Foundations of the UML

Lecture 14: Statecharts Semantics (1)

Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

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What are Statecharts?

$$\begin{aligned}\text{Statecharts} := {} & \text{Mealy machines} \\ & + \text{State hierarchy} \\ & + \text{Broadcast communication} \\ & + \text{Orthogonality}\end{aligned}$$

Statecharts

Definition (Statecharts)

A statechart SC is a triple $(N, E, Edges)$ with:

1. N is a set of nodes (or: states) structured in a tree
2. E is a set of events
   - pseudo-event $after(d)$ denotes a delay of $d \in \mathbb{R}_{\geqslant 0}$ time units
   - $\perp \notin E$ stands for "no event available"
3. $Edges$ is a set of (hyper-) edges, defined later on.

Definition (System)

A system is described by a finite collection of statecharts $(SC_1, \ldots, SC_k)$.

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Tree structure

Function *children*

Nodes obey a tree structure defined by function $\mathit{children} : N \rightarrow 2^N$ where $x \in \mathit{children}(y)$ means that x is a child of y, or equivalently, y is the parent of x.

Partial order $\trianglelefteq$

The partial order $\trianglelefteq \subseteq N \times N$ is defined by:

- $\forall x \in N.\, x \trianglelefteq x$
- $\forall x, y \in N.\, x \trianglelefteq y$ if $x \in \mathit{children}(y)$
- $\forall x, y, z \in N.\, x \trianglelefteq y \wedge y \trianglelefteq z \Rightarrow x \trianglelefteq z$

$x \trianglelefteq y$ means that x is a descendant of y, or equivalently, y is an ancestor of x. If $x \trianglelefteq y$ or $y \trianglelefteq x$, nodes x and y are ancestrally related.

Root node

There is a unique root with no ancestors, and $\forall x \in N.\, x \trianglelefteq \text{root}$.

Functions on nodes

The type of nodes

Nodes are typed, $\mathit{type}(x) \in \{\, \textsc{basic}, \textsc{and}, \textsc{or} \,\}$ such that for $x \in N$:

- $\mathit{type}(\text{root}) = \textsc{or}$
- $\mathit{type}(x) = \textsc{basic}$ iff $\mathit{children}(x) = \varnothing$, i.e., x is a leaf
- $\mathit{type}(x) = \textsc{and}$ implies $(\forall y \in \mathit{children}(x).\, \mathit{type}(y) = \textsc{or})$

Default nodes

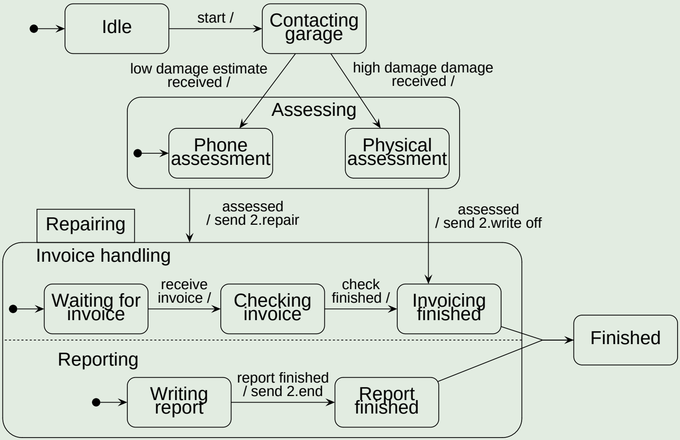
$\mathit{default} : N \rightarrow N$ is a partial function on $\{\, x \in N \mid \mathit{type}(x) = \textsc{or} \,\}$ with

$$\mathit{default}(x) = y \quad \text{implies} \quad y \in \mathit{children}(x).$$

The function *default* assigns to each OR-node x one of its children as default node that becomes active once x becomes active.

Example

A damage assessor



Edges

Definition (Edges)

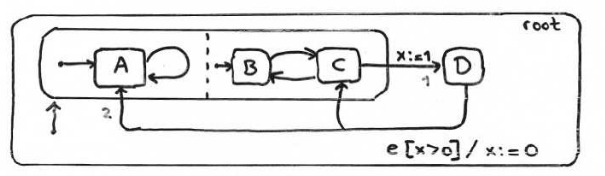
An edge is a quintuple (X, e, g, A, Y), denoted $X \xrightarrow{e[g]/A} Y$ with:

- $X \subseteq N$ is a set of source nodes with $X \neq \varnothing$
- $e \in E \cup \{ \perp \}$ is the trigger event
- $A \subseteq Act$ is a set of actions
  - such as $v :=$ expr or local variable v and expression expr
  - or *send* $j.e$, i.e., send event e to statechart SC_j
- Guard g is a Boolean expression over all variables in $(SC_1, \ldots, SC_k)$
- $Y \subseteq N$ is a set of target nodes with $Y \neq \varnothing$

The sets X and Y may contain nodes at different depth in the node tree.

Example

Example statechart



$$\text{edge 1: } \{ C \} \xrightarrow{\perp[true]/\{ x:=1 \}} \{ D \}$$

$$\text{edge 2: } \{ D \} \xrightarrow{e[x>0]/\{ x:=0 \}} \{ A, C \}$$

Towards a statechart semantics

- Formal semantics: map $(SC_1, \ldots, SC_k)$ onto a single Mealy machine
- This is done using a step semantics distinguishing macro and micro steps
- Macro steps are "observable" and are subdivided into a finite number of micro steps that cannot be prolonged
- In a macro step, a maximal set of edges is performed
- Events generated in macro step n are only available in macro step $n+1$
  - If such event is not "consumed" in step $n+1$, it dies, and is not available in step $n+2$, $n+3$, ...

Assumptions [Eshuis & Wieringa, 2000]

- Input to a macro step is a set of events (and not a queue)

  the order of event generation is ignored, i.e., if e and e' are generated in macro step i, the order in which they are generated is irrelevant in step $i+1$

- A macro step reacts to all available events

  events can only be used in macro step immediately following their generation

- Instantaneous edges and actions

- Unlimited concurrency

  there is no limit on the number of events that can be consumed in a macro step

- Perfect communication, i.e., messages are not lost

What does a single StateChart mean?

Intuitive semantics as a transition system:

- State = a set of nodes ("current control") + the values of variables
- Edge is enabled if guard holds in current state
- Executing edge $X \xrightarrow{e[g]/A} Y$ = perform actions A, consume event e
  - leave source nodes X and switch to target nodes Y

  $\Rightarrow$ events are unordered, and considered as a set
- Principle: execute as many edges at once (without conflict)

  $\Rightarrow$ the total execution of such maximal set is a macro step

States and configurations

Definition (Configuration)

A configuration of $SC = (N, E, Edges)$ is a set $C \subseteq N$ of nodes satisfying:

- root $\in C$
- $x \in C$ and $type(x) = \text{OR}$ implies $|children(x) \cap C| = 1$
- $x \in C$ and $type(x) = \text{AND}$ implies $children(x) \subseteq C$

Let $Conf$ denote the set of configurations of SC.

Definition (State)

State of $SC = (N, E, Edges)$ is a triple (C, I, V) where

- C is a configuration of SC
- $I \subseteq V$ is the set of events to be processed
- V is a valuation of the variables.

Example

Enabling of an edge

Definition (Enabledness)

Edge $X \xrightarrow{e[g]/A} Y$ is enabled in state (C, I, V) whenever:

- $X \subseteq C$, i.e. all source nodes are in configuration C
- $(\underbrace{(C_1, \ldots, C_n)}_{\text{configurations}}, \underbrace{(V_1, \ldots, V_n)}_{\text{variable valuations}}) \models g$, i.e., guard g is satisfied
- $e \neq \bot$ implies $e \in I$, or $e = \bot$

Let $En(C, I, V)$ denote the set of enabled edges in state (C, I, V).

Macro steps

- On receiving an input e, several edges in SC may become enabled
- Then, a maximal and consistent set of enabled edges is taken
- If there are several such sets, choose one nondeterministically
- Edges in concurrent components can be taken simultaneously
- But edges in other components cannot; they are inconsistent
- To resolve nondeterminism (partly), priorities are used

Consistency: examples

Least common ancestor

Definition (Least common ancestor)

For $X \subseteq N$, the least common ancestor, denoted $lca(X)$, is the node $y \in N$ such that:

$$(\forall x \in X.\, x \trianglelefteq y) \quad \text{and} \quad \forall z \in N.\, (\forall x \in X.\, \trianglelefteq z) \text{ implies } y \trianglelefteq z.$$

Intuition

Node y is an ancestor of any node in X (first clause), and is a descendant of any node which is an ancestor of any node in X (second clause).

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Orthogonality of nodes

Definition (Orthogonality of nodes)

Nodes $x, y \in N$ are orthogonal, denoted $x \perp y$, if

$$\neg(x \trianglelefteq y) \quad \text{and} \quad \neg(y \trianglelefteq x) \quad \text{and} \quad \mathit{type}(\mathit{lca}(\{\, x, y \,\})) = \textsc{and}.$$

Scope

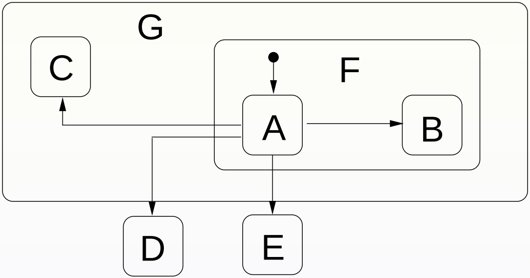
Definition (Scope of edge)

The scope of edge $X \overset{\cdots}{\longrightarrow} Y$ is the most nested OR-node that is an ancestor of both X and Y.

Intuition

The scope of edge $X \overset{\cdots}{\longrightarrow} Y$ is the most nested OR-node that is unaffected by executing the edge $X \overset{\cdots}{\longrightarrow} Y$.

Scope: example



$$scope(A \longrightarrow D) = \text{root} \quad \text{and} \quad scope(A \longrightarrow C) = G \quad \text{and} \quad scope(A \longrightarrow B) = F$$

Consistency

Definition (Consistency)

1. Edges $ed, ed' \in Edges$ are consistent if:

$$ed = ed' \quad \text{or} \quad scope(ed) \perp scope(ed').$$

2. $T \subseteq Edges$ is consistent if all edges in T are pairwise consistent.

Example

On the black board.

What is now a macro step?

A macro step is a set T of edges such that:

- all edges in step T are enabled
- all edges in T are pairwise consistent
  - they are identical or
  - scopes are (descendants of) different children of the same AND-node
- enabled edge ed is not in step T implies

  there exists $ed' \in T$ such that ed is inconsistent with ed', and
  the priority of ed' is not smaller than ed
- step T is maximal (wrt. set inclusion)

Next lecture: January 11