Foundations of the UML

Lecture 5: Compositional Message Sequence Graphs

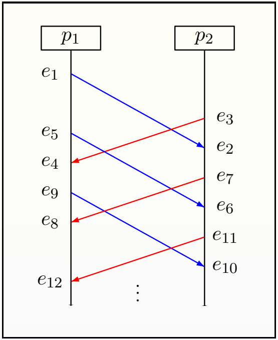
Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

http://moves.rwth-aachen.de/i2/370

2. November 2009

Restriction of MSGs [Yannakakis 1999]



This MSC cannot be decomposed as

$$M_1 \bullet M_2 \bullet \ldots \bullet M_n \quad \text{for } n > 1$$

This can be seen as follows:

- e_1 and $e_2 = m(e_1)$ must reside in same M_i
- $e_3 < e_2$ and $e_1 < e_4$ thus
  $e_3, e_4 \notin M_j$, $j < i$ or $j > i$
  $\Longrightarrow e_3, e_4 \in M_i$
- by similar reasoning: $e_5, e_6 \in M_i$ etc.

Problem:

Compulsory matching between send and receive in same MSG vertex (i.e., send e and receive $m(e)$)

Compositional MSCs [Gunter, Muscholl, Peled 2001]

Solution: drop restriction that e and $m(e)$ belong to the same MSC (= allow for incomplete message transfer)

Definition

$M = (\mathcal{P}, E, \mathcal{C}, l, m, <)$ is a compositional MSC (CMSC, for short) where $\mathcal{P}, E, \mathcal{C}$ and l are as before, and

- $m : E_! \to E_?$ is a partial, injective function such that (as before):

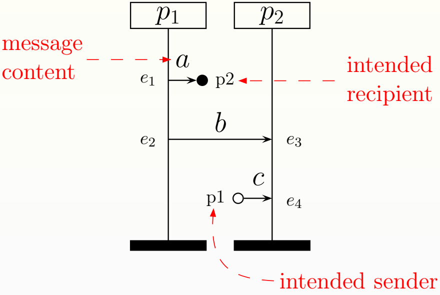
$$m(e) = e' \wedge l(e) = !(p,q,a) \implies l(e') = ?(q,p,a)$$

- $< \; = \left(\bigcup_{p \in \mathcal{P}} <_p \quad \cup \quad \underbrace{\{(e, m(e)) \mid e \in \underbrace{dom(m)}_{\text{domain of } m}\}}_{\text{"}m(e)\text{ is defined"}}\right)^*$

Note:

An MSC is a CMSC where m is total and bijective.

CMSC example



$$m(e_2) = e_3$$
$$e_1 \notin dom(m)$$
$$e_4 \notin rng(m)$$

Definition

A compositional MSG (CMSG) $G = (V, \rightarrow, v_0, F, \lambda)$ with $\lambda : V \rightarrow \mathbb{CM}$, where $\mathbb{CM}$ is the set of all CMSCs, and $V, \rightarrow, v_0,$ and F as before.

Concatenation of CMSCs (1)

Let $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, <_i) \in \mathbb{CM} \quad i \in \{1, 2\}$
be CMSCs with $E_1 \cap E_2 = \varnothing$

The concatenation of CMSCs M_1 and M_2 is the CMSC
$M_1 \bullet M_2 = (\mathcal{P}_1 \cup \mathcal{P}_2,\ E,\ \mathcal{C}_1 \cup \mathcal{C}_2, l, m, <)$ with:

- $E = E_1 \cup E_2$
- $l(e) = l_1(e)$ if $e \in E_1$, $l_2(e)$ otherwise
- $m(e) = E_! \rightarrow E_?$ satisfies:
  1. m extends m_1 and m_2, i.e., $e \in dom(m_i)$ implies $m(e) = m_i(e)$
  2. m matches unmatched send events in M_1 with unmatched receive events in M_2 according to order on process (matching from top to bottom)
     the k-th unmatched send in M_1 is matched with
     the k-th unmatched receive in M_2 (of the same "type")
  3. $M_1 \bullet M_2$ is FIFO (when restricted to matched events)

RWTH AACHEN UNIVERSITY

Concatenation of CMSCs (2)

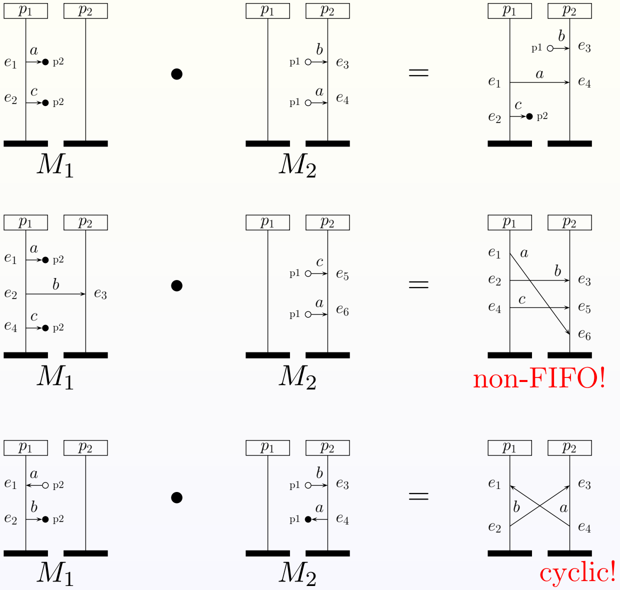
Let $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, <_i) \in \mathbb{CM} \qquad i \in \{1, 2\}$
be CMSCs with $E_1 \cap E_2 = \varnothing$

The concatenation of CMSCs M_1 and M_2 is the CMSC
$M_1 \bullet M_2 = (\mathcal{P}_1 \cup \mathcal{P}_2, E_1 \cup E_2, \mathcal{C}_1 \cup \mathcal{C}_2, l, m, <)$ with:

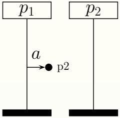
- $<$ is the reflexive and transitive closure of:

$$\left(\bigcup_{p \in \mathcal{P}} <_{p,1} \cup <_{p,2}\right) \quad \cup \quad \{(e, e') \mid e \in E_1 \cap E_p\,,\ e' \in E_2 \cap E_p\}$$
$$\cup \quad \{(e, m(e) \mid e \in dom(m)\}$$

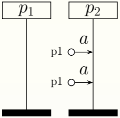
Examples



Associativity

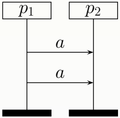


M

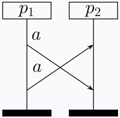


M'

$(M \bullet M) \bullet M'$:



$M \bullet (M \bullet M')$:



$\Longrightarrow$ this is non-FIFO (and thus undefined)

Note:

Concatenation of CMSCs is <u>not</u> associative.

Paths

Let $G = (V, \rightarrow, v_0, F, \lambda)$ be a CMSG.

Definition

A path π of G is a finite sequence

$$\pi = u_0 \; u_1 \; \ldots \; u_n \text{ with } u_i \in V \;\; (0 \leq i \leq n) \text{ and } u_i \rightarrow u_{i+1} \;\; (0 \leq i < n)$$

Definition

Path $\pi = u_0 \; \ldots \; u_n$ is accepting if: $u_0 = v_0$ and $u_n \in F$.

Definition

The CMSC of a path $\pi = u_0 \; \ldots \; u_n$ is:

$$M(\pi) \; = \; (\ldots(\lambda(u_0) \bullet \lambda(u_1)) \bullet \lambda(u_2) \ldots) \bullet \lambda(u_n) \; = \; \prod_{i=0}^{n} \lambda(u_i)$$

where CMSC concatenation is left associative.

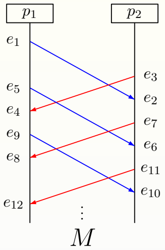
The MSC language of a CMSG

Definition

The (MSC) language of CMSG G is defined by:

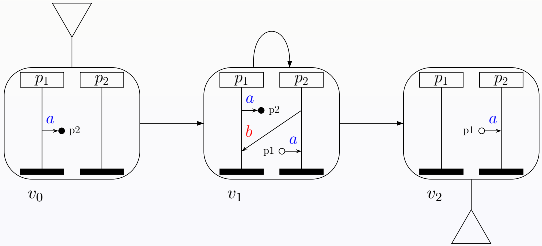
$$L(G) = \{ \underbrace{M(\pi) \in \mathbb{M}}_{\text{only MSCs are considered}} \mid \pi \text{ is an accepting path of } G\}.$$

Consider again



Recall: this behavior cannot be modeled for $n > 1$ by:

$$M = M_1 \bullet M_2 \bullet \ldots \bullet M_n \quad \text{with} \quad M_i \in \mathbb{M}$$



The (safe) CMSG for the above MSC.

Safe paths and CMSGs

Definition

Path π of CMSG G is safe whenever $M(\pi) \in \mathbb{M}$.

Definition (safeness)

CMSG G is safe if for any accepting path π of G, $M(\pi)$ is an MSC.

So:

CMSG G is safe if on any of its accepting paths there are no unmatched sends and receipts, i.e., if any of its executions is an MSC.

Properties on CMSG safeness

Theorem:

The decision problem "does CMSG G have at least one safe, accepting path?" is undecidable.

Theorem:

The decision problem "is CMSG G safe?" is decidable in PTIME.

Existence of safe accepting paths

Theorem

The decision problem:

does CMSG G have a safe, accepting path?

is undecidable.

Proof.

Reduction from Post's Correspondence Problem (PCP)

… black board …

□

All accepting paths are safe

Theorem

The decision problem:

are all accepting paths of CMSG G safe?

is decidable.

Proof.

Polynomial reduction to reachability problem in pushdown automata

... black board ...

□

Pushdown automata

Definition

A pushdown automaton (PDA, for short) $K = (Q, q_0, \Gamma, \Sigma, \Delta)$ with

- Q, a finite set of control states
- $q_0 \in Q$, the initial state
- Γ, a finite stack alphabet
- Σ, a finite input alphabet
- $\Delta \subseteq Q \times \Sigma \times \Gamma \times Q \times \{\text{push}, \text{pop}, \text{skip}\}$, the transition relation.

Example

$(q, a, \gamma, q', \text{pop}) \in \Delta$ means: in state q, on reading input symbol a and top of stack is symbol γ, change to q' and pop γ.

Reachability in pushdown automata

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

Definition

Given a transition in Δ, a (direct) successor configuration c' of c is obtained: $c \vdash c'$.

Reachability problem

For configuration c, and initial configuration c_0: $c_0 \vdash^* c$?

Theorem: [Esparza et al. 2000]

The reachability problem for PDA is decidable in PTIME.

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G

- Proof idea: construct a PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ such that

$$\text{CMSG } G \text{ is not safe wrt. } (p_i, p_j) \quad \text{iff} \quad \text{PDA } K_{i,j} \text{ accepts}$$

- For accepting path $u_0 \ldots u_k$ in G, feed $K_{i,j}$ with

$$\rho_0 \ldots \rho_k \quad \text{where} \quad \rho_i \in Lin(\lambda(u_i))$$

- Possible violations that $K_{i,j}$ may encounter:
  1. $\#$ unmatched $!(p_i, p_j, \cdot) \; > \; \#$ unmatched $?(p_j, p_i, \cdot)$
  2. type of k-th unmatched send $\neq$ type of k-th unmatched receive
  3. non-FIFO communication

The nondeterministic PDA $K_{i,j}$

Let $\{a_1, \ldots, a_k\}$ be the message contents in CMSG G for (p_i, p_j).

Nondeterministic PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ where:

- Control states $Q = \{q_0, q_{a_1}, \ldots, q_{a_k}, q_{err}, q_F\}$
- Stack alphabet $\Gamma = \{1, \bot\}$

  1 counts # unmatched $!(p_i, p_j, a_m)$, and $\bot$ is stack bottom
- Input alphabet $\Sigma = \begin{cases} \text{unmatched action } !(p_i, p_j, a_m) \\ \text{unmatched action } ?(p_j, p_i, a_m) \\ \text{matched actions } !?(p_i, p_j, a_m), !?(p_j, p_i, a_m) \end{cases}$
- Transition function Δ is described on next slide

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \perp, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G
- On reading $!(p_i, p_j, a_m)$ in q_0, push 1 on stack
  - nondeterministically move to state q_{a_m} or stay in q_0
- On reading $?(p_j, p_i, a_m)$ in q_0, proceed as follows:
  - if 1 is on stack, pop it
  - otherwise, i.e., if stack is empty, accept (i.e., move to q_F)
- On reading matched send $!?(p_i, p_j, a_k)$ in q_0
  - stack empty? ignore input; otherwise, accept
- Do nothing in q_0,
  - on reading matched send events $!?(p_j, p_i, a_k)$, or
  - on reading unmatched sends or receipts not related to p_i and p_j
- Input empty? Accept, if stack non-empty; otherwise reject

Safeness of CMSGs (3)

The behaviour in state q_{a_m} for $0 < m \leqslant k$:

- Ignore all actions except $?(p_i, p_j, a_\ell)$
- On reading $?(p_i, p_j, a_\ell)$ in q_{a_m} proceed as follows
  - if 1 is on top of stack, pop it
- If stack is empty:
  - if last receive differs from a_m, accept
  - otherwise reject, while ignoring the rest (if any) of the input

RWTH AACHEN UNIVERSITY

Safeness of CMSGs (4)

It follows: PDA $K_{i,j}$ accepts iff CMSG G is not safe wrt. (p_i, p_j)

$\Longrightarrow$ CMSG G is not safe wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

$\Longrightarrow$ reachability of a configuration in a PDA is in PTIME, hence checking safeness wrt. (p_i, p_j) is in PTIME.

Time complexity

The time complexity of checking whether CMSG G is safe is in $\mathcal{O}(k^2 \cdot N^2 \cdot M \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $M = |\mathcal{C}|$.

Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(M \cdot |E|^2)$. The number of PDAs is k^2, as we consider ordered pairs. The number of paths that need to be considered in the CMSG is in $\mathcal{O}(N^2)$, as it suffices to consider a single traversal for each loop in the CMSG. □