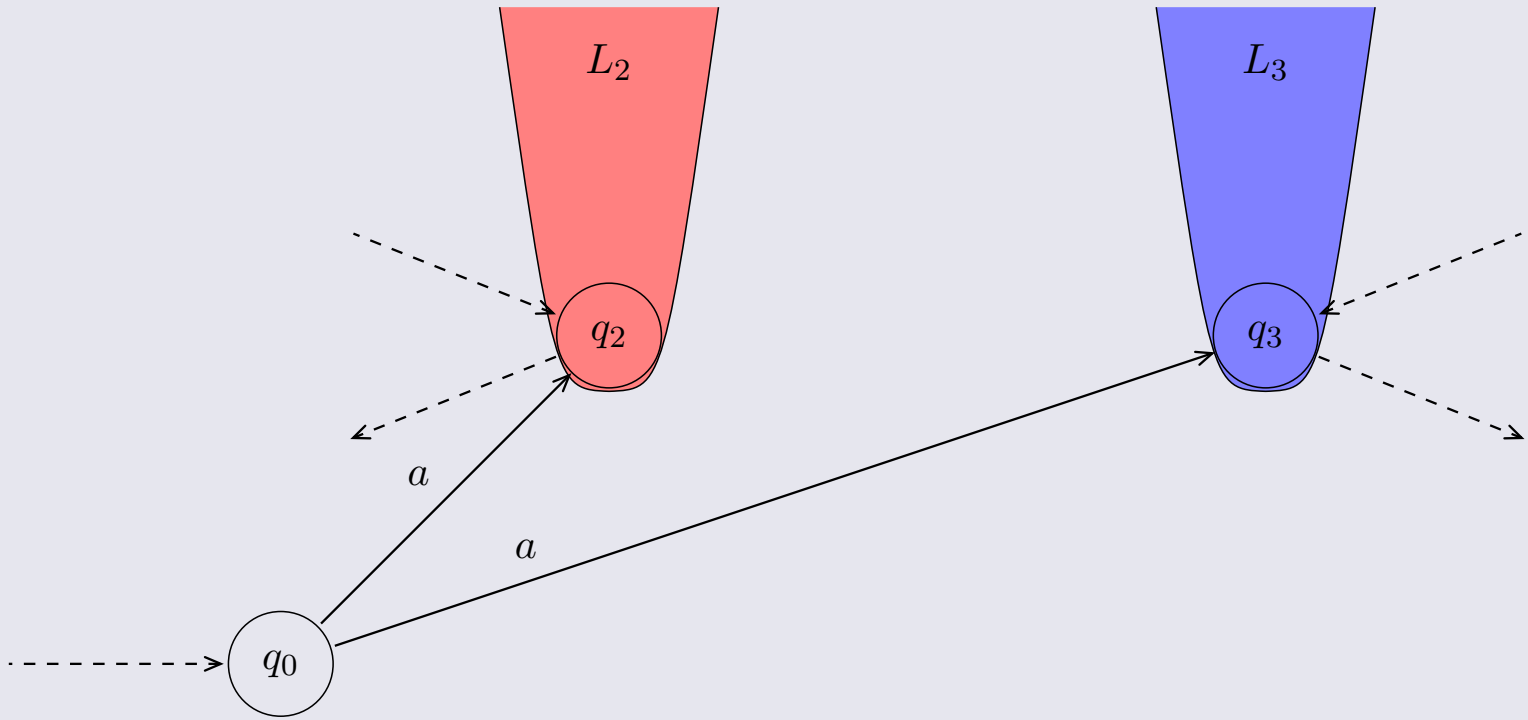
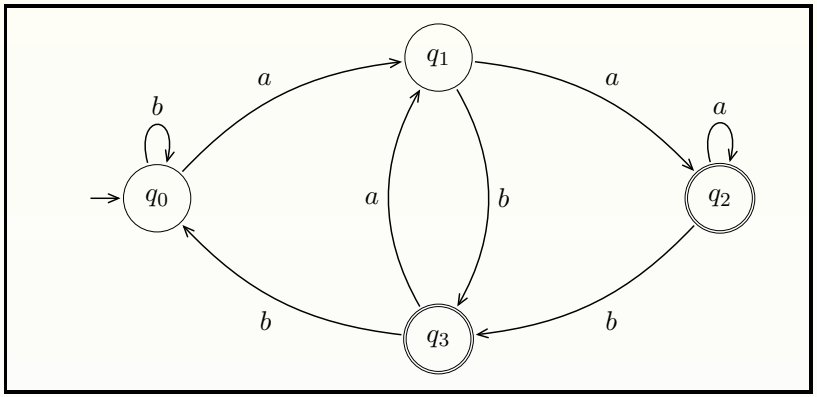


Example



Residual languages for $L = \Sigma^* a \Sigma$

$$L_{q_0} = \Sigma^* a \Sigma$$
$$L_{q_1} = \Sigma^* a \Sigma \cup \Sigma$$
$$L_{q_2} = \Sigma^* a \Sigma \cup \Sigma \cup \{\varepsilon\}$$
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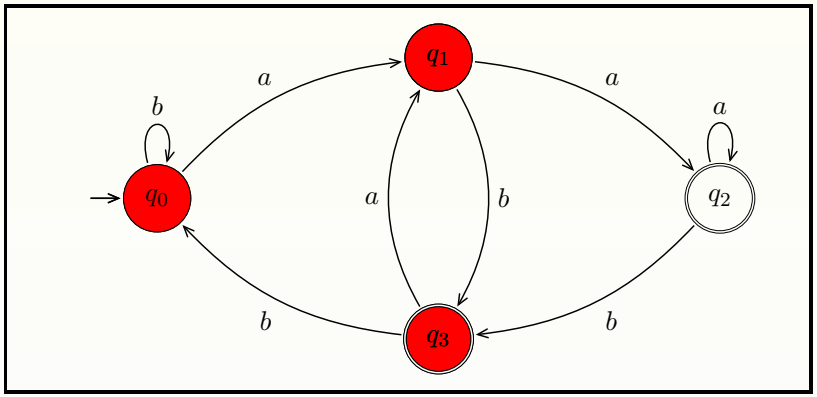

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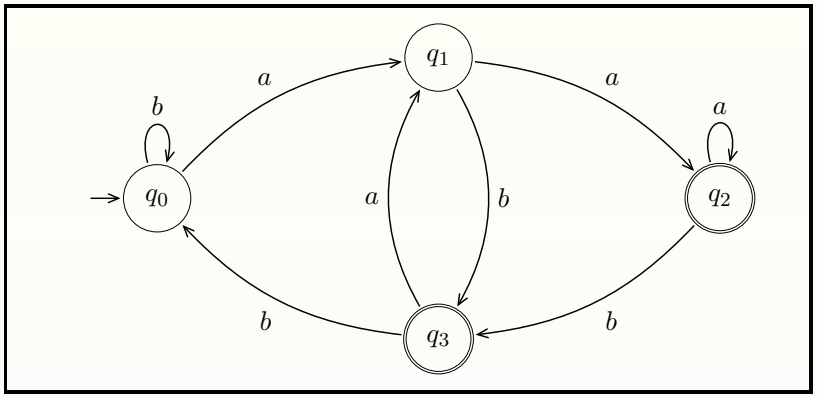

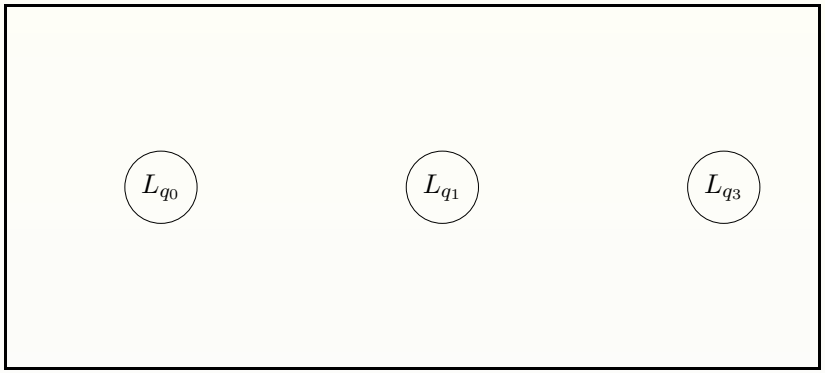

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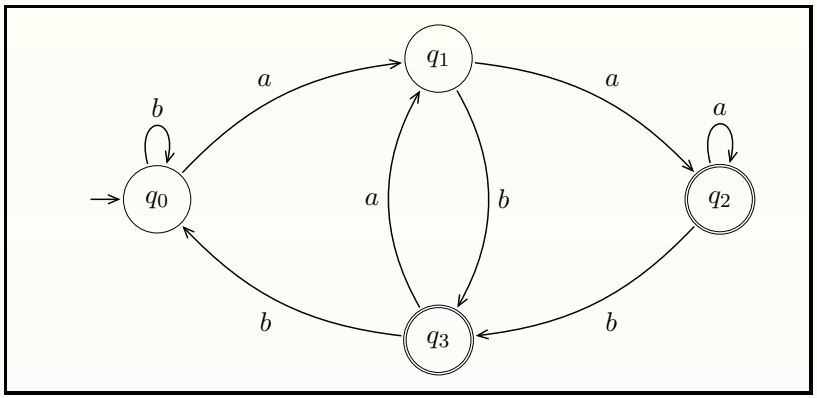

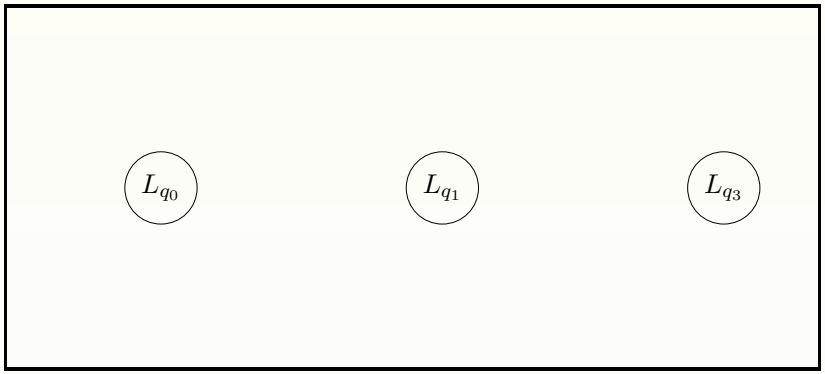

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$$L_{q_0} = \Sigma^* a \Sigma \qquad \text{(initial state)}$$

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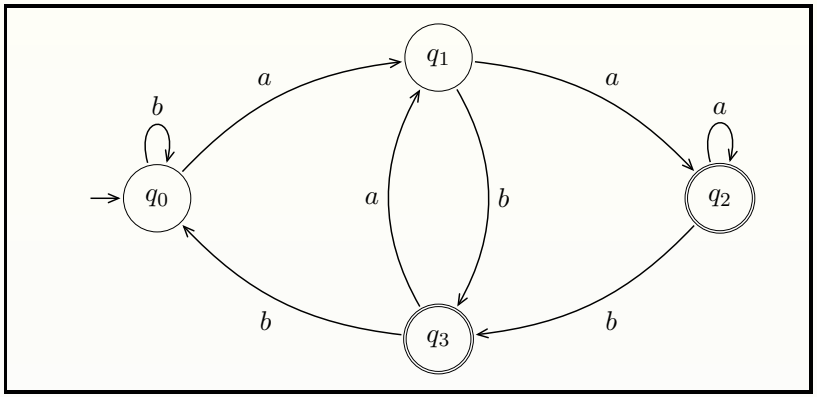

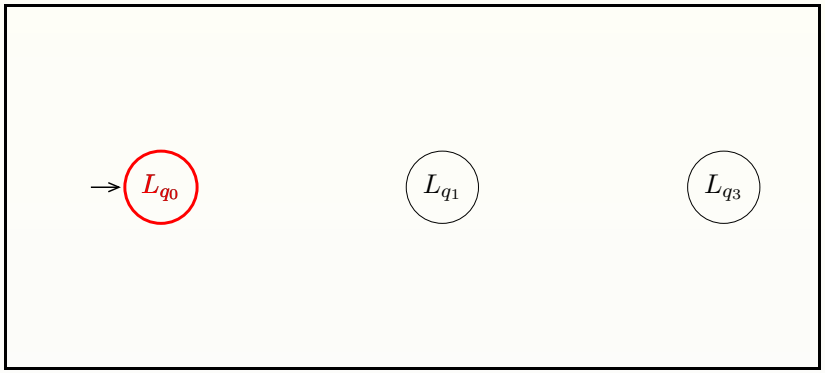

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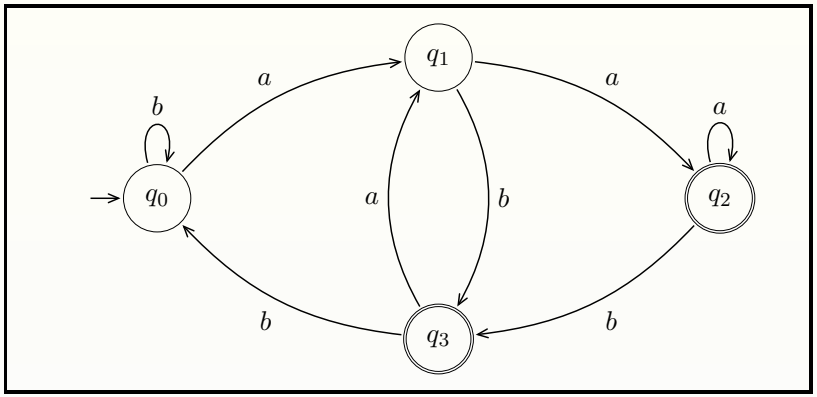

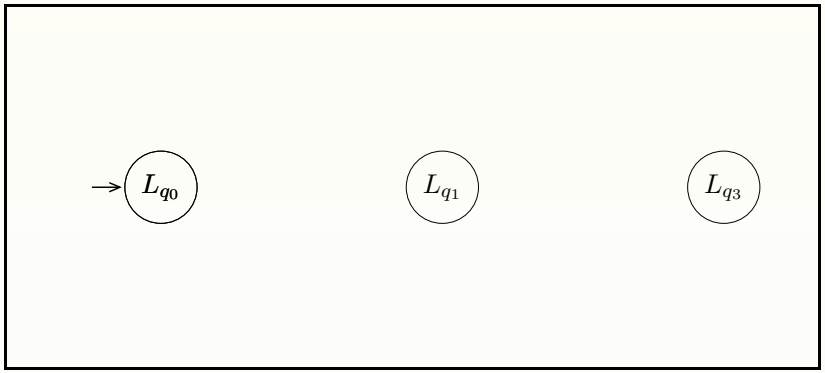

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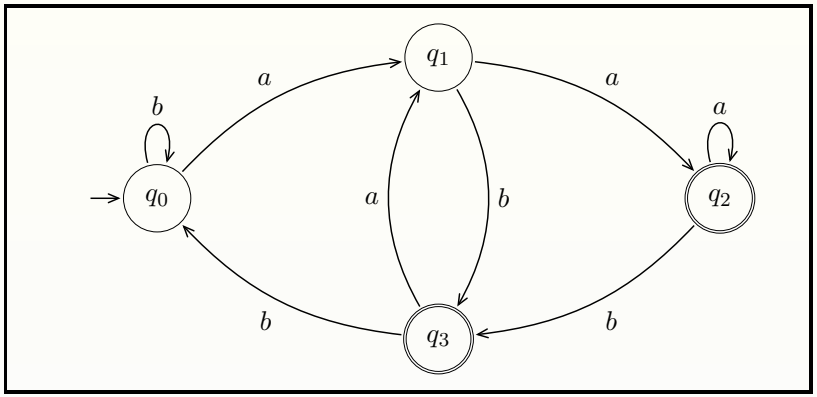
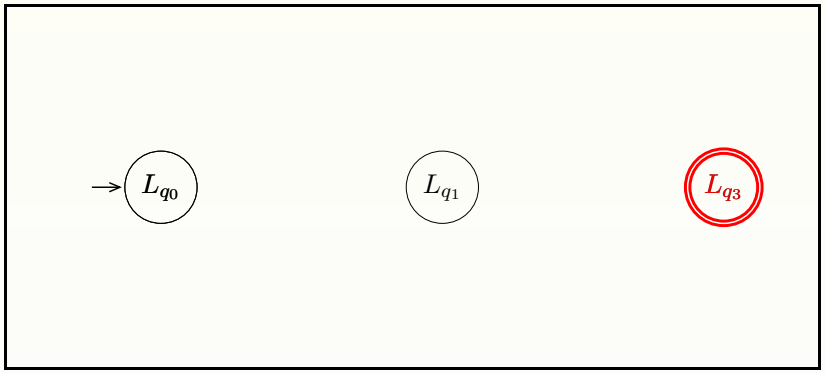
$$L_{q_3} = \Sigma^* a \Sigma \cup \{\varepsilon\} \qquad \text{(final state)}$$

Example





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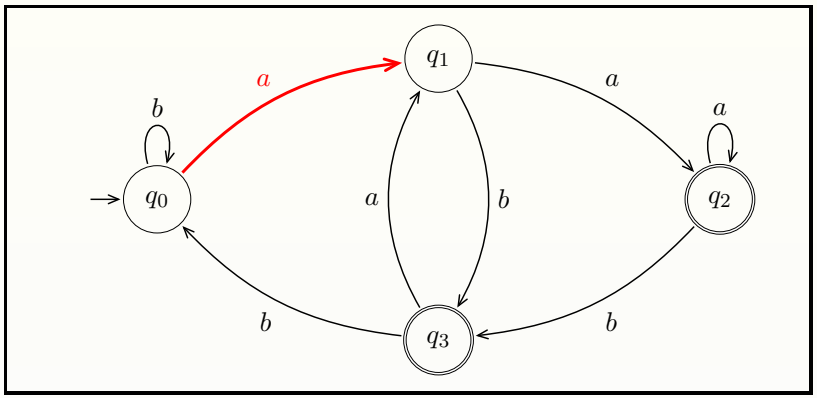

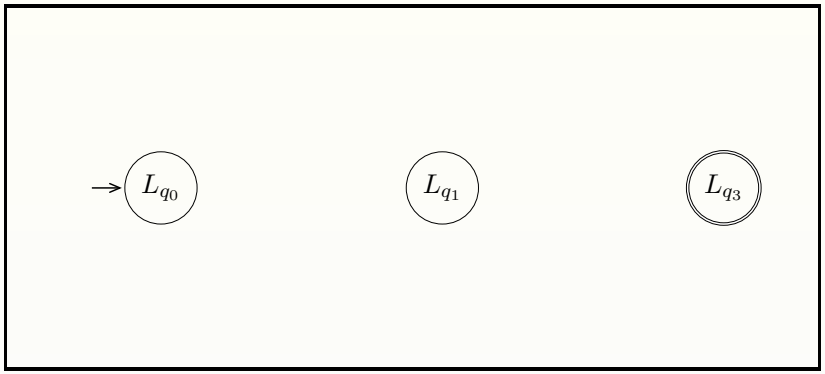

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$$L_{q_0} = \Sigma^* a \Sigma \qquad (a\text{-transitions})$$

$$L_{q_1} = \Sigma^* a \Sigma \cup \Sigma$$

$$L_{q_2} = \Sigma^* a \Sigma \cup \Sigma \cup \{\varepsilon\}$$

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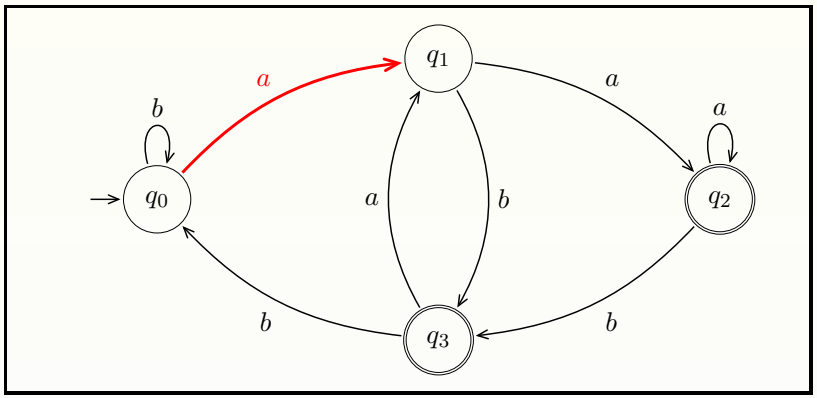
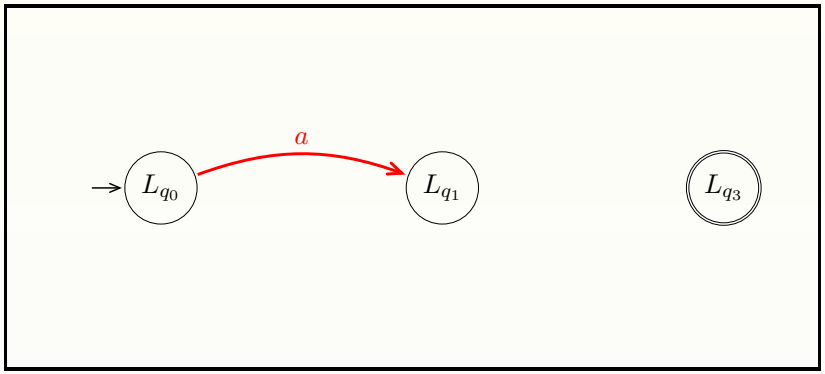
Example





Residual languages for $L = \Sigma^* a \Sigma$

$$L_{q_0} = \Sigma^* a \Sigma \qquad (a\text{-transitions})$$

$$L_{q_1} = \Sigma^* a \Sigma \cup \Sigma$$

$$L_{q_2} = \Sigma^* a \Sigma \cup \Sigma \cup \{\varepsilon\}$$

$$L_{q_3} = \Sigma^* a \Sigma \cup \{\varepsilon\}$$

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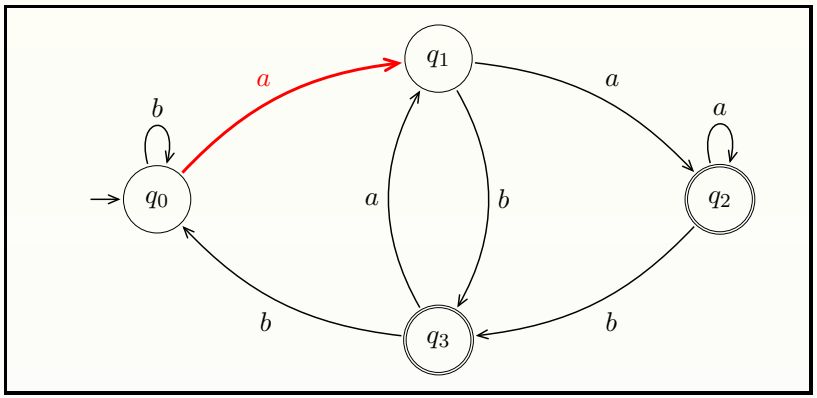

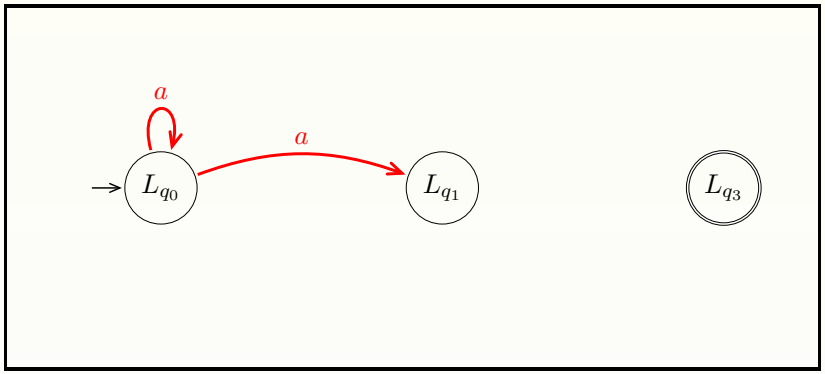

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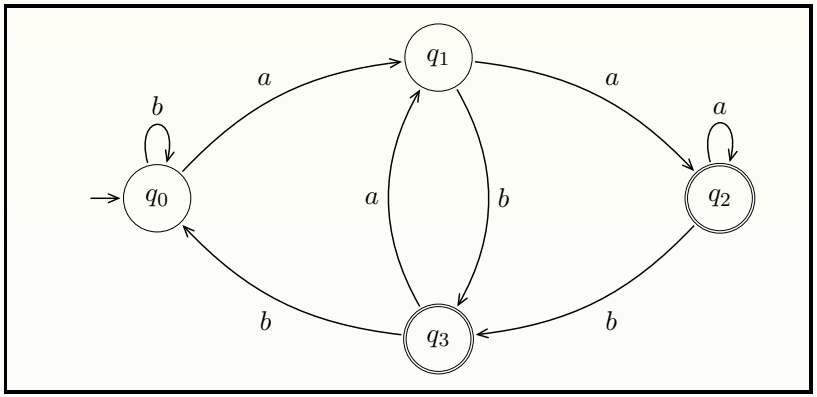

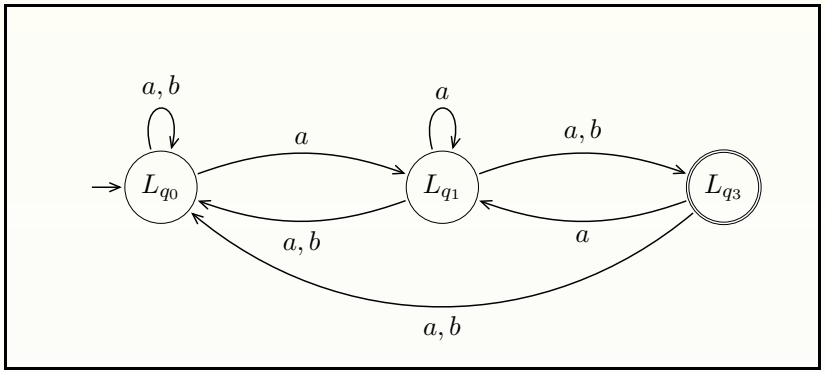

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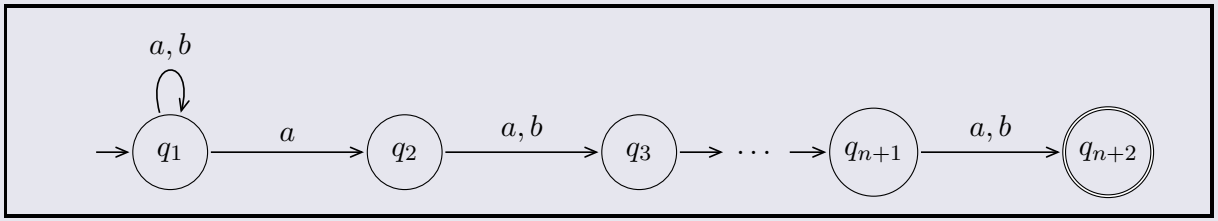
$L_n = \{w \in \Sigma^* | w$ has an a at the $(n+1)$-last position$\}$



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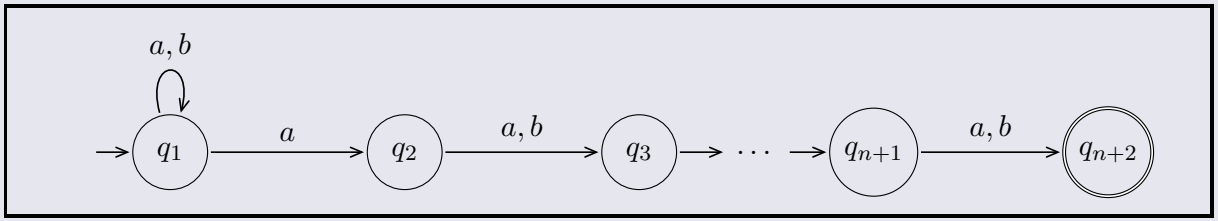
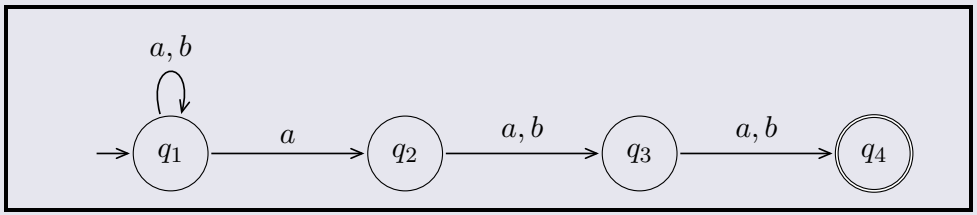
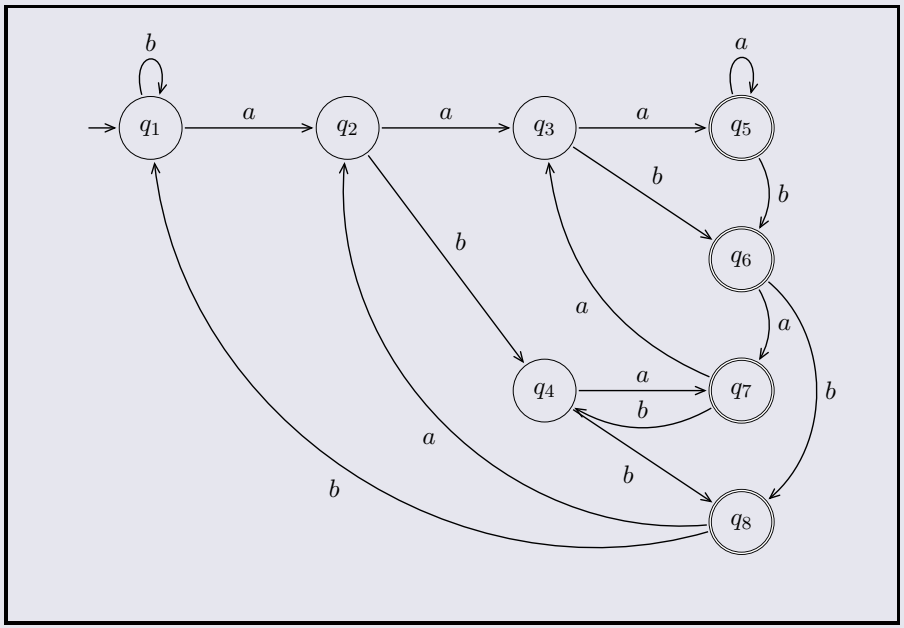


$L_2 = \{w \in \Sigma^* | w$ has an a at the 3^{rd}-last position$\}$



Minimal DFA and RFSA

Minimal DFA and RFSA for L_2:



Automata for language L_n:

- minimal DFA general case: 2^{n+1} states

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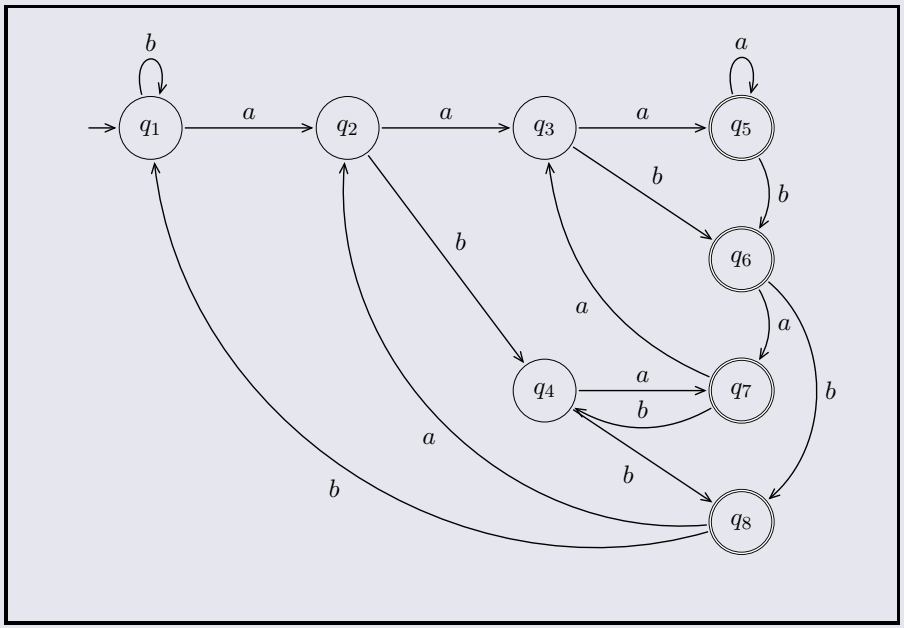

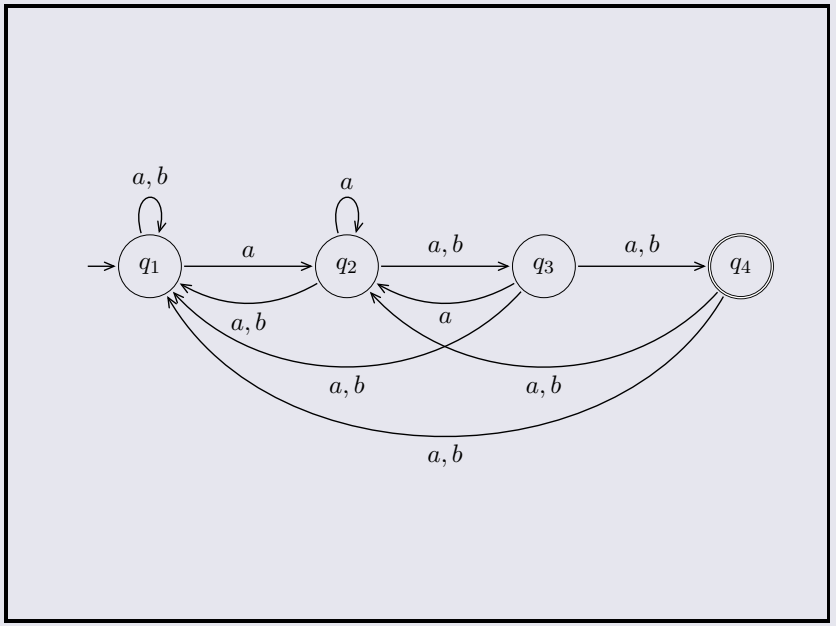

Automata for language L_n:

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- *canonical* RFSA general case: $n + 2$ states

Presentation outline

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Tables in NL*

$\mathcal{T}$	ε	a	aa
ε	$-$	$-$	$+$
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From tables to RFSA

- we deal with tables

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- we deal with tables
- table rows approximate residual languages
- not all rows represent states
- as long as there is no other evidence: equal rows represent equal residual languages
- transition relation respects language inclusion
- treatment of counterexamples:
  - add to columns
  - otherwise non-termination

Table properties

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Towards correctness

Theorem (Complexity of NL*)

Let:

- n: number of states of minimal DFA $\mathcal{A}_L$ for regular language L,
- m: length of the biggest counterexample

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the canonical RFSA $\mathcal{R}(L)$.

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- $O(m|\Sigma|n^3)$ membership queries

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Presentation outline

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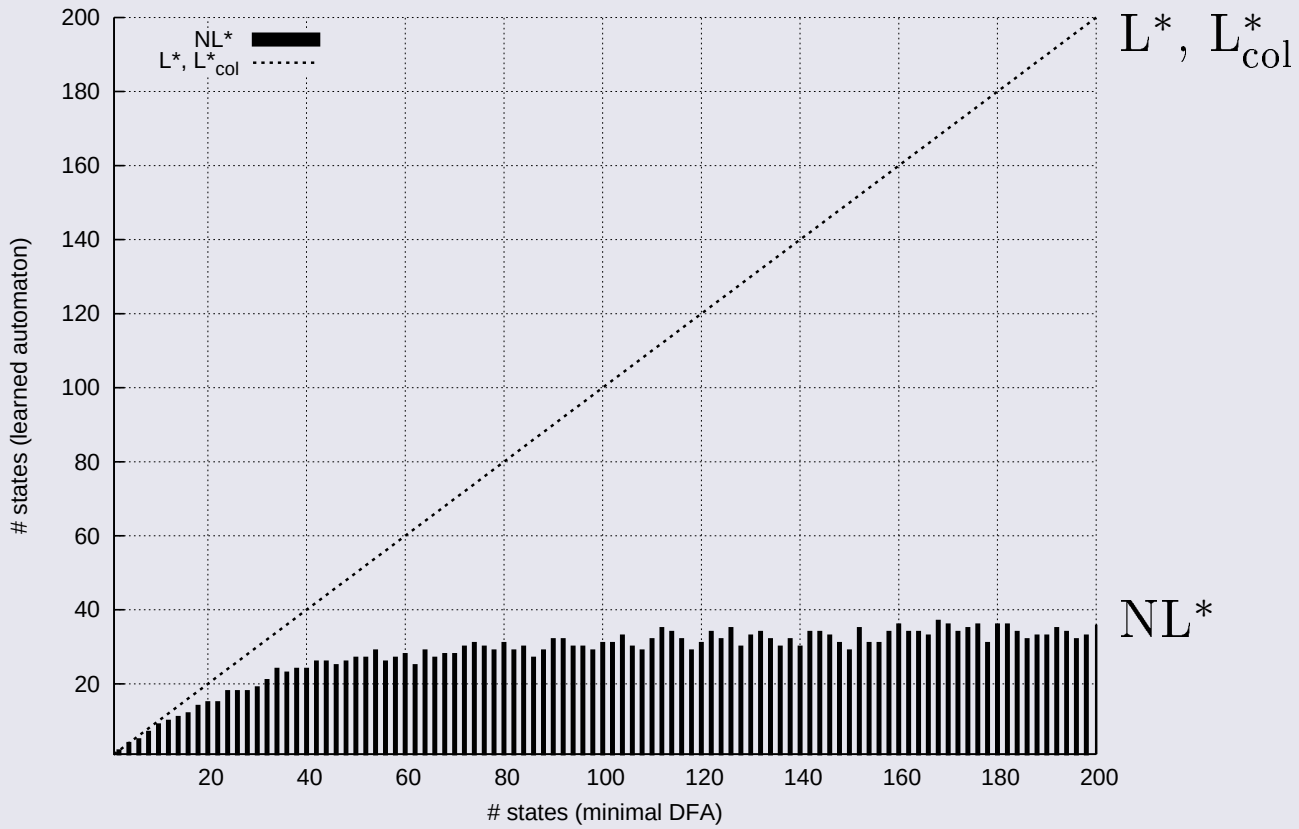
Algorithm - Overview

Number of states (L^*, L^*_{col} vs. NL^*)



- $\approx$ 3200 reg. exp. with minimal DFA of 1 to 200 states

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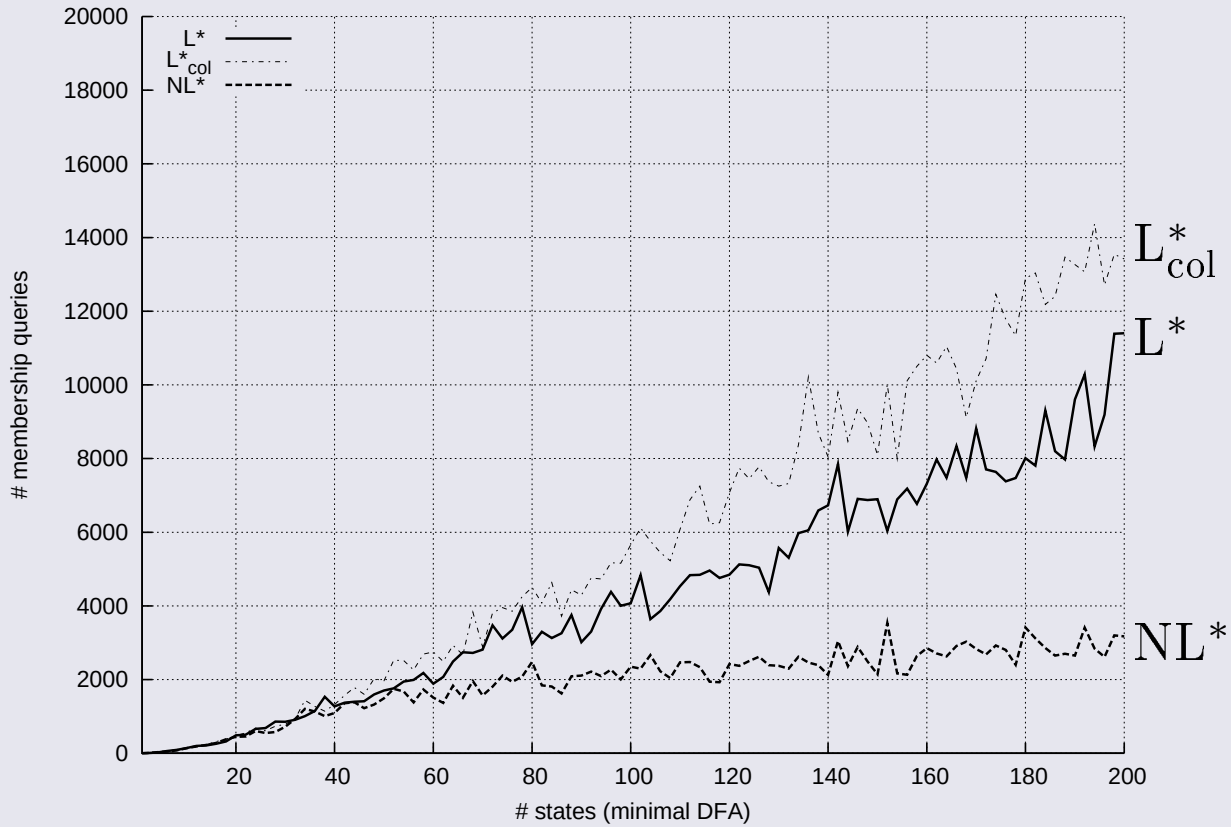

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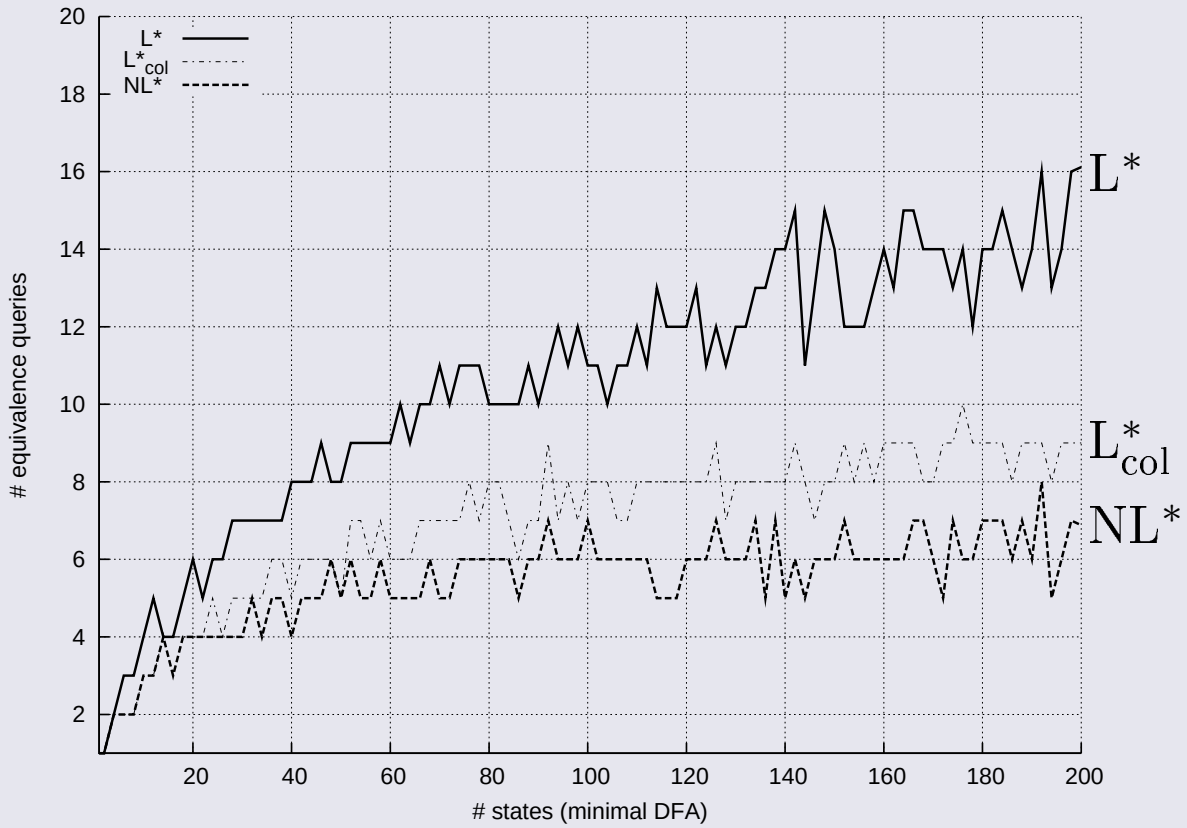

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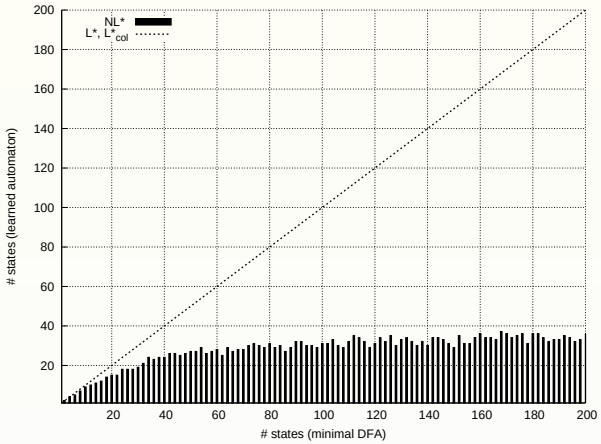

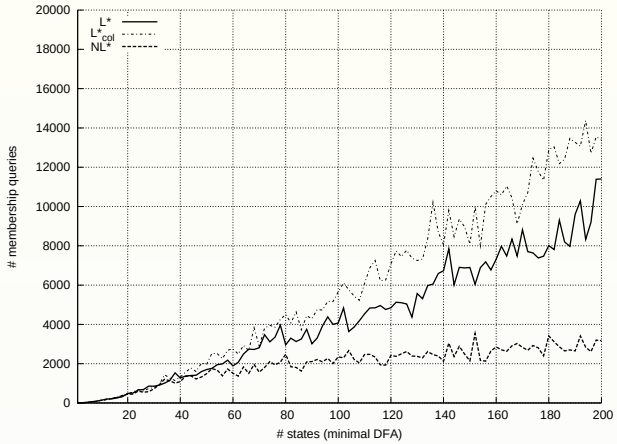

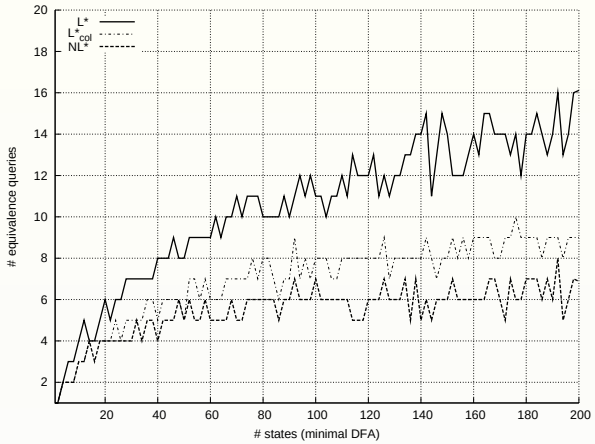

What did *we learn* ?

- NL* outperforms L* by far

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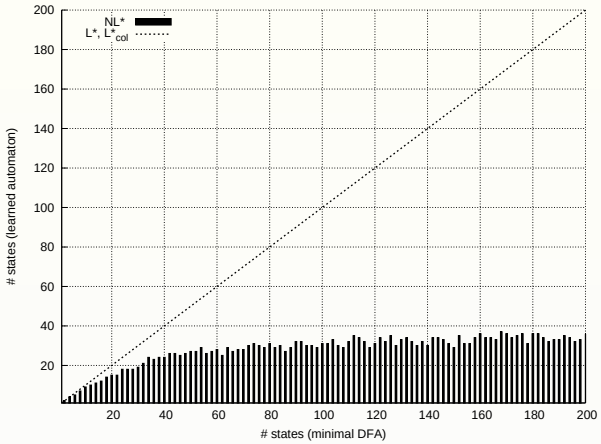

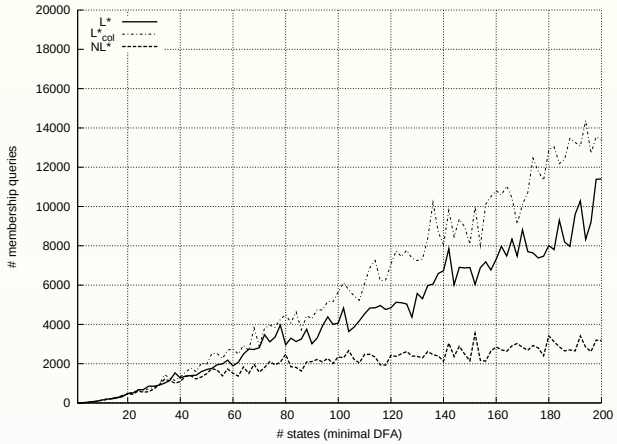

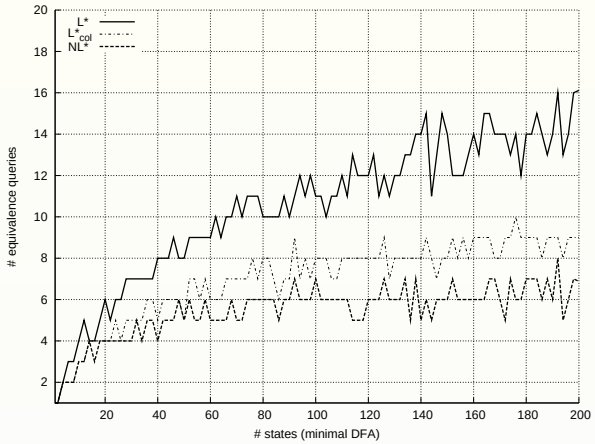

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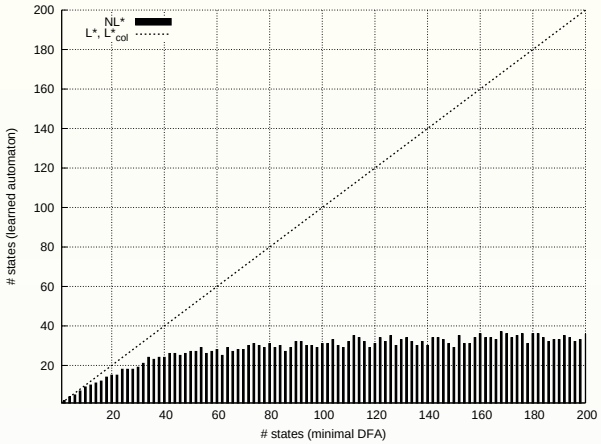

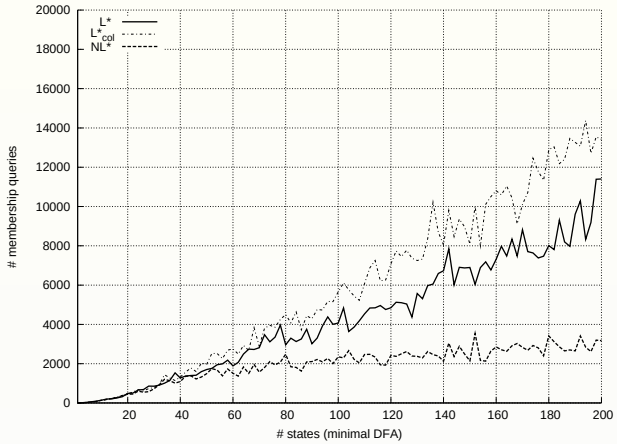

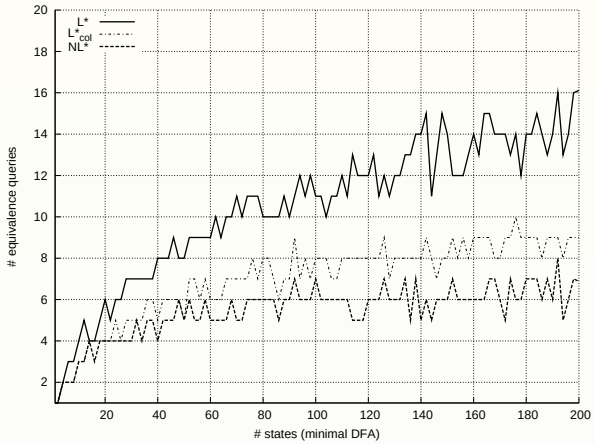

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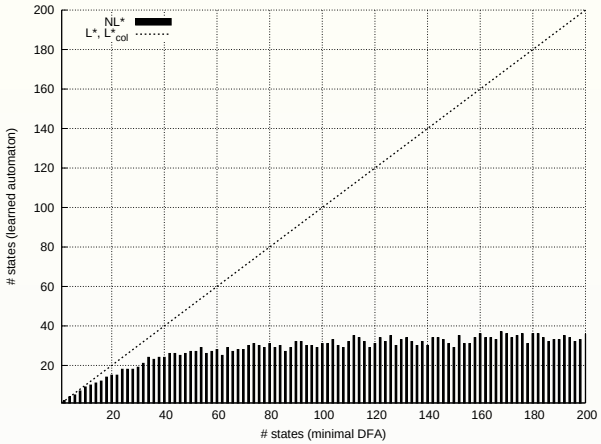

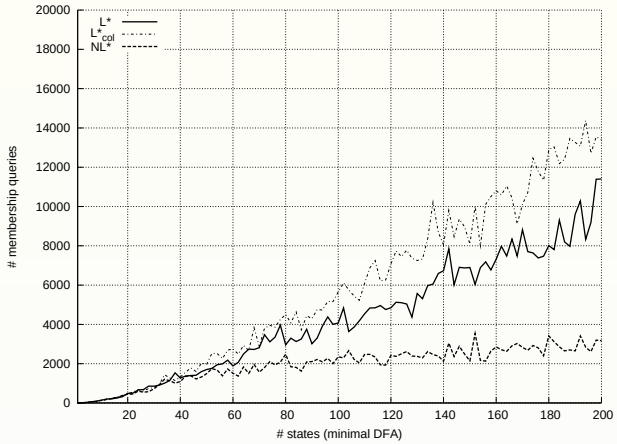

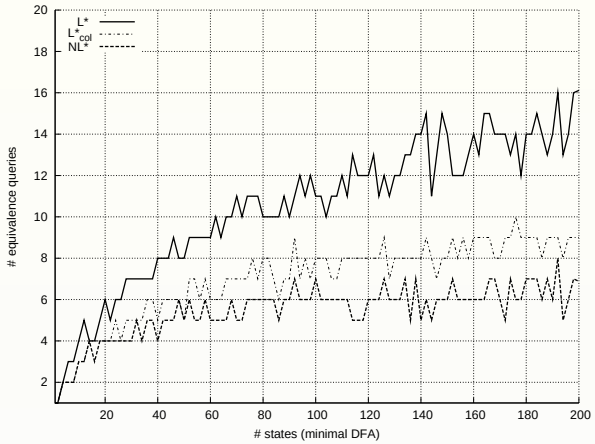

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