FOUNDATIONS OF THE UML

lecture 6 : REGULARITY

Joost-Pieter Katoen

MOVES @ RWTH

moves.rwth-aachen.de/i2/192

WS 2007 / 2008

December 3, 2007

SUMMARY

- Conditions for (safe) realizability for languages obtained by finite set of MSCs
- Checking realizability is coNP-complete; safe realizability is in decidable in PTIME

But......

- Can results be obtained for larger class of MSGs ? e.g., certain infinite ones?
- What happens if we allow synchronization messages ?
- How do we obtain an MPA that realizes an MSG ?
- Conditions ABS, AB′, AB are semantic notions. Can we find — possibly only necessary — simple syntactic conditions that guarantee realizability ?

REGULAR MSCs

- The set of MSCs $\{M_1, \ldots, M_k\}$ is <u>regular</u> if $\bigcup_{i=1}^{k} Tr(M_i)$ is a regular language.

  possibly $k = \infty$!

  traces

- MSG G is <u>regular</u> if Tr (G) is a regular language.
- MPA A is <u>regular</u> if L(A) is regular

Every ∀bounded MPA is regular

The decision problem "is $L \subseteq Act^*$ with L regular realizable by a set of MSCs?" is <u>decidable</u>

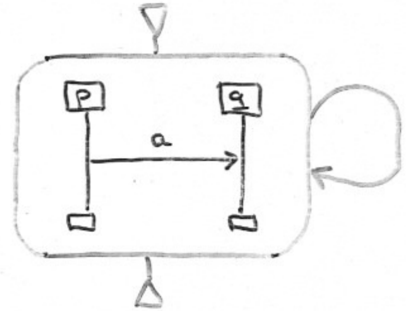
i.e., do MSCs $M_1, \ldots, M_k$ exist such that $\bigcup_{i=1}^{k} Tr(M_i) = L$ for regular language L?

But:

The decision problem "is MSG G regular"? is undecidable [Henriksen et al. 2005]

EXAMPLES

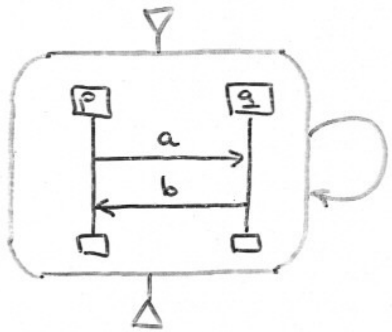
G_1:



not regular

$Tr(G_1)$ = Dyck languag

G_2:

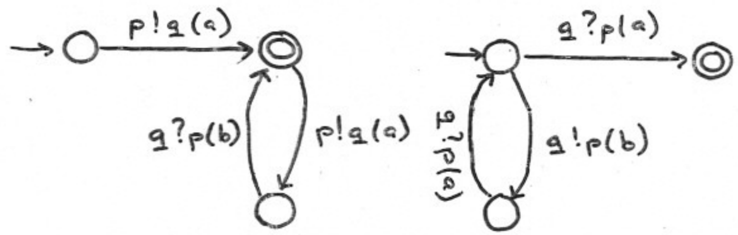


regular

Tr G_2) =

$(p!q(a)\ q?p(a)\ q!p(b)\ p?q(b))^*$

A:



is regular

is ∀3-bounded

REGULARITY AND REALIZABILITY

$L \subseteq Act^*$ is an MSC language if

$L = Tr(\{M_1, \ldots, M_k\})$ for some MSCs M_i.

Then: [Henriksen et. al '03]:

$$\boxed{\begin{array}{c} L \text{ is a regular MSC language} \\ \text{iff} \\ L \text{ is realizable by a } \forall\text{-bounded } \overbrace{\text{MPA}}^{\text{deterministic}} \end{array}}$$

MPA A is $\forall$-bounded if there exists $B \in \mathbb{N}$ such that for each reachable configuration of A, $|\eta(c)| \leq B$ for each channel c

(equivalently: any MSC in $L(A)$ is $\forall B$-bounded)

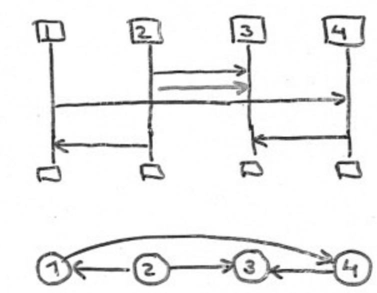
REGULAR MSGs

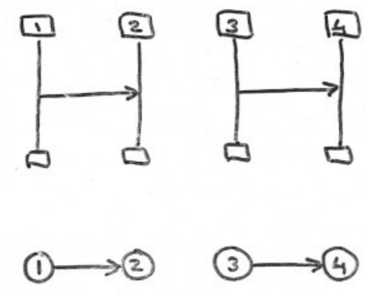
- MSG G is regular if Traces (G) is a regular language
- "is MSG G regular?" is undecidable
- Now try to impose structural conditions on G that guarantee that G is regular
- Use: so-called communication graph

The communication graph of the MSC $M = (P, E, \mathbb{C}, \ell, m, <)$ is the digraph $(V, \rightarrow)$

with:
- $V = P \setminus \underbrace{\{p \in P \mid E_p = \emptyset\}}_{\text{"inactive" processes}}$
- $p \rightarrow q$ iff $\left.\begin{array}{l} p!q(a) \in E \\ q?p(a) \in E \end{array}\right\}$ for some $a \in \mathbb{C}$

Examples:

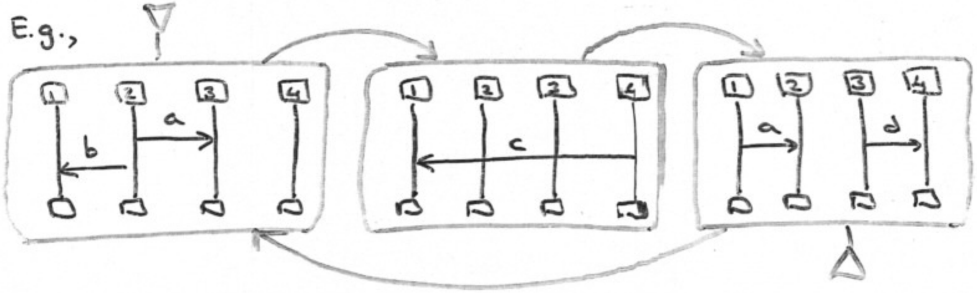




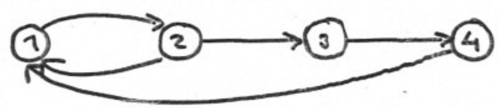
COMMUNICATION - CLOSED MSGs

MSG G is communication-closed if for any loop $\underbrace{v_1 v_2 \ldots v_n}_{=\pi}$ in G, the MSC $M(\pi)$ has a strongly connected communication graph.

E.g.,



is communication-closed since for the only loop, the communication graph is:



is strongly connected

Note: checking whether MSG G is communication closed is in PTIME (determine all loops in G, all graphs, check strong connectedness)

COMMUNICATION-CLOSED VS. REGULARITY

For any MSG G:

G is communication-closed $\rightarrow$ G is regular

Notes: 1. the reverse does not hold;

2. not every regular language can be represented by an MSG (see Yannakakis examp.

But:

For $\underbrace{}_{\text{set of}}$ MSCs $\{M_1, \ldots, M_k\}$ which is regular: possibly infinite

L can be represented by an MSG

iff

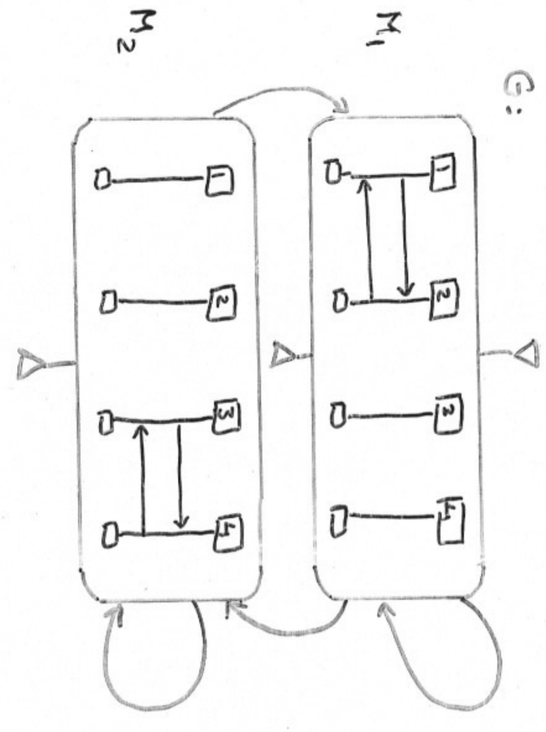
L can be represented by a comm.-closed MSG

where $L = \bigcup_{i=1}^{k} Tr(M_i)$

COMMUNICATION-CLOSEDNESS VS. REGULARITY

Communication closedness is a sufficient, but <u>not</u> necessary condition for regularity

G:



$M_1 \cdot (M_1 + M_2)^*$

Communication graph:

Not communication closed.

But Tr(G) is regular:

$(1!2\ 2?1\ 2!1\ 1?2)\,.$

$(1!2\ 2?1\ 2!1\ 1?2\ +\ 3!4\ 4?3\ 4!3\ 3?4)^*$

SUMMARY OF REALIZABILITY

[Alur et al. '00]
[Lohrey '03]

	finite MSG*	comm. - closed MSG	general MSG	
realizability	coNP - complete	undecidable	undecidable	FIFO communication
safe realizability	PTIME	EXPSPACE complete	undecidable	
realizability	coNP - complete	PSPACE hard	undecidable	<u>non</u>-FIFO communication
safe realizability	PTIME	EXSPACE complete	undecidable	

* G is finite if #MSCs defined by G is finite, i.e., L(G) is finite

REGULAR EXPRESSIONS OVER MSCs

Let $M \in \mathbb{M}$ be an MSC

The set of regular expressions over $\mathbb{M}$ is given by the grammar:

$$\alpha ::= \emptyset \mid M \mid \alpha_1 \cdot \alpha_2 \mid \alpha_1 + \alpha_2 \mid \alpha^*$$

Semantics of regular expressions is given by $L : RegExp_{\mathbb{M}} \longrightarrow 2^{\mathbb{M}}$ and defined as:

$L(\emptyset) = \emptyset$

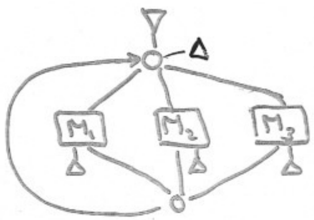
$L(M) = \{M\}$

$L(\alpha_1 \cdot \alpha_2) = L(\alpha_1) \cdot L(\alpha_2)$ ← concatenation of sets of MSCs

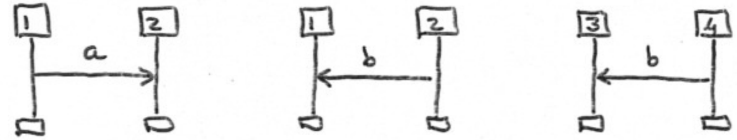
$L(\alpha_1 + \alpha_2) = L(\alpha_1) \cup L(\alpha_2)$

$L(\alpha^*) = L(\alpha)^*$ ← Kleene star over sets of MSCs

e.g. $(\{M_1, M_2, M_3\})^*$ is to be read as:



REGULAR EXPRESSIONS FOR MSCs



A B C

Consider the regular expressions:

$\alpha_1 = (A \cdot B)^*$ deterministic safe product MPA $\forall 1$-bounded

$\alpha_2 = (A + B)^*$ deterministic $\exists 1$-bounded MPA

$\alpha_3 = (A \cdot C)^*$ not realizable

$\alpha_4 = A \cdot (A + B)^*$ $\exists 1$-bounded safe MPA

How about the realizability of $L(\alpha_i)$?

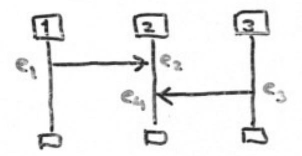
(Note: all realizable $L(\alpha_i)$ have as possible realization a locally accepting MPA, i.e., $F = \prod_{p \in P} F_p$ for some $F_p \subseteq S_p$)

Can we obtain a simple criterion on regular expressions that guarantees realizability?

REGULAR EXPRESSIONS FOR MSCs

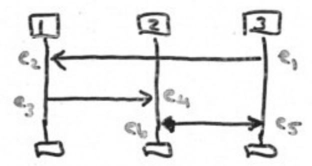
- MSC $M = (P, E, \mathcal{C}, \ell, m, <)$ is <u>connected</u> if:

$$\forall e, e' \in E \,.\quad e <^{+} e' \quad \text{or} \quad e' <^{+} e$$



not connected

since $e_1 \not<^{+} e_3 \wedge e_3 \not<^{+} e_1$



connected

- Regular expression α is <u>connected</u> if for any subexpression β^* of α, $L(\beta)$ is a set of connected MSCs

- Let $\underbrace{\{M_1, \ldots, M_k\}}_{D}$ be MSCs such that $D \subseteq E^*$ for some finite set E of MSCs.

Then:

[Genest et al. 2006]

$$\boxed{D \text{ is realizable iff there exists a } \underline{\text{connected}} \text{ reg. expr. } \alpha \text{ such that } L(\alpha) = D}$$