Foundations of the UML

Lecture 11: Realising local choice MSGs

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http://moves.rwth-aachen.de/i2/370

29. November 2009

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Safe realisability

Definition (Realisability)

1. MSG G is realisable whenever $L(G) = L(\mathcal{A})$ for some CFM $\mathcal{A}$.
2. MSG G is safely realisable whenever $L(G) = L(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.

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Questions

- Can results be obtained for larger classes of MSGs?
- What happens if we allow synchronisation messages?
  - recall that weak CFMs do not involve synchronisation messages
- How do we obtain a CFM realising an MSG algorithmically?
  - in particular, for non-local choice MSGs
- Are there simple conditions on MSGs that guarantee realisability?
  - e.g., easily identifiable subsets of (safe) realisable MSGs

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Summary of results

Results so far:

1. Conditions for (safe) realisability for finite sets of MSCs.
2. Checking these conditions is co-NP complete (in P).
3. Regular MSGs are (safely) realisable by $\forall$-bounded CFMs.
4. Checking regularity of MSGs is undecidable.
5. Communication-closedness implies regularity, but its check is co-NP complete.
6. Local communication-closedness implies regularity, and can be checked in P.

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Today's topics

- Today we focus on safe realisability of local-choice MSGs.
- By local-choice property, no deadlocks can occur in realising CFM.

Today's topics:

1. Realisability for constrained regular expressions of local-choice MSGs
2. An algorithm that generates a CFM (with synchronisation messages) from a local-choice MSG

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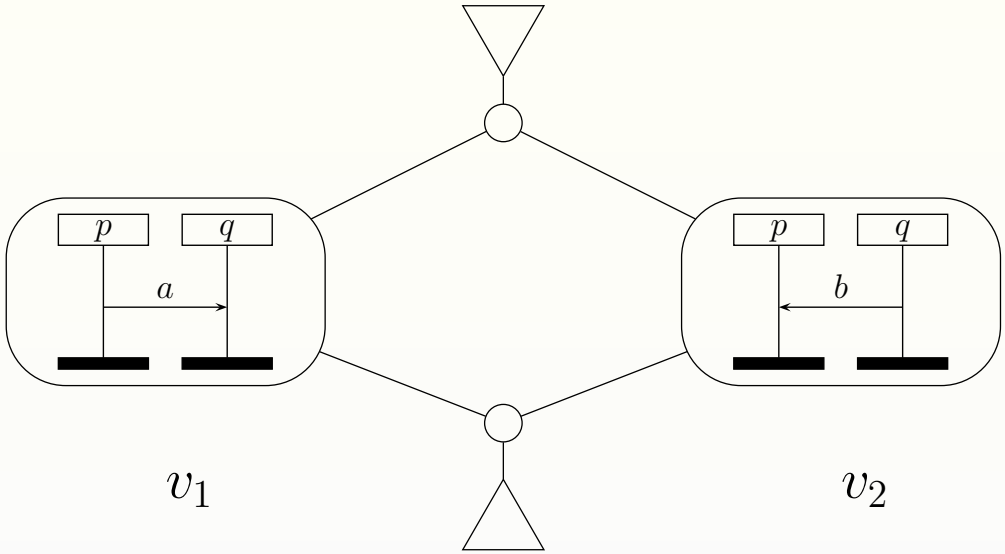
Local choice property (1)

G:



Inconsistency if process p behaves according to v_1 and process q behaves according to v_2

$\Longrightarrow$ possible distributed realization may yield a deadlock

Problem:

Subsequent behavior is determined by distinct processes

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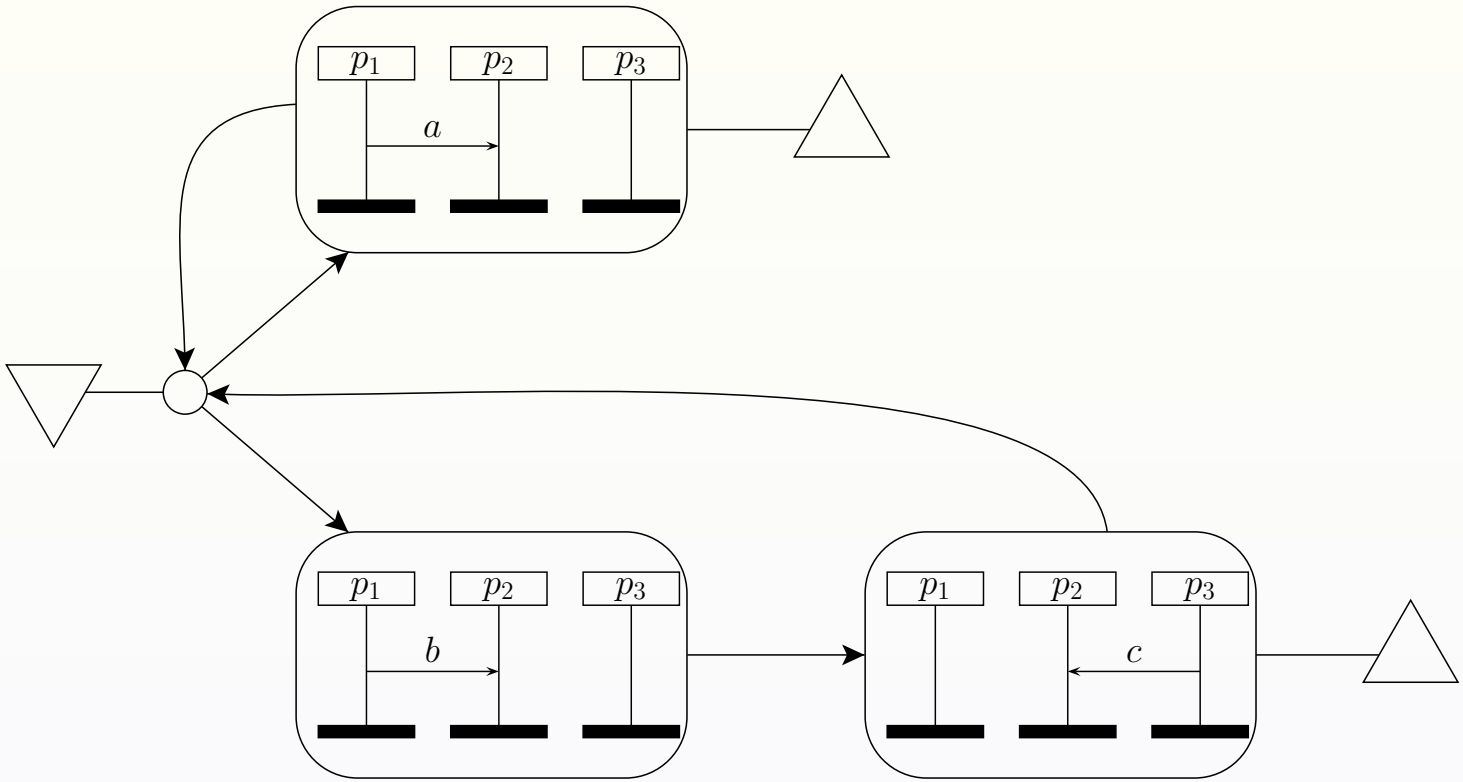
Example of local-choice MSG



Inconsistency if p_1 sends a and p_3 sends c.

Local choice property (2)

- e is a minimal event wrt. $\preceq$ if $\neg(\exists e' \neq e.\ e' \preceq e)$
- p is active in MSC M if $E_p \neq \varnothing$
- p is active in path $v_1 \ldots v_n$ in MSG G if p is active in $\lambda(v_i)$ for some i

Definition (local choice MSG)

MSG $G = (V, \rightarrow, v_0, F, \lambda)$ is local choice if:

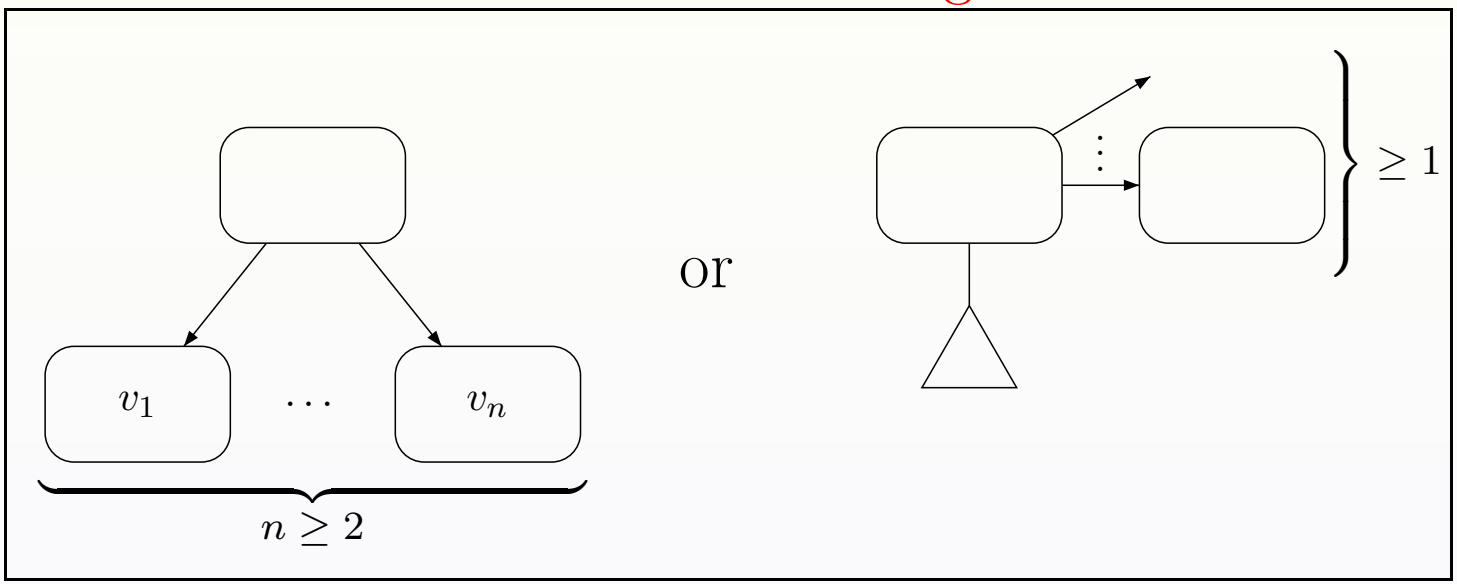
1. $\exists$ active p. $\forall \pi \in \text{Paths}(v_0)$.
   π contains a single minimal event $e \in E_p$
2. $\forall$ branching vertex $v \in V$. with $v \rightarrow w$
   $\exists$ active p . $\forall \pi \in \text{Paths}(w)$.
   π contains a single minimal event $e \in E_p$

Intuition:

Along every path from an initial or branching vertex there is a single process deciding how to proceed which can inform the other processes how to proceed.

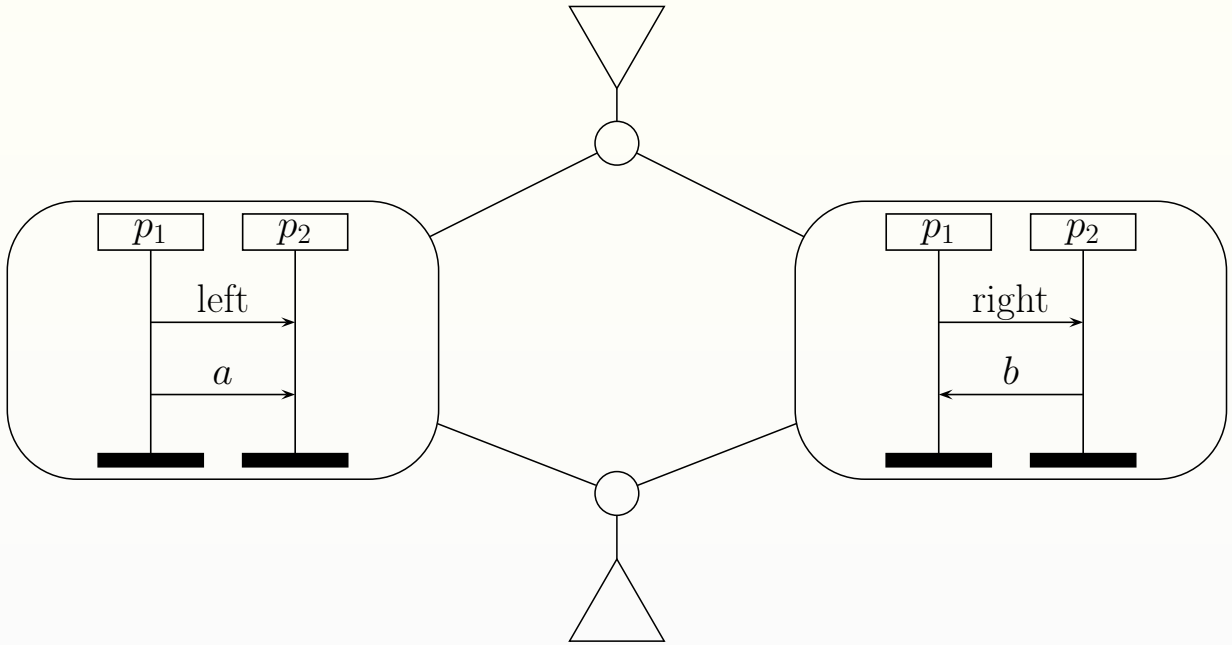
Branching vertices

A vertex is branching if:



Local choice

G:



Note:

Checking whether an MSG is local choice can be done in PTIME.

How can non-local choice be resolved?

Refine your MSG and add control messages (cf. above example)

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Regular expressions over MSCs

Definition (Asynchronous iteration)

For $\mathcal{M}_1, \mathcal{M}_2 \subseteq \mathbb{M}$ sets of MSCs, let:

$$\mathcal{M}_1 \bullet \mathcal{M}_2 \;=\; \{\, M_1 \bullet M_2 \mid M_1 \in \mathcal{M}_1, M_2 \in \mathcal{M}_2 \,\}$$

For $\mathcal{M} \subseteq \mathbb{M}$ let

$$\mathcal{M}^i \;=\; \begin{cases} \{M_\epsilon\} & \text{if } i{=}0, \text{ where } M_\epsilon \text{ denotes the empty MSC} \\ \mathcal{M} \bullet \mathcal{M}^{i-1} & \text{if } i > 0 \end{cases}$$

The asynchronous iteration of $\mathcal{M}$ is now defined by:

$$\mathcal{M}^* = \bigcup_{i \geqslant 0} \mathcal{M}^i.$$

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Regular expressions over MSCs

Definition (Regular expressions over MSCs)

The set $\text{REX}_{\mathbb{M}}$ of regular expressions over $\mathbb{M}$ is given by the grammar:

$$\alpha ::= \varnothing \mid M \mid \alpha_1 \cdot \alpha_2 \mid \alpha_1 + \alpha_2 \mid \alpha^*$$

where MSC $M \in \mathbb{M}$.

Definition (Semantics of regular expressions, $L(.) : \text{REX}_{\mathbb{M}} \rightarrow 2^{\mathbb{M}}$)

- $L(\varnothing) = \varnothing$
- $L(M) = \{ M \}$
- $L(\alpha_1 \cdot \alpha_2) = L(\alpha_1) \bullet L(\alpha_2)$
- $L(\alpha_1 + \alpha_2) = L(\alpha_1) \cup L(\alpha_2)$
- $L(\alpha^*) = L(\alpha)^*$

Locally accepting CFMs

Definition (Locally accepting CFM)

CFM $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ is locally accepting if

$$F = \prod_{p \in \mathcal{P}} F_p \quad \text{where} \quad F_p \subseteq S_p.$$

An la CFM abbreviates a locally accepting CFM.

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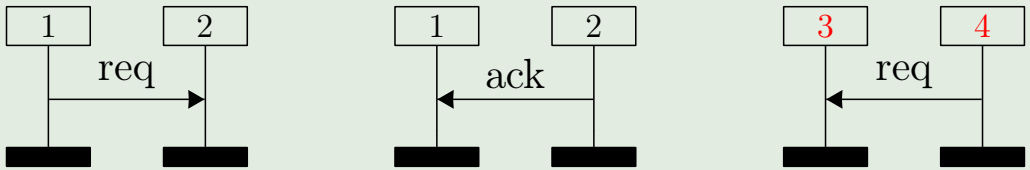
Regular expressions for MSCs

Let $\mathcal{P} = \{1, 2, 3, 4\}$ and $\mathcal{C} = \{\text{req}, \text{ack}\}$.

Example



A B C

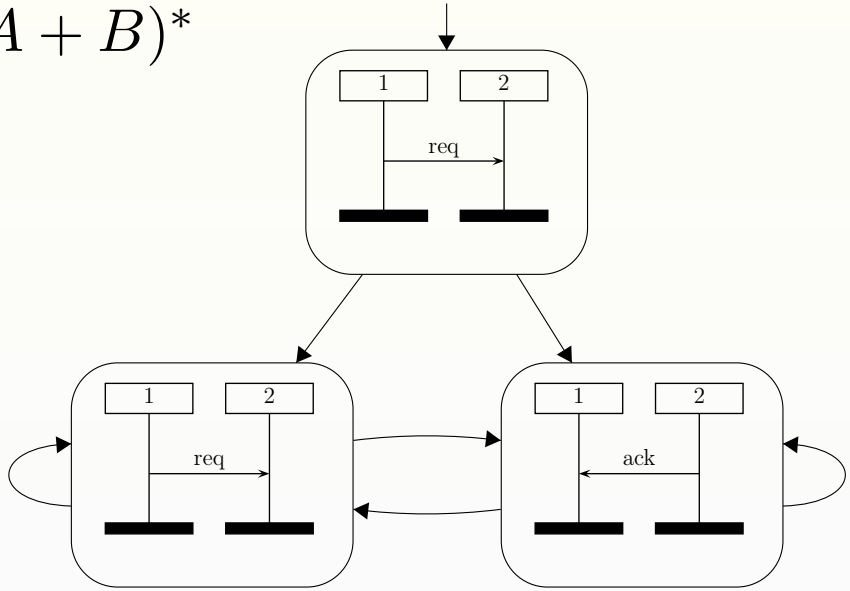
Consider the following regular expressions over $\mathbb{M}$:

- $\alpha_1 = (A \cdot B)^*$ det. $\forall$1-bounded deadlock-free weak la CFM
- $\alpha_2 = (A + B)^*$ det. $\exists$1-bounded la CFM
- $\alpha_3 = (A \cdot C)^*$ not realisable
- $\alpha_4 = A \cdot (A + B)^*$ $\exists$1-bounded deadlock-free la CFM

How about realisability of $L(\alpha_i)$?

Realising local-choice expressions by deadlock-free CFMs

$A \cdot (A + B)^*$



Realising local-choice expressions by deadlock-free CFMs

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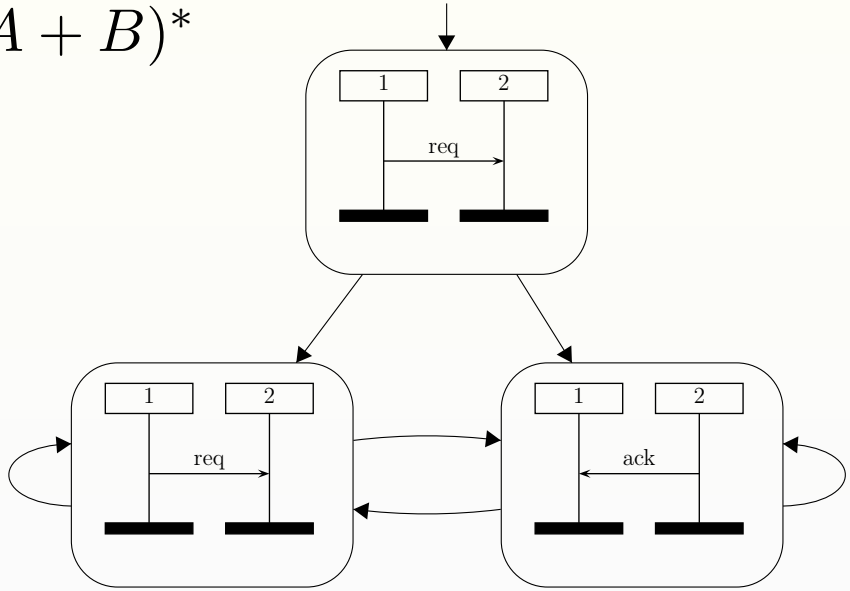
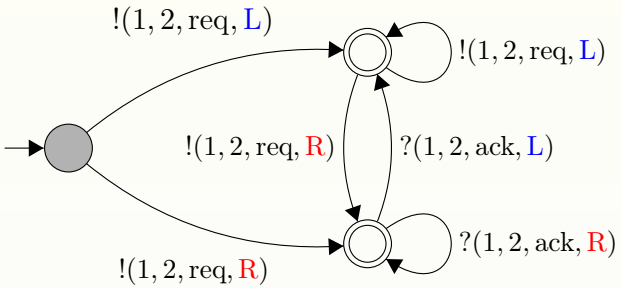
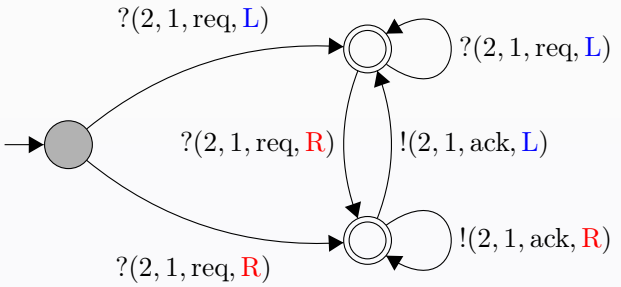


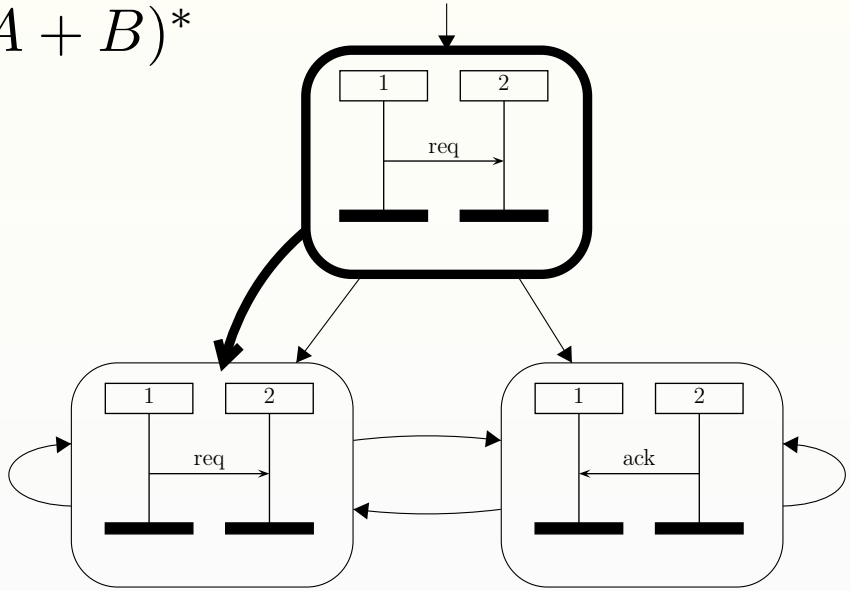


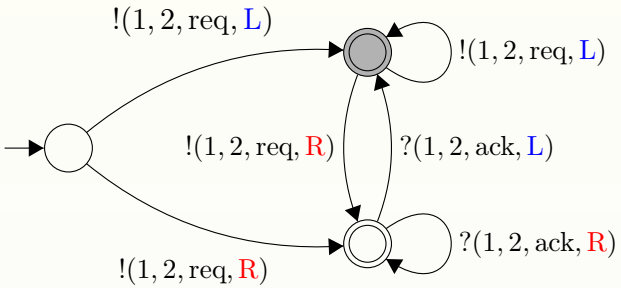


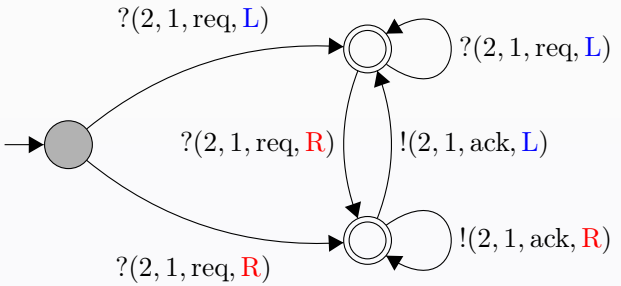
$1 \rightarrow 2$:
$2 \rightarrow 1$:

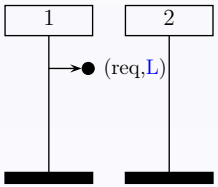
Realising local-choice expressions by deadlock-free CFMs

$A \cdot (A + B)^*$









$1 \rightarrow 2 :$ (req,L)
$2 \rightarrow 1 :$

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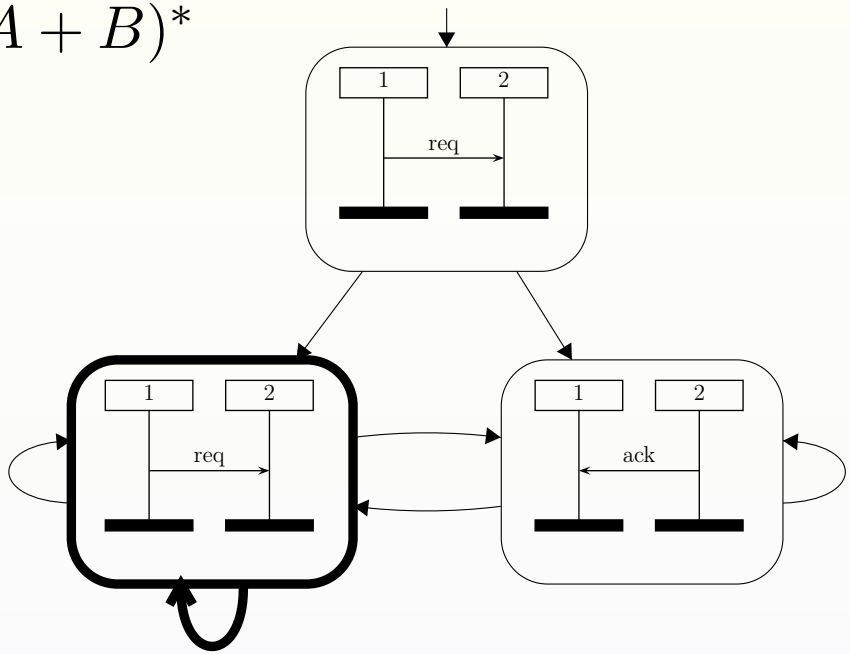
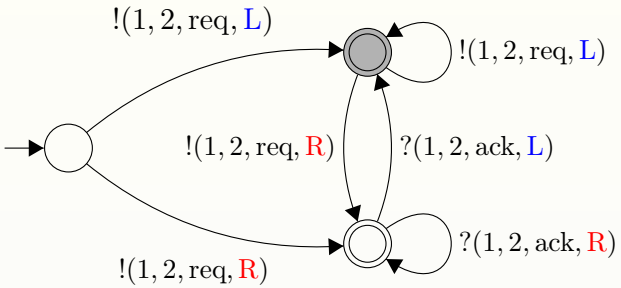
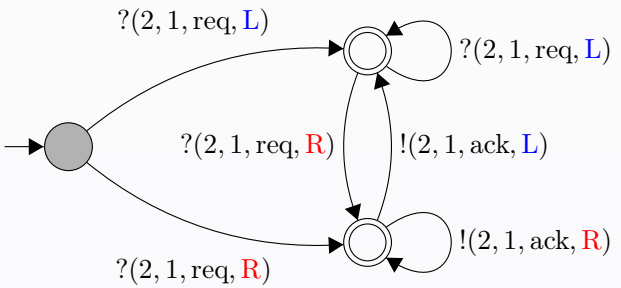
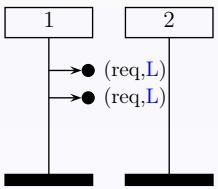
Realising local-choice expressions by deadlock-free CFMs

$A \cdot (A + B)^*$





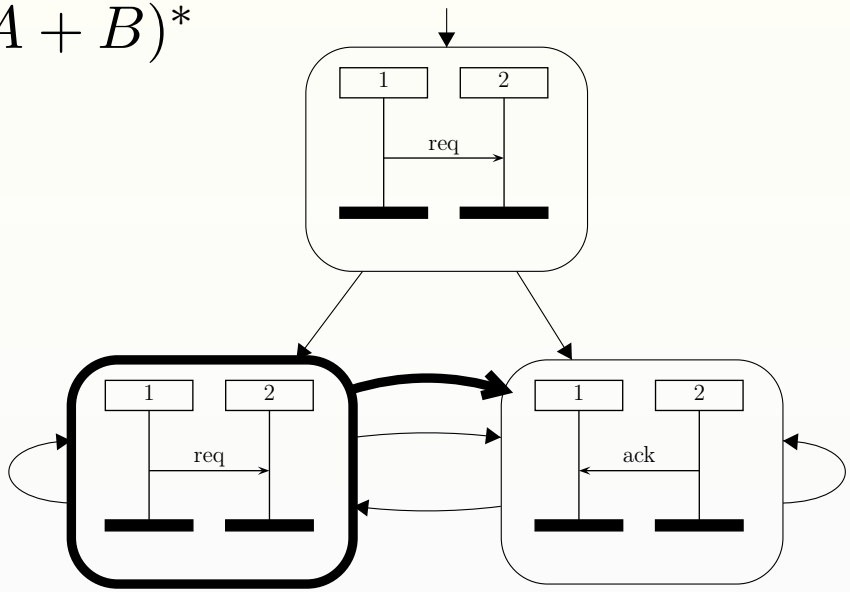


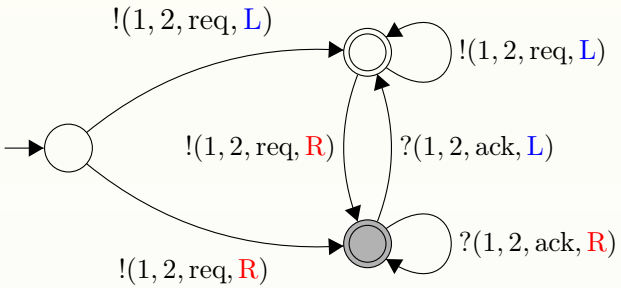


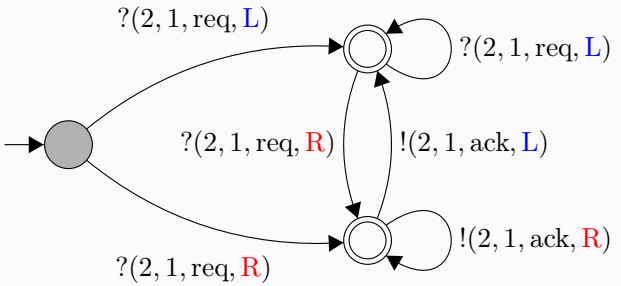
$1 \rightarrow 2 :$ (req,L) (req,L)
$2 \rightarrow 1 :$

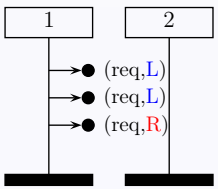
Realising local-choice expressions by deadlock-free CFMs

$A \cdot (A + B)^*$









$1 \rightarrow 2$: (req,L) (req,L) (req,R)
$2 \rightarrow 1$:

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Realising local-choice expressions by deadlock-free CFMs

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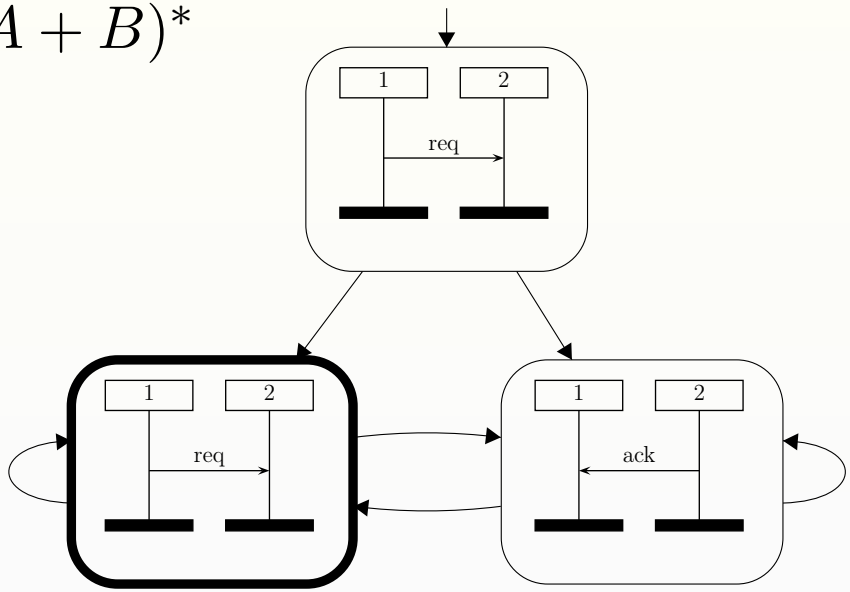

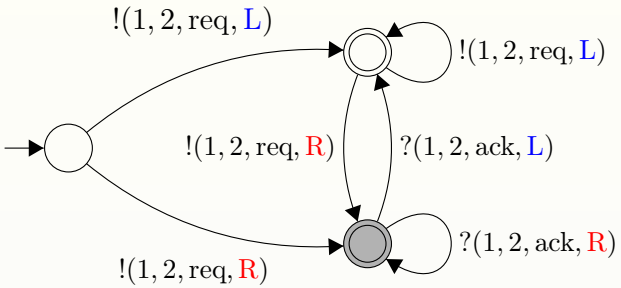

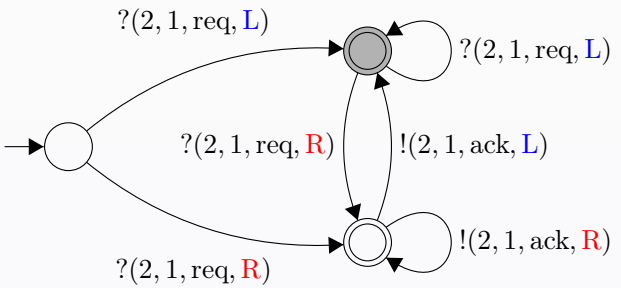

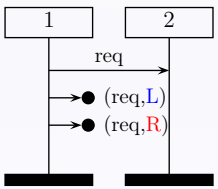

$1 \to 2 :$ (req,L) (req,R)
$2 \to 1 :$

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Realising local-choice expressions by deadlock-free CFMs

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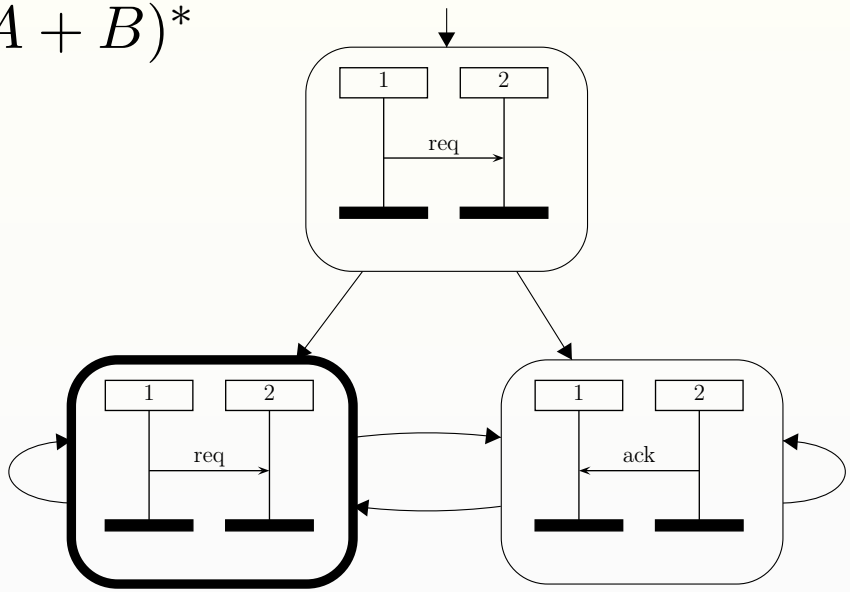

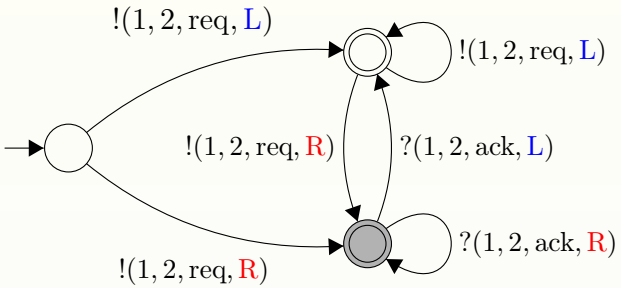

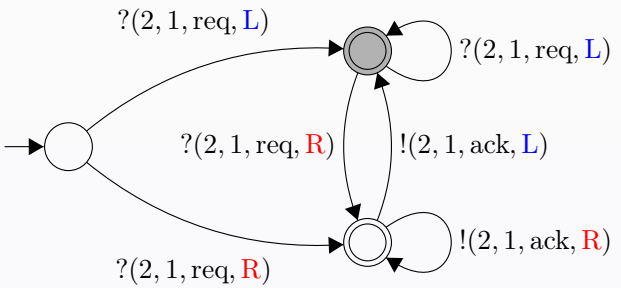

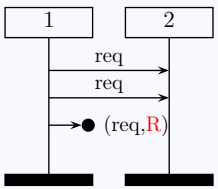

$1 \rightarrow 2 : (\text{req},\text{R})$
$2 \rightarrow 1 :$

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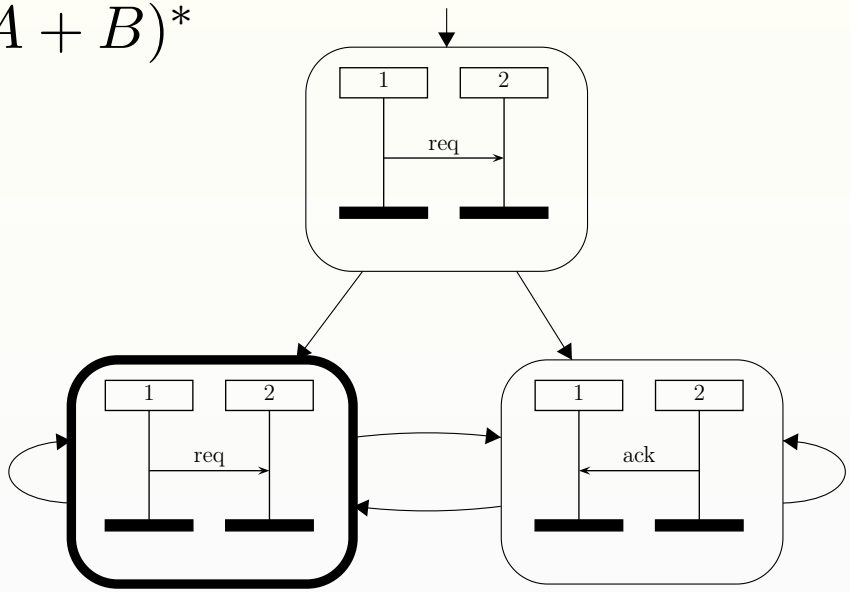
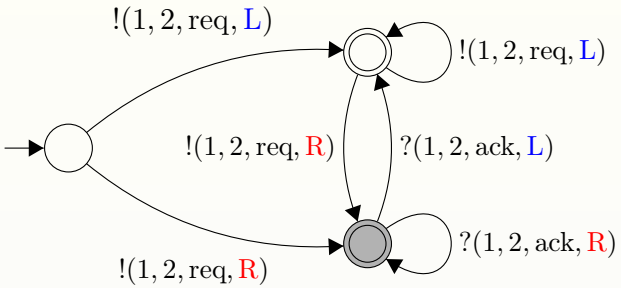
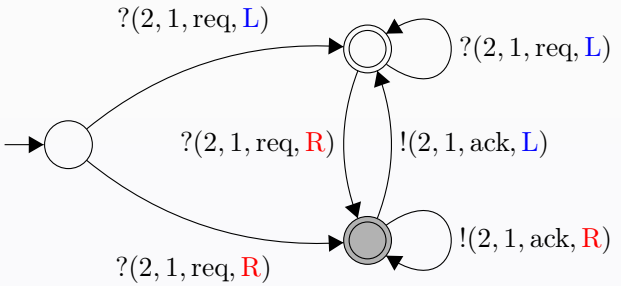
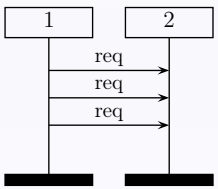
Realising local-choice expressions by deadlock-free CFMs

$A \cdot (A + B)^*$









$1 \rightarrow 2$:
$2 \rightarrow 1$:

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Realising local-choice expressions by deadlock-free CFMs

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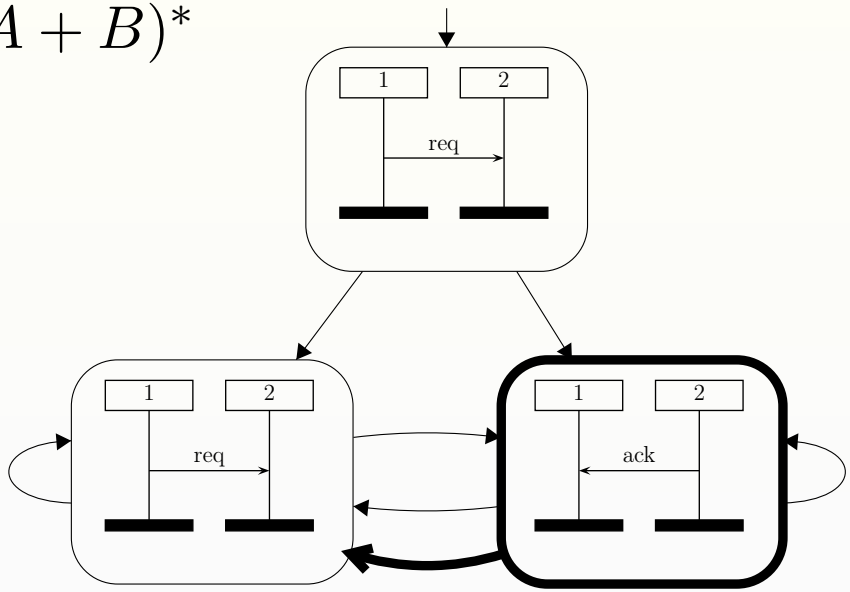

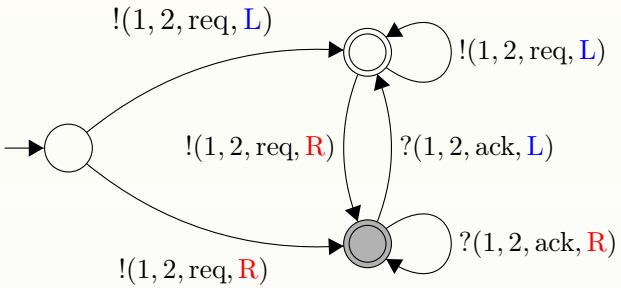

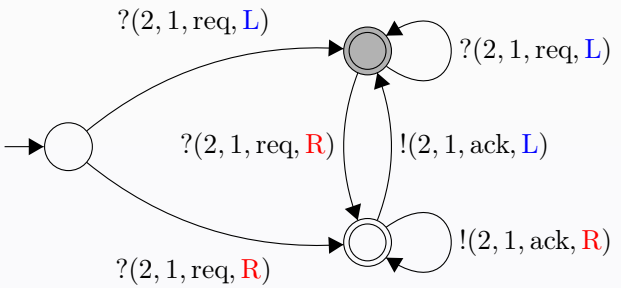

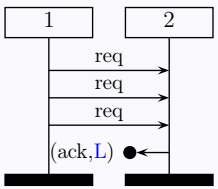

$1 \rightarrow 2$: req
$2 \rightarrow 1$: (ack,L)

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Realising local-choice expressions by deadlock-free CFMs

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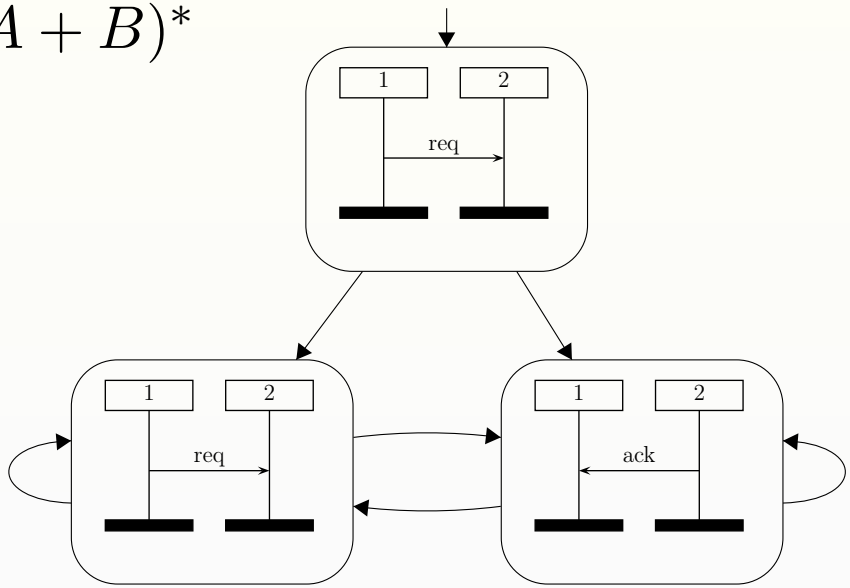

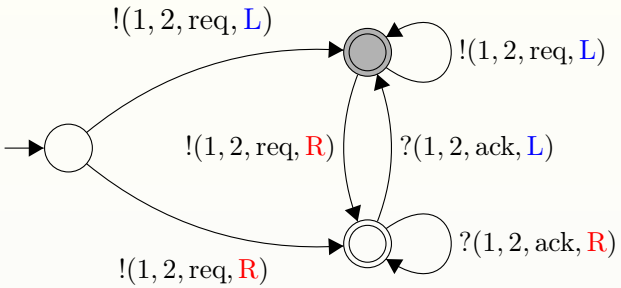

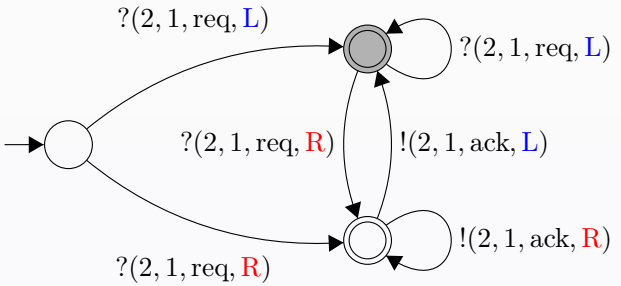

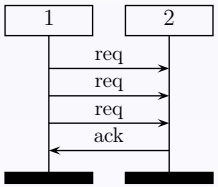

$1 \rightarrow 2$:
$2 \rightarrow 1$:

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Star-connected

Definition (Connected MSC)

An MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ is connected if:

$$\forall e, e' \in E.\, e <^* e' \quad \text{or} \quad e' <^* e.$$

Examples on the black board.

Definition (Star-connected)

We call $\alpha \in \text{REX}_{\mathbb{M}}$ star-connected if, for any subexpression β^* of α, $L(\beta)$ is a set of connected MSCs.

Examples on the black board.

Regular expressions vs. CFMs

Definition (Finitely generated)

Set of MSCs $\mathcal{M} \subseteq \mathbb{M}$ is finitely generated if there is a finite set of MSCs $\widehat{\mathcal{M}} \subseteq \mathbb{M}$ such that $\mathcal{M} \subseteq \widehat{\mathcal{M}}^*$.

Theorem [Genest et al. 2006]

Finitely generated $\mathcal{M}$ is realisable

iff

there exists a star-connected regular expression α with $L(\alpha) = \mathcal{M}$.

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Local choice MSGs

An example local-choice MSG on black board.

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Realising local choice (C)MSGs

Theorem [Genest et. al., 2005]

Any local-choice (C)MSG G is safely realisable by a CFM with additional synchronisation data which is of size linear in G.

Proof

As MSG G is local choice, at every branch v of G there is a unique process, $p(v)$, say, such that on every path from v the unique minimal event occur at $p(v)$. Then:

1. Process $p(v)$ determines the successor vertex of v.
2. Process $p(v)$ informs all other processes about its decision by adding synchronisation data to the exchanged messages.
3. Synchronisation data is the path (in G) from v to the next branching vertex along the direction chosen by $p(v)$.

Maximal non-branching paths

Definition (Maximal non-branching paths)

For MSG $G = (V, \rightarrow, v_0, F, \lambda)$, let $nbp : V \rightarrow V^*$ be defined by:

$$nbp(v) = \begin{cases} v & \text{if } v \in F \text{ or } v \text{ is a branching vertex} \\ v_1 \ldots v_n & \text{otherwise} \end{cases}$$

where the path $v_1 \ldots v_n \in V^*$ satisfies:

1. $v_i = v$ for some i, $0 < i \leqslant n$, and
2. $v_n \in F$ or is a branching vertex, and
3. v_1 is a direct successor of a branching vertex, and
4. $v_2, \ldots, v_{n-1} \notin F$ and are all non-branching vertices

Intuition

$nbp(v)$ is the maximal non-branching path to which v belongs.

Structure of the CFM of local choice MSG G

Let MSG $G = (V, \rightarrow, v_0, F, \lambda)$ be local choice.

Define the CFM $\mathcal{A}_G = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F')$ with:

1. Local automaton $\mathcal{A}_p = (S_p, \Delta_p)$ as defined on next slides

2. $\mathbb{D} = \{ npb(v) \mid v \in V \}$
   synchronisation data = maximal non-branching paths in G

3. $s_{init} = \{ (v_0, \varnothing) \}^n$ where $n = |\mathcal{P}|$
   each local automaton $\mathcal{A}_p$ starts in initial state $(v_0, \varnothing)$, i.e., in initial vertex v_0 while no events of p have been performed

4. $\overline{s} \in F'$ iff for all $p \in \mathcal{P}$, local state $\overline{s}[p] = (v, E)$ with $E \subseteq E_p$ and:
   1. $v \in F$ and E contains a maximal event wrt. $<_p$ in MSC $\lambda(v)$, or
   2. $v \notin F$ and $\pi = v \ldots w$ is a path in G and E contains a maximal event wrt. $<_p$ in MSC $\lambda(\pi)$.

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State space of local automaton $\mathcal{A}_p$

- $S_p = V \times E_p$ such that for any $s = (v, E) \in S_p$:

$$\forall e, e' \in \lambda(v).\ \big(e <_p e' \text{ and } e' \in E \quad \text{implies} \quad e \in E\big)$$

  that is, E is downward-closed with respect to $<_p$ in MSC $\lambda(v)$

- Intuition: a state (v, E) means that process p is currently in vertex v of G and has already performed the events E of $\lambda(v)$

- Initial state of $\mathcal{A}_p$ is $\overline{s_{init}}[p] = (v_0, \varnothing)$

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Transition relation of local automaton $\mathcal{A}_p$

- Executing events within a vertex of the MSG G:

$$\frac{e \in E_p \cap \lambda(v)}{(v, E) \xrightarrow{l(e), nbp(v)}_p (v, E \cup \{ e \})}$$

  Note: since $E \cup \{e\}$ is downward-closed wrt. $<_p$, e is enabled

- Changing vertex of the MSG G:

$$\frac{\begin{array}{c} E = E_p \cap \lambda(v) \text{ and } e \in E_p \cap \lambda(w) \text{ and} \\ vu_0 \ldots u_n w \in V^* \text{ with } p \text{ not active in } u_0 \ldots u_n \end{array}}{(v, E) \xrightarrow{l(e), nbp(w)}_p (w, \{e\})}$$

  Note: vertex w is the first successor vertex of v on which p is active

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Examples

A couple of examples on the black board.

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