

Theoretical Foundations of the UML

Lecture 9: Safe Realisability

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- 1 Safe realisability
- 2 Closure and inference revisited
- 3 Characterisation and complexity of safe realisability

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Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM \mathcal{A} such that $L(\mathcal{A})$ equals the set of input MSCs.

Questions:

- 1 Is this possible? (That is, is this decidable?)
- 2 If so, how complex is it to obtain such CFM?
- 3 If so, how do such algorithms work?

Problem variants (1)

Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM \mathcal{A} such that $L(\mathcal{A})$ equals the set of input MSCs.

Different forms of requirements

- Consider finite sets of MSCs, given as an enumerated set.
- Consider MSGs, that may describe an infinite set of MSCs.
- Consider MSCs whose set of linearisations is a regular word language.
- Consider MSGs that are non-local choice.

Problem variants (2)

Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM \mathcal{A} such that $L(\mathcal{A})$ equals the set of input MSCs.

Different system models

- Consider CFMs without synchronisation messages.
- Allow CFMs that may deadlock. Possibly, a realisation deadlocks.
- Forbid CFMs that deadlock. No realisation will ever deadlock.
- Consider CFMs that are deterministic.
- Consider CFMs that are bounded.
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Today's lecture

Today's setting

Realisation of a finite set of MSCs by a **deadlock-free weak** CFM.

Realisation of a finite set of well-formed words (= language) by a **deadlock-free weak** CFM.

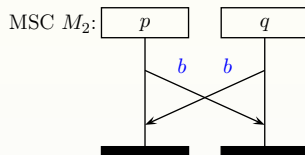
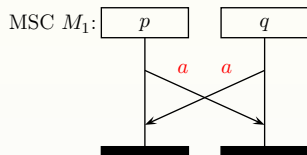
This is known as **safe realisability**.

Results:

- ① Conditions for realisability of a finite set of MSCs by a deadlock-free weak CFM.
- ② Checking safe realisability by deadlock-free CFMs is in P.
(Realisability for weak CFMs that may deadlock is co-NP complete.)

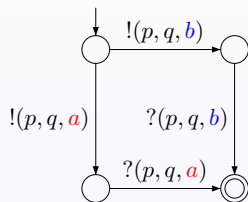
Safe realisability

Possibly a set of MSCs is realisable only by a CFM that may deadlock

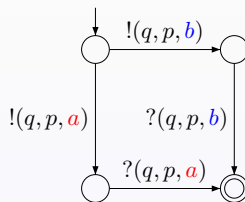


process p and q have to agree on either a or b

Realisation of $\{ M_1, M_2 \}$ by a weak CFM:



process p



process q

Deadlock occurs when, e.g.,
 p sends a and q sends b

Definition (Safe realisability)

- 1 MSC M is **safely realisable** whenever $\{M\} = L(\mathcal{A})$ for some **deadlock-free** CFM \mathcal{A} .
- 2 A finite set $\{M_1, \dots, M_n\}$ of MSCs is **safely realisable** whenever $\{M_1, \dots, M_n\} = L(\mathcal{A})$ for some **deadlock-free** CFM \mathcal{A} .
- 3 MSG G is **safely realisable** whenever $L(G) = L(\mathcal{A})$ for some **deadlock-free** CFM \mathcal{A} .

Phrased using linearisations

$L \subseteq \text{Act}^*$ is **safely realisable** if $L = \text{Lin}(\mathcal{A})$ for some deadlock-free CFM \mathcal{A} .

Note:

Safe realisability implies realisability, but the converse does not hold.

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Definition (Inference relation and closure)

For well-formed $L \subseteq Act^*$, and well-formed word $w \in Act^*$, let:

$$L \models w \quad \text{iff} \quad (\forall p \in \mathcal{P}. \exists v \in L. w \upharpoonright p = v \upharpoonright p)$$

Language L is **closed** under \models whenever $L \models w$ implies $w \in L$.

Definition (Weak closure)

Language L is **weakly closed** under \models whenever for every prefix w of some word in L , it holds $L \models w$ implies $w \in L$.

Weak closure thus restricts closure under \models to prefixes in L only. Previously, closure was required for all $w \in Act^*$.

Deadlock-free closure

For language L , let $\text{pref}(L) = \{w \mid \exists u. w \cdot u \in L\}$ the set of **prefixes** of L .

Definition ((Deadlock-free) Inference relation)

For well-formed $L \subseteq \text{Act}^*$, and proper word $w \in \text{Act}^*$, i.e., w is a **prefix of a well-formed word**, let:

$$L \models^{df} w \quad \text{iff} \quad (\forall p \in \mathcal{P}. \exists v \in L. w \upharpoonright p \text{ is a prefix of } v \upharpoonright p)$$

Definition (Closure under \models^{df})

Language L is **closed** under \models^{df} whenever $L \models^{df} w$ implies $w \in \text{pref}(L)$.

Intuition

The closure condition says that the set of partial MSCs (i.e., prefixes of L) can be constructed from the projections of the MSCs in L onto individual processes.

Deadlock-free weak CFM are closed under \models^{df}

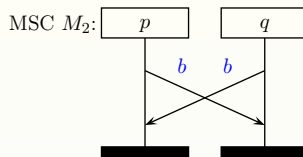
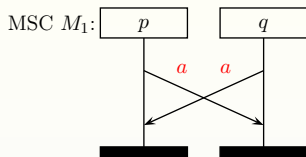
Lemma:

For any **deadlock-free** weak CFM \mathcal{A} , $Lin(\mathcal{A})$ is closed under \models^{df} .

Proof

Similar proof strategy as for the closure of weak CFMs under \models . Basic intuition is that if $w \upharpoonright p$ is a prefix of $v^p \upharpoonright p$, then from the point of view of process p , w can be prolonged with the word u , say, such that $w \cdot u = v^p$. This applies to all processes, and as the weak CFM is deadlock-free, such continuation is always possible.

Example



Example

$L = Lin(\{M_1, M_2\})$ is not closed under \models^{df} :

$$w = !(p, q, a)!(q, p, b) \notin \text{pref}(L)$$

But: $L \models^{df} w$ since w is a proper prefix of a well-formed word, and

- for process p , there exists $u \in L$ with $w \upharpoonright p = !(p, q, a) \in \text{pref}(\{u \upharpoonright p\})$, and
- for process q , there exists $v \in L$ with $w \upharpoonright q = !(q, p, b) \in \text{pref}(\{v \upharpoonright q\})$.

Note that L is closed under \models . So this shows that closure under \models does not imply closure under \models^{df} .

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Theorem:

[Alur et al., 2001]

$L \subseteq Act^*$ is **safely** realisable iff L is **weakly** closed under \models and closed under \models^{df} .

Proof

On the black board.

Corollary

The finite set of MSCs $\{M_1, \dots, M_n\}$ is safely realisable iff $\bigcup_{i=1}^n Lin(M_i)$ is closed under \models and \models^{df} .

Theorem

For any well-formed $L \subseteq Act^*$:

L is **regular**, weakly closed under \models and closed under \models^{df}
if and only if

$L = Lin(\mathcal{A})$ for some **\forall -bounded** deadlock-free weak CFM \mathcal{A} .

Theorem:

[Alur et al., 2001]

The decision problem “is a given set of MSCs safely realisable?” is in P.

Proof

- 1 For a given finite set of MSCs, safe realisability can be checked in time $\mathcal{O}((k^2 + r) \cdot n)$ where n is the number of processes, k the number of MSCs, and r the number of events in all MSCs together.
- 2 If the MSCs are not safely realisable, the algorithm returns an MSC which is implied, but not included in the input set of MSCs.

(We skip the details in this lecture.)