Foundations of the UML

Lecture 7: Languages and Subclasses of CFMs

Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

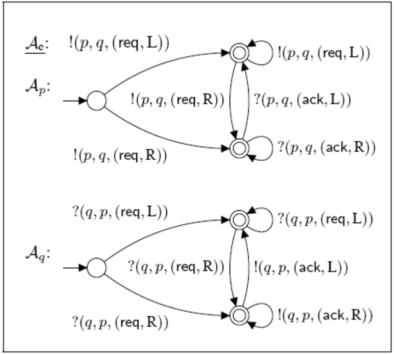
http://moves.rwth-aachen.de/i2/370

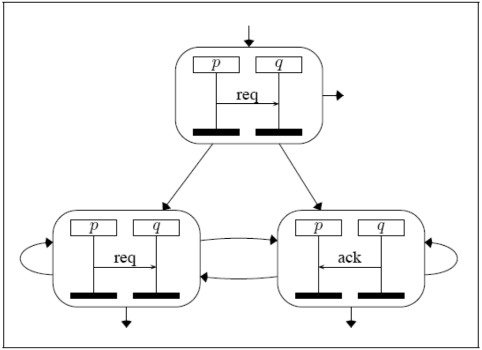
16. November 2009

Communicating finite-state machines

- A communicating finite-state machine (CFM) is a collection of finite-state machines, one for each process
- Communication between these machines takes place via (a priori) unbounded reliable FIFO channels
- The underlying system architecture is parametrised by the set $\mathcal{P}$ of processes and the set $\mathcal{C}$ of messages
- Action $!(p, q, m)$ puts message m at the end of the channel (p, q)
- Action $?(q, p, m)$ is enabled only if m is at head of buffer, and its execution by process q removes m from the channel (p, q)
- Synchronisation messages are used to avoid deadlocks

Example communicating finite-state machine





Formal definition

Definition

A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a structure

$$\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$$

where

- $\mathbb{D}$ is a nonempty finite set of synchronization messages (or data)
- for each $p \in \mathcal{P}$:
  - S_p is a non-empty finite set of local states (the S_p are disjoint)
  - $\Delta_p \subseteq S_p \times Act_p \times \mathbb{D} \times S_p$ is a set of local transitions
- $s_{init} \in S_{\mathcal{A}}$ is the global initial state
  - where $S_{\mathcal{A}} := \prod_{p \in \mathcal{P}} S_p$ is the set of global states of $\mathcal{A}$
- $F \subseteq S_{\mathcal{A}}$ is the set of global final states

We often write $s \xrightarrow{\sigma, m}_p s'$ instead of $(s, \sigma, m, s') \in \Delta_p$

Formal semantics of CFMs

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : Ch \rightarrow (\mathcal{C} \times \mathbb{D})^*\}$

Definition (global step)

$\Longrightarrow_{\mathcal{A}} \subseteq Conf_{\mathcal{A}} \times Act \times \mathbb{D} \times Conf_{\mathcal{A}}$ is defined as follows:

- sending a message: $((\overline{s}, \eta), !(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], !(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta' = \eta[(p, q) := (a, m) \cdot \eta((p, q))]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$
- receipt of a message: $((\overline{s}, \eta), ?(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], ?(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta((q, p)) = w \cdot (a, m) \neq \epsilon$ and $\eta' = \eta[(q, p) := w]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

Linearizations of a CFM

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition

A run of $\mathcal{A}$ on $\sigma_1 \ldots \sigma_n \in Act^*$ is a sequence $\rho = \gamma_0\, m_1\, \gamma_1 \ldots \gamma_{n-1}\, m_n\, \gamma_n$ such that

- $\gamma_0 = (s_{init}, \eta_\varepsilon)$ with η_ε mapping any channel to ε
- $\gamma_{i-1} \xRightarrow{\sigma_i, m_i}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \ldots, n\}$

Run ρ is accepting if $\gamma_n \in F \times \{\eta_\varepsilon\}$.

Definition

The set of linearizations of CFM $\mathcal{A}$:

$Lin(\mathcal{A}) := \{w \in Act^* \mid \text{there is an accepting run of } \mathcal{A} \text{ on } w\}$

Well-formedness

Let $Ch := \{(p,q) \mid p \neq q\,,\, p,q \in \mathcal{P}\}$ be a set of channels over $\mathcal{P}$.

We call $w = a_1 \ldots a_n \in Act^*$ proper if

1. every receive in w is preceded by a corresponding send, i.e.:
   $\forall (p,q) \in Ch$ and prefix u of w, we have:

$$\underbrace{\sum_{m \in \mathcal{C}} |u|_{!(p,q,m)}}_{\#\text{ sends from } p \text{ to } q} \quad \geqslant \quad \underbrace{\sum_{m \in \mathcal{C}} |u|_{?(q,p,m)}}_{\#\text{ receipts by } q \text{ from } p}$$

   where $|u|_a$ denotes the number of occurrences of action a in u

2. the FIFO policy is respected, i.e.:
   $\forall 1 \leqslant i < j \leqslant n$, $(p,q) \in Ch$, and $a_i = !(p,q,m_1)$, $a_j = ?(q,p,m_2)$:

$$\sum_{m \in \mathcal{C}} |a_1 \ldots a_{i-1}|_{!(p,q,m)} \; = \; \sum_{m \in \mathcal{C}} |a_1 \ldots a_{j-1}|_{?(q,p,m)} \quad \text{implies} \quad m_1 = m_2$$

A proper word w is well-formed if $\sum_{m \in \mathcal{C}} |w|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |w|_{?(q,p,m)}$

Well-formedness and CFMs

Proposition:

For any CFM $\mathcal{A}$ and $w \in Lin(\mathcal{A})$, w is well-formed.

From linearizations to partial orders

Associate to $w = a_1 \ldots a_n \in Act^*$ an Act-labelled poset

$$M(w) = (E, \prec, \ell)$$

such that:

- $E = \{1, \ldots, n\}$ are the positions in w labelled with $\ell(i) = a_i$
- $\prec = \left(\prec_{\text{msg}} \ \cup \ \bigcup_{p \in \mathcal{P}} \prec_p \right)^*$ where
  - $i \prec_p j$ if and only if $i < j$ for any $i, j \in E_p$
  - $i \prec_{\text{msg}} j$ if for some $(p, q) \in Ch$ and $m \in \mathcal{C}$ we have:

$$\ell(i) = !(p, q, m) \text{ and } \ell(j) = ?(q, p, m) \text{ and}$$
$$\sum_{m \in \mathcal{C}} |a_1 \ldots a_{i-1}|_{!(p,q,m)} \ = \ \sum_{m \in \mathcal{C}} |a_1 \ldots a_{j-1}|_{?(q,p,m)}$$

Example

construct $M(w)$ for $w = !(r, q, m)!(p, q, m_1)!(p, q, m_2)?(q, p, m_1)?(q, p, m_2)?(q, r, m)$

CFMs and well-formed words

Relating well-formed words to MSCs

For any well-formed $w \in Act^*$, $M(w)$ is an MSC.

Definition (MSC language of a CFM)

For CFM $\mathcal{A}$, let $L(\mathcal{A}) = \{ M(w) \mid w \in Lin(\mathcal{A}) \}$.

Relating well-formed words to CFMs

For any well-formed words u and v with $M(u)$ is isomorphic to $M(v)$:

$$\text{for any CFM } \mathcal{A}: \; u \in L(\mathcal{A}) \quad \text{iff} \quad v \in L(\mathcal{A}).$$

Elementary questions are undecidable for CFMs

Theorem: [Brand & Zafiropulo 1983]

The following problem:

INPUT:	CFM $\mathcal{A}$ over processes $\mathcal{P}$ and message contents $\mathcal{C}$
QUESTION:	Is $L(\mathcal{A})$ empty?

is undecidable (even if $\mathcal{C}$ is a singleton).

Proof (sketch)

Reduction from halting problem for nondeterministic Turing machine to emptiness for a CFM with two processes.

Bounded words

Definition (B-bounded words)

Let $B \in \mathbb{N}$ and $B > 0$. A word $w \in Act^*$ is called B-bounded if for any prefix u of w and any channel $(p, q) \in Ch$:

$$0 \;\leqslant\; \sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \;\leqslant\; B$$

Intuition

Word w is B-bounded if for any pair of processes (p, q), the number of sends from p to q cannot be more than B ahead of the number of receipts by q from p (for every message a).

Example

$!(1, 2, a)\ !(1, 2, b)\ ?(2, 1, a)\ ?(2, 1, b)$ is 2-bounded but not 1-bounded.

Bounded MSCs

Definition (Universally bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called universally B-bounded ($\forall B$-bounded, for short) if

$$Lin(M) \;=\; Lin^B(M)$$

where $Lin^B(M) := \{w \in Lin(M) \mid w \text{ is } B\text{-bounded}\}$.

Intuition

MSC M is $\forall B$-bounded if all its linearizations are B-bounded.

Consequence

All runs of MSC M can be realised with a buffer capacity B.

Bounded MSCs

Definition (Existentially bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called existentially B-bounded ($\exists B$-bounded, for short) if $Lin(M) \cap Lin^B(M) \neq \varnothing$.

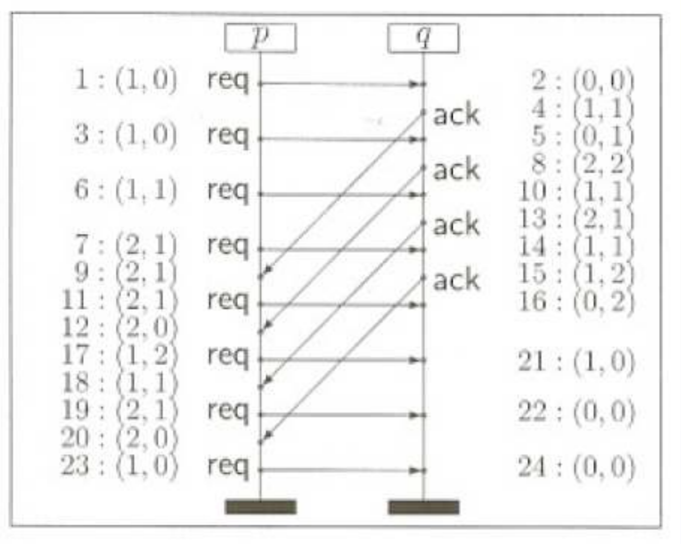
Intuition

MSC M is $\exists B$-bounded if at least one linearization is B-bounded.

Consequence

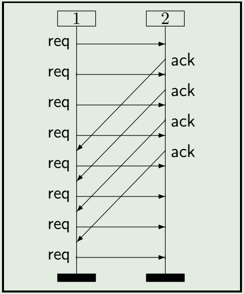
At least one run of MSC M can be realised with a buffer capacity B.

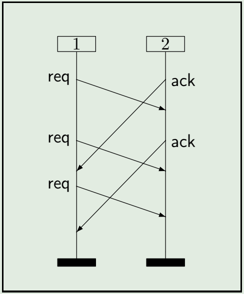
Bounded MSCs

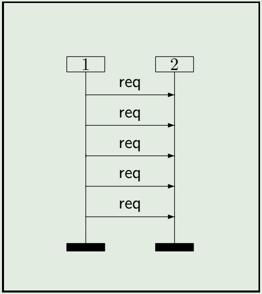


Bounded MSCs

Example







$\forall 4$-bounded
$\exists 2$-bounded
not $\exists 1$-bounded

$\forall 3$-bounded
$\exists 1$-bounded

$\forall 5$-bounded
$\exists 1$-bounded

Bounded CFMs [Henriksen et. al 2005]

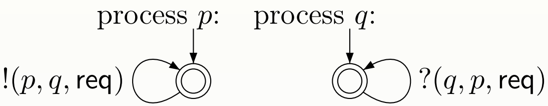
Definition (Universally bounded CFM)

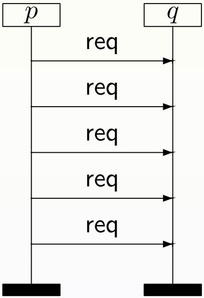
1. Let $B \in \mathbb{N}$ and $B > 0$. CFM $\mathcal{A}$ is *universally B-bounded* if any MSC in $L(\mathcal{A})$ is $\forall B$-bounded.
2. CFM $\mathcal{A}$ is *universally bounded* if it is $\forall B$-bounded for some $B \in \mathbb{N}$ and $B > 0$.

Definition (Existentially bounded CFM)

Let $B \in \mathbb{N}$ and $B > 0$. CFM $\mathcal{A}$ is *existentially B-bounded* if any MSC in $L(\mathcal{A})$ is $\exists B$-bounded.

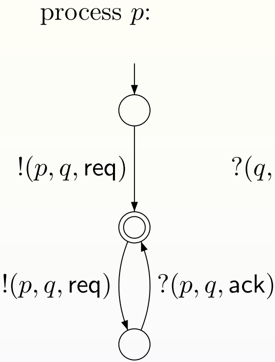
Example (1)

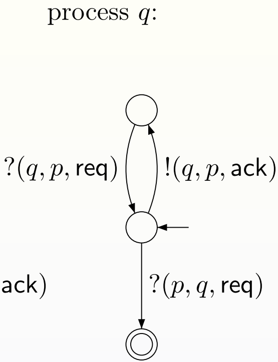


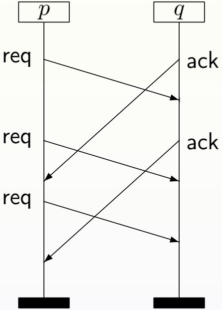


existentially 1-bounded, but not $\forall B$-bounded for any B

Example (2)

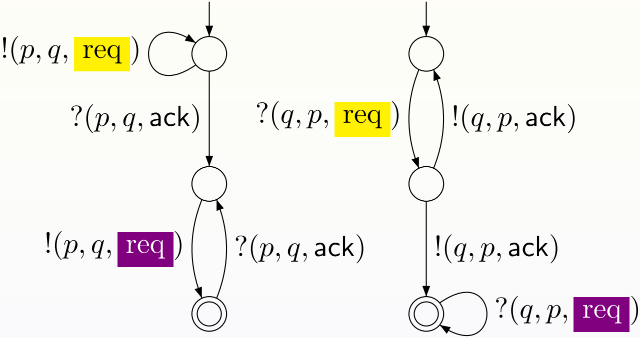






existentially 1-bounded, and $\forall$3-bounded

Example (3)



existentially $\lceil \frac{n}{2} \rceil$-bounded, but not $\forall B$-bounded for any B

Justification

- Phase 1: process p sends n messages to q
  - messages of phase 1 are tagged with data req
- … and waits for the first acknowledgement of q
- Phase 2: each ack is directly answered by p by another message
  - messages of phase 2 are tagged with data req
- So, p sends $2n$ reqs to q and q sends n acks
  - existentially $\lceil \frac{n}{2} \rceil$-bounded, but not $\forall$-bounded
- The CFM is also non-deterministic, and may deadlock

Determinism

Definition (Deterministic CFM)

A CFM $\mathcal{A}$ is *deterministic* if for all $p \in \mathcal{P}$, the transition relation Δ_p satisfies the following two conditions:

1. $(s, !(p, q, (a, m_1)), s_1) \in \Delta_p$ and $(s, !(p, q, (a, m_2)), s_2) \in \Delta_p$ implies $m_1 = m_2$ and $s_1 = s_2$
2. $(s, ?(p, q, (m, \lambda)), s_1) \in \Delta_p$ and $(s, ?(p, q, (m, \lambda)), s_2) \in \Delta_p$ implies $s_1 = s_2$

Example:

Example CFM (1) and (2) are deterministic, while (3) is not.

Deadlock-freeness

Definition (Deadlock-free CFM)

A CFM $\mathcal{A}$ is *deadlock-free* if, for all $w \in Act^*$ and all runs γ of $\mathcal{A}$ on w, there exist $w' \in Act^*$ and run γ' in $\mathcal{A}$ such that $\gamma \cdot \gamma'$ is an accepting run of $\mathcal{A}$ on $w \cdot w'$.

Example:

Example CFM (1) and (2) are deadlock-free, while (3) is not.

Definition (Product CFM)

A CFM is called a *product* CFM if $|\mathbb{D}| = 1$.

RWTH AACHEN UNIVERSITY

CFM vs. product CFM

Theorem:

Product CFM are less expressive than CFM.

Proof.

For $m, n \geqslant 1$, let $M(m,n) \in \mathbb{M}$ over $\{1,2\}$ and $\{\text{req}, \text{ack}\}$ be given by:

- $M \upharpoonright 1 = (!(1,2,\text{req}))^m \; (?(1,2,\text{ack}) \; !(1,2,\text{req}))^n$
- $M \upharpoonright 2 = ?(2,1,\text{req}) \; !(2,1,\text{ack}))^n \; (?(2,1,\text{req}))^m$

Claim: there is no product CFM over $\{1,2\}$ and $\{\text{req}, \text{ack}\}$ whose language is $L = \{M(n,n) \mid n > 0\}$. By contraposition. Suppose there is a product CFM $\mathcal{A} = ((\mathcal{A}_1, \mathcal{A}_2), \mathbb{D}, s_{init}, F)$ with $L(\mathcal{A}) = L$. For any $n > 0$, there is an accepting run of $\mathcal{A}$ on $M(n,n)$. If n is sufficiently large, then

- $\mathcal{A}_1$ visits a cycle of length $i > 0$ to read the first n letters of $M(n,n) \upharpoonright 1$
- $\mathcal{A}_2$ visits a cycle of length $j > 0$ to read the last n letters of $M(n,n) \upharpoonright 2$

But then, there is an accepting run of $\mathcal{A}$ on $M(n + (i \cdot j), n) \notin L$. □