Software Modeling and Verification
Lehrstuhl für Informatik 2
RWTH Aachen University

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Foundations of the UML

Winter Term 07/08

– Assignment 2 –

Hand in until November 21st before the exercise class.

Exercise 1 (5 points)

Formally prove or disprove the correctness of the following statements for **CMSGs** (i.e., $M_i \in \mathbb{CM}$, $i \in \{1, 2, 3\}$):
(remember: $|$ $\widehat{=}$ choice, $\times$ $\widehat{=}$ (weak) sequence, * $\widehat{=}$ iteration)

a) $M_1|M_2 = M_2|M_1$

b) $M_1 \times M_2 = M_2 \times M_1$

c) $(M_1 \times M_2) \times M_3 = M_1 \times (M_2 \times M_3)$

d) $(M_1|M_2)|M_3 = M_1|(M_2|M_3)$

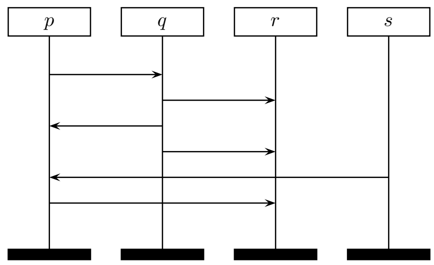
e) $(M_1 \times M_2)|M_3 = (M_1|M_3) \times (M_2|M_3)$

f) $(M_1|M_2) \times M_3 = (M_1 \times M_3)|(M_2 \times M_3)$

g) $M_1^*|M_2^* = (M_1|M_2)^*$

Exercise 2 (5 points)

Consider the MSC M:



a) Draw the Hasse diagram of M.

b) Determine all races in the MSC M and justify your answer (e.g., by means of another Hasse diagram for $\ll$).

Exercise 3 (10 points)

Let $\mathcal{W} = ActLin(\mathbb{M})$ (cf. Definition 1) be the set of so-called *well-formed and complete* words over Act.

Define a function $MSC : \mathcal{W} \to \mathbb{M}$ which determines for $w \in \mathcal{W}$ the corresponding MSC $MSC(w)$ such that, for any MSC M and any $w \in ActLin(M)$, we have $MSC(w) = M$

(Hint: the function MSC has to calculate all MSC components $\langle \mathcal{P}, E, \mathcal{C}, \ell, m, < \rangle$ subject to w).

Exercise 4 **(14 points)**

Consider the (C)MSGs from figure 1 (last page):

a) Prove or disprove the following properties for the MSGs $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$:

   (a) local-choice (as defined in the lecture)

   (b) regularity (as defined in Definition 3 at the end of this assignment)

b) Prove or disprove the following property for the CMSG $\mathcal{G}_4$:

   (a) safety (as defined in Definition 4 at the end of this assignment)

In each case justify your answer in detail. If there are several reasons why a property does not hold, state at least two of them.

Exercise 5 **(16 points)**

Consider the following two properties of CMSGs (**note:** $\lambda(v)$ contains at least one event for $v \in V$)):

(P_1) : A CMSG $\mathcal{G} = \langle V, \rightarrow, v_0, F, \lambda \rangle$ satisfies (P_1) if for every transition $(v, w) \in \rightarrow$ the communication graph of $\lambda(v)$, $\lambda(w)$ and $\lambda(v) \cdot \lambda(w)$ is weakly connected.

(P_2) : A CMSG $\mathcal{G} = \langle V, \rightarrow, v_0, F, \lambda \rangle$ satisfies (P_2) if every CMSC labeling a simple loop (i.e., a loop where each edge is exactly traversed once) of the graph $\langle V, \rightarrow \rangle$ has a weakly connected communication graph.

a) Show that (P_1) implies (P_2).

b) Prove that every local-choice MSG fulfills property (P_2).

c) Show that the other direction of 5b) does not hold in general.

Explanation: a graph is called *strongly connected* if every node can be reached by every other node. If the direction of edges is disregarded it is called *weakly connected.*

Definition 1: Let $Act = \biguplus_{p \in \mathcal{P}} Act_p$ be the set of actions for an MSC M. If $w = w_1 \dots w_n$ is a linearization of M then we call $w' \in Act^*$ with $w' = l(w_1) \dots l(w_n)$ an *action linearization* of M.
The set of all action linearizations of an MSC M is called $ActLin(M)$.

Definition 2: The *communication graph* CG of a CMSC $M = \langle \mathcal{P}, E, \mathcal{C}, \ell, m, < \rangle$ is defined as the graph $CG(M) = \langle V, \rightarrow \rangle$ (with the set of nodes $V := \mathcal{P} \setminus \{p \in \mathcal{P} \mid E_p = \emptyset\}$ and the edge relation $\rightarrow := \{(p_1, p_2) \mid p_1!p_2(c),\ p_2?p_1(c) \in \ell(E),\ c \in \mathcal{C}\}$).

Definition 3: A Message Sequence Graph $\mathcal{G}$ is *regular* if each MSC labeling a loop in $\mathcal{G}$ has a strongly connected communication graph.

Definition 4: A compositional Message Sequence Graph $\mathcal{G}$ is called *safe* if every sequence of CMSCs (using the concatenation defined in the lecture) describing an accepting path of $\mathcal{G}$ results in an MSC.

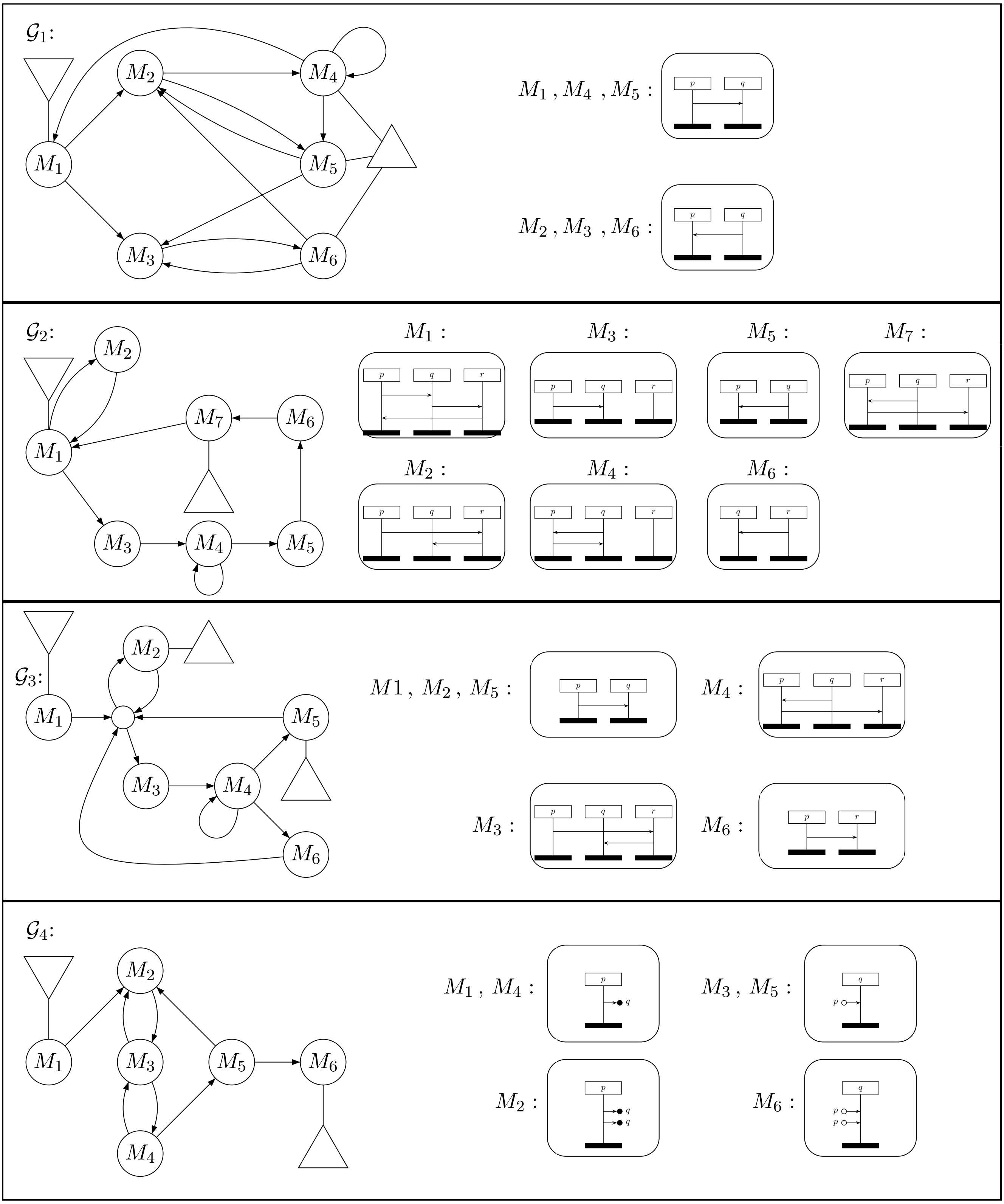


Figure 1: (C)MSGs for exercise 3