Software Modeling and Verification
Lehrstuhl für Informatik 2
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Foundations of the UML

Winter Term $07/08$

– Lecture 6: Regularity –

(February 8, 2008)
summarized by *Andreas Hackelöer*

Summary

- Conditions for (safe) realizability for languages have been obtained for a finite set of MSCs
- Checking realizability is coNP-complete; safe realizability is decidable in PTIME

But..

- Can results be obtained for a larger class of MSGs? E.g., MSGs that specify an infinite set of MSCs?
- What happens if we allow synchronization messages?
- How do we obtain an MPA that realizes an MSG?
- Conditions ABS, AB', AB are semantic notions. Can we find - possibly only necessary - simple syntactic conditions that guarantee realizability?

Regular MSCs

Let $Tr(M_i)$ be the traces of MSC M_i.

- The set of MSCs $\{M_1, .., M_k\}$, with possibly $k = \infty$, is regular if

$$\bigcup_{i=1}^{k} Tr(M_i)$$

  is a regular language.
- MSC G is regular if $Tr(G)$ is a regular language.
- MPA A is regular if $L(A)$ is regular.

Facts:

Every $\forall$-bounded MPA is regular.

The decision problem "Is $L \subseteq Act^\star$ $with$ L $regular$ realizable by a set of MSCs?" is decidable.

I.e., do MSCs $M_1, .., M_k$ exist such that $\bigcup_{i=1}^{k} Tr(M_i) = L$ for a regular language L?

Proof:

Let $L \subseteq Act^\star$ be regular. Let DFA $A = (S, \Sigma, s_0, \delta, F)$ be the minimal DFA for L, i.e. $L(A) = L$. Assume w. log. that A has no dead states (i.e., from every $s \in S$, it is possible to reach some $s' \in F$). Associate with each state $s \in S$ a channel-capacity function $cp : S \times Ch \rightarrow \mathbb{N}$ satisfying:

1. $cp(s, c) = 0$ if $s = s_0$ or $s \in F$, for any $c \in Ch$

2. If $\delta(s, p!q) = s$ then $cp(s', c) = \begin{cases} cp(s,c)\ if\ c \neq (p,q) \\ cp(s,c)+1\ if\ c = (p,q) \end{cases}$

3. If $\delta(s, q?p) = s'$ then $cp(s', c) = \begin{cases} cp(s,c)\ if\ c \neq (p,q) \\ cp(s,c)-1\ if\ c = (p,q) \end{cases}$

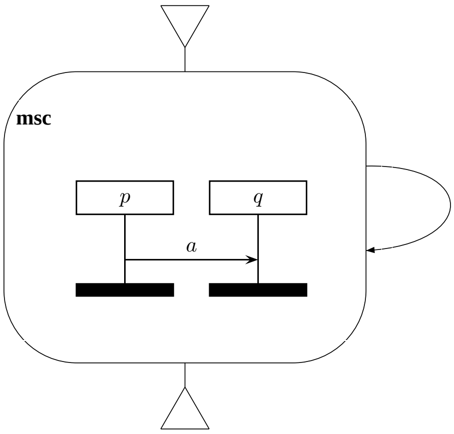
4. If $\delta(s, a) = s_1$ and $\delta(s_1, b) = \underline{s_2}$ with $a \in \Sigma_p$, $b \in \Sigma_q$, $p \neq q$ then
if not $(a = p!q\ and\ b = q?p)$ or $cp(s, (p, q)) > 0$ then:
$\exists s' \in S$: $\delta(s, b) = s_1'$ and $\delta(s_1', a) = \underline{s_2}$.

Claim: $L \subseteq Act^\star$ with L regular is realizable by a set of MSCs if and only if there exists function cp satisfying 1-4.

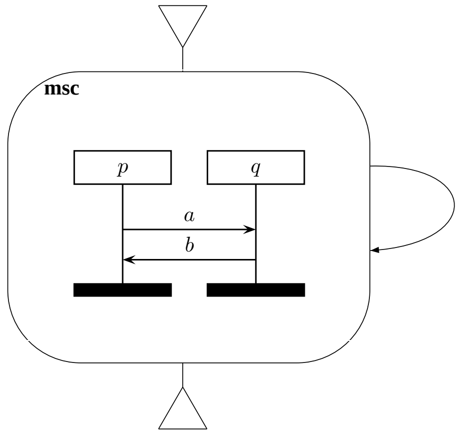
But..

The decision problem "Is MSG G regular?" is undecidable [Henriksen et al., 2005]

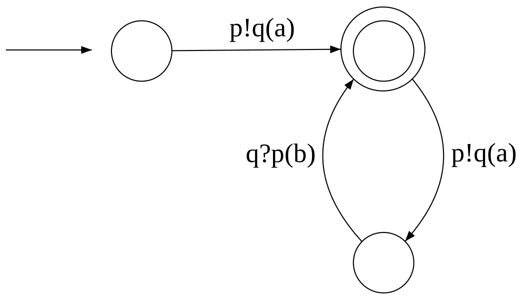
Examples

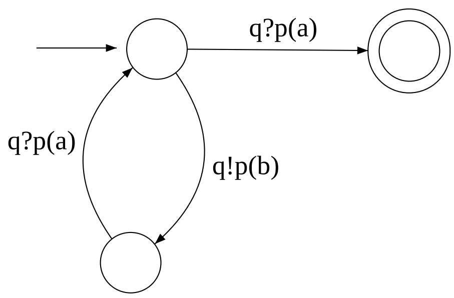


MSG G_1: Not regular
$Tr(G_1) =$ Dyck language



MSG G_2: Not regular
$Tr(G_2) = \big(p!q(a)\ q?p(a)\ q!p(b)\ p?q(b)\big)$





Regularity and Realizability

$L \subseteq Act^\star$ is an <u>MSC language</u> if

$$L = Tr(M_1, ..., M_k) = \bigcup_{i=1}^{k} Tr(M_i)$$

for some MSCs M_i.

Then ([Henriksen et al. 03]:

L is a regular language <u>if and only if</u> L is realizable by a $\forall$-bounded deterministic MPA.

An MPA A is $\forall$-bounded if there exists $B \in \mathbb{N}$ such that for each reachable configuration of A, $|\eta(c)| \leq B$ for each channel c.

Equivalently: Any MSC in $L(A)$ is ∀B - bounded.

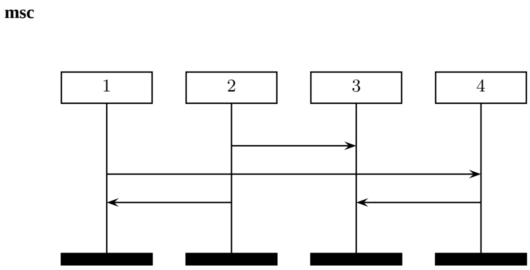
Regular MSGs

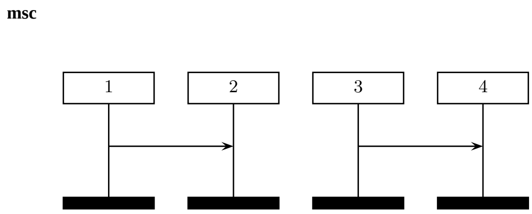
- MSG G is regular if $Traces(G)$ is a regular language
- The decision problem "Is MSG G regular?" is undecidable
- Now try to impose structural conditions on G that guarantee that G is regular
- Use: So-called communication graph

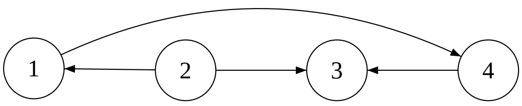
The communication graph of the MSC $M = (P, E, \mathbb{C}, l, m, <)$ is the digraph $(V, \to)$ with:

- $V = P \setminus \underbrace{\{p \in P | E_p = \varnothing\}}_{"inactive"processes}$
- $p \to \underline{q}$ if and only if for some $a \in \mathbb{C}$ $\begin{cases} p!\underline{q}(a) \in E \ or \\ \underline{q}?p(a) \in E \end{cases}$

Examples:



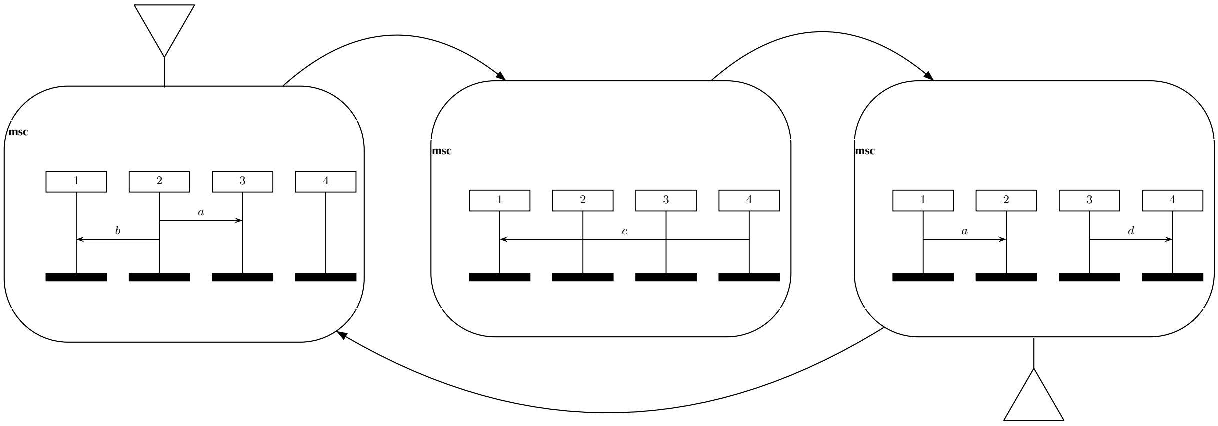




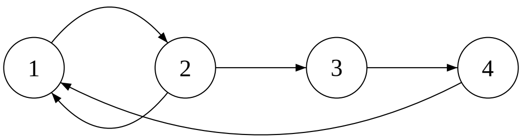
Communication-closed MSGs

MSG G is communication-closed if for any loop $\underbrace{v_1, v_2, .., v_n}_{=\pi}$ in G, the MSC $M(\pi)$ has a strongly connected communication graph.

E.g.,



is communication-closed since for the only loop, the communication graph is:



which is strongly connected.

Note: Checking whether MSG G is communication-closed is in PTIME (determine all loops in G, construct for every loop its communication graph, check strong connectedness).

Communication-Closedness vs. Regularity

For any MSG G: G is communication-closed $\Rightarrow$ G is regular

Notes:

1. The reverse does not hold (see example below);
2. Not every language can be represented by an MSG (see Yannakakis exam).

But:

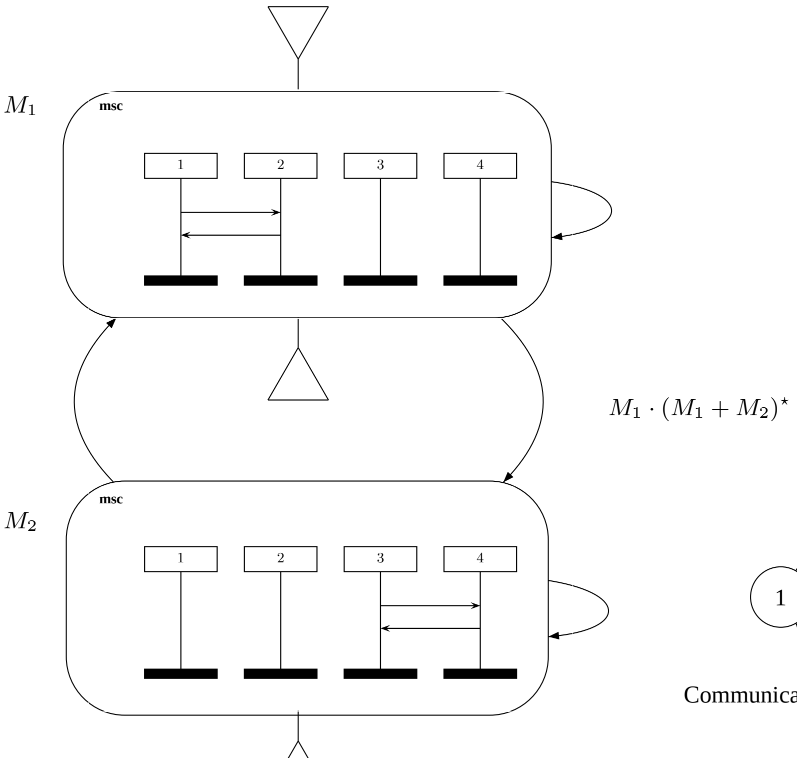
For a set of MSCs $M_1, .., M_k | k\ possibly\ infinite$ which is regular:

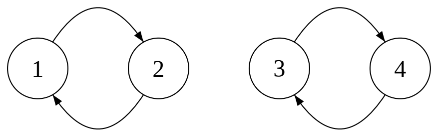
L can be represented by an MSG
if and only if
L can be represented by an communication-closed MSG

where

$$L = \bigcup_{i=1}^{k} Tr(M_i)$$

Communication-closedness is a sufficient, but not necessary condition for regularity. Consider the following graph G:





Communication graph not communication-closed.

But $Tr(G)$ is regular:

$(1!2\ 2?1\ 2!1\ 1?2) \cdot (1!2\ 2?1\ 2!1\ 1!2\ + 3!4\ 4?3\ 4!3\ 3?4)^\star$

Summary of Realizability

	finite MSG*	comm.-closed MSG	general MSG	
realizability	coNP-complete	undecidable	undecidable	FIFO communication
safe realizability	PTIME	EXPSPACE-complete	undecidable	FIFO communication
realizability	coNP-complete	PSPACE-hard	undecidable	non-FIFO communication
safe realizability	PTIME	EXPSPACE-complete	undecidable	non-FIFO communication

* G is finite if the number of MSGs defined by G is finite, i.e., $L(G)$ is finite.

Regular Expressions over MSCs

Let $M \in \mathbb{M}$ be an MSC.

The set of regular expressions over $\mathbb{M}$ is given by the grammar:

$$\alpha := \varnothing | M | \alpha_1 * \alpha_2 | \alpha_1 + \alpha_2 | \alpha^\star$$

Semantics of regular expressions is given by

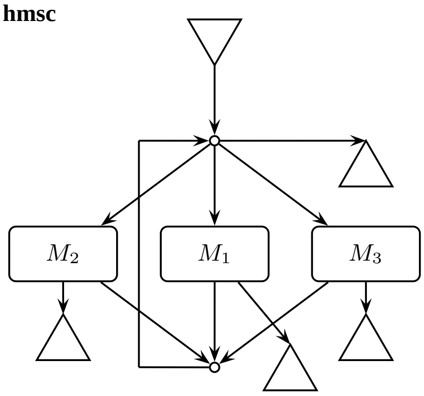
$$L: \ RegExp_{\mathbb{M}} \longrightarrow 2^{\mathbb{M}}$$

and defined as:

$L(\varnothing) = \varnothing$
$L(M) = \{M\}$
$L(\alpha_1 * \alpha_2) = L(\alpha_1) * L(\alpha_2)$, a concatenation of sets of MSCs
$L(\alpha_1 + \alpha_2) = L(\alpha_1) \cup L(\alpha_2)$
$L(\alpha^\star) = L(\alpha)^\star$, where * is the Kleene star over sets of MSCs

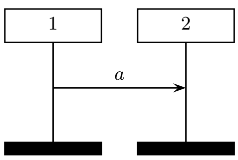
E.g.,$(\{M_1, M_2, M_3\})^\star$ is to be read as:

hmsc

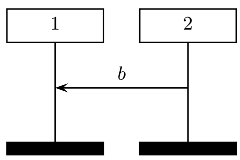


Regular Expressions for MSCs

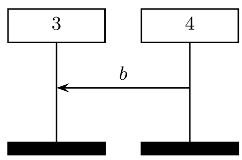
msc A



msc B



msc C



Consider the regular expressions:

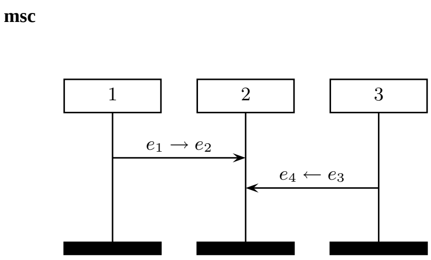
$\alpha_1 = (A * B)^\star$	deterministic safe product MPA $\forall$1-bounded
$\alpha_2 = (A + B)^\star$	deterministic $\exists$1-bounded MPA
$\alpha_3 = (A * C)^\star$	not realizable
$\alpha_4 = A * (A + B)^\star$	$\exists$1-bounded safe MPA

How about the realizability of $L(\alpha_i)$?

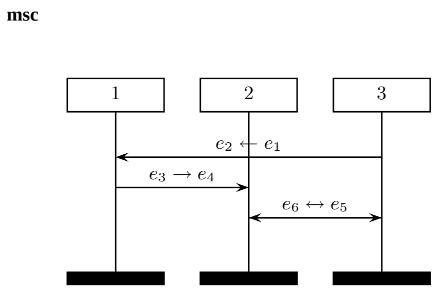
(Note: All realizable $L(\alpha_i)$ have as possible realization a locally accepting MPA, i.e., $F = \prod_{p \in P} F_p$ for some $F_p \subseteq S_p$)

Can we obtain a simple criterion on regular expressions that guarantees realizability?

- MSC $M = (P, E, \mathbb{C}, l, m, <)$ is connected if: $\forall e, e\prime \in E : e <^+ e\prime \ or \ e\prime <^+ e$



Not connected, since $e_1 \not\leq^+ e_3 \wedge e_2 \not\leq^+ e_1$



Connected

- Regular expression α is <u>connected</u> if for any subexpression $\beta^\star$ of α, $L(\beta)$ is a set of connected MSCs
- Let $\underbrace{\{M_i, .., M_k\}}_{D}$ be MSCs such that $D \subseteq E^\star$ for some finite set E of MSCs.

  Then [Genest et al. 2006]:

D is realizable if and only if there exists a <u>connected</u> regular expression α such that $L(\alpha) = D$.