Theoretical Foundations of the UML
Lecture 10: Regular MSCs

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Outline

Overview

Realisabiliy and safe realisability

Definition (Realisability)

1. MSC M is realisable whenever $\{M\} = L(\mathcal{A})$ for some CFM $\mathcal{A}$.

2. A finite set $\{M_1, \ldots, M_n\}$ of MSCs is realisable whenever $\{M_1, \ldots, M_n\} = L(\mathcal{A})$ for some CFM $\mathcal{A}$.

3. MSG G is realisable whenever $L(G) = L(\mathcal{A})$ for some CFM $\mathcal{A}$.

Definition (Safe realisability)

Same as above except that the CFM should be deadlock-free.

Summary of results

Approach so far:

The (safe) realisation of a (finite) set of MSCs by a weak CFM is the one where the automaton $\mathcal{A}_p$ of process p generates the projections of these MSCs on p.

Results so far:

1. Conditions for (safe) realisability for languages obtained by finite sets of MSCs.
2. Checking safe realisability for such languages is in P.
3. Checking realisability for such languages is co-NP complete.

Some remaining questions

- Can results be obtained for larger classes of MSGs?

- What happens if we allow synchronisation messages?
  - recall that weak CFMs do not involve synchronisation messages

- How do we obtain a CFM realising an MSG algorithmically?
  - in particular, for non-local choice MSGs

- Are there simple conditions on MSGs that guarantee realisability?
  - e.g., easily identifiable subsets of (safe) realisable MSGs

Today's lecture

Today's setting

(Safe) Realisability of a regular set of MSCs.

Or, erquivalently: (safe) realisability of a regular set of well-formed words (that is, a regular language).

Results:

1. Checking whether a regular language L is well-formed is decidable.
2. For well-formed language L:
   - L is regular iff it is (safely) realisable by a $\forall$-bounded CFM.
3. Checking whether an MSG is regular is undecidable.
4. If an MSG is (locally) communication-closed, it is regular.
5. Checking whether an MSG is comm.-closed is coNP-complete.
6. Checking whether an MSG is locally communication-closed is in P.

Overview

Regular MSCs

Let $\mathbb{M}$ be the set of MSCs over $\mathcal{P}$ and $\mathcal{C}$.

Definition (Regular)

1. $\mathcal{M} = \{M_1, \ldots, M_n\}$ with $n \in \mathbb{N} \cup \{\infty\}$ is called regular if $Lin(\mathcal{M}) = \bigcup_{i=1}^{n} Lin(M_i)$ is a regular word language over Act^*.
2. MSG G is regular if $Lin(G)$ is a regular word language over Act^*.
3. CFM $\mathcal{A}$ is regular if $Lin(\mathcal{A})$ is a regular word language over Act^*.

Here, Act is the set of actions in $\mathcal{M}$, G, and $\mathcal{A}$, respectively.

Lemma:

Every $\forall$-bounded CFM is regular.   Why?

On the black board.

Regularity and well-formedness

Theorem

The decision problem "is a regular language $L \subseteq Act^*$ well-formed"?
—that is, does L represent a set of MSCs?— is decidable.

Proof

Since L is regular, there exists a minimal DFA $\mathcal{A} = (S, Act, s_0, \delta, F)$
that accepts L. Consider the productive states in this DFA, i.e., all
states from which some state in F can be reached. We label any
productive state s with a channel-capacity function $K_s : Ch \to \mathbb{N}$ such
that four constraints (cf. next slide) are fulfilled. Then: L is well-formed
iff each productive state in the DFA $\mathcal{A}$ can be labeled with K_s
satisfying these constraints. In fact, if a state-labeling violates any of
these constraints, it is due to a word that is not well-formed.

Constraints on state-labelling

1. $s \in F \cup \{s_0\}$, implies $K_s((p, q)) = 0$ for every channel (p, q).

2. $\delta(s, !(p, q, a)) = s'$ implies

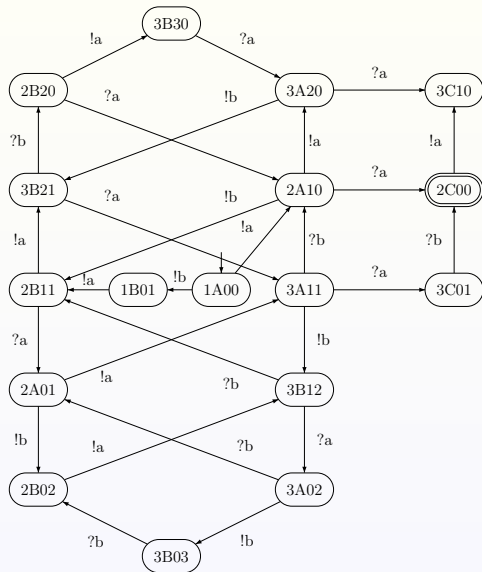$$K_{s'}(c) = \begin{cases} K_s(c) + 1 & \text{if } c = (p, q) \\ K_s(c) & \text{otherwise.} \end{cases}$$

3. $\delta(s, ?(p, q, a)) = s'$ implies $K_s((q, p)) > 0$ and

$$K_{s'}(c) = \begin{cases} K_s(c) - 1 & \text{if } c = (q, p) \\ K_s(c) & \text{otherwise.} \end{cases}$$

4. $\delta(s, \alpha) = s_1$ and $\delta(s_1, \beta) = s_2$ with $\alpha \in Act_p$ and $\beta \in Act_q$, $p \neq q$, implies

   not $(\alpha = !(p, q, a)$ and $\beta = ?(q, p, a))$, or $K_s((p, q)) > 0$
   implies $\delta(s, \beta) = s_1'$ and $\delta(s_1', \alpha) = s_2$ for some $s_1' \in S$.

These constraints can be checked in linear time in the size of δ.

Yannakakis' example

Boundedness and regularity

Definition (B-bounded words)

Let $B \in \mathbb{N}$ and $B > 0$. A word $w \in Act^*$ is called B-bounded if for any prefix u of w and any channel $(p, q) \in Ch$:

$$0 \leqslant \sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \leqslant B$$

Corollary:

For any regular, well-formed language L, there exists $B \in \mathbb{N}$ and $B > 0$ such that every $w \in L$ is B-bounded.

Proof

The bound is the largest value attained by the channel-capacity functions assigned to productive states in the proof of the previous theorem.

Overview

Regularity and realisability

Theorem: [Henriksen et al., 2005, Baudru & Morin, 2007]

For any set L of well-formed words, the following statements are equivalent:

1. L is regular.
2. L is realisable by a $\forall$-bounded CFM.
3. L is realisable by a deterministic $\forall$-bounded CFM.
4. L is safely realisable by a $\forall$-bounded CFM.

Lemma:

The maximal size of the CFM realising L is such that for each process p, the number $|Q_p|$ of states of local automaton $\mathcal{A}_p$ is:

1. double exponential in the bound B, and n^2 where $n = |\mathcal{P}|$, and
2. exponential in $m \log m$ where m is the size of a minimal DFA for L.

Overview

Regularity for MSGs is undecidable

Theorem [Henriksen et. al, 2005]

The decision problem "is MSG G regular"? is undecidable.

Proof

Outside the scope of this lecture.

Towards structural conditions for regular MSGs

- MSG G is regular if $Lin(G)$ is a regular language

- Regularity yields deterministic, or safe, but bounded CFMs

- But, "is MSG G regular"? is unfortunately undecidable

- Is it possible to impose structural conditions on MSGs that guarantee regularity?

- Yes we can. For instance, by constraining:
  1. the communication structure of the MSCs in loops of G, or
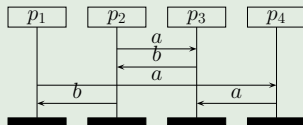 2. the structure of expressions describing the MSCs in G

Communication graph

Definition (Communication graph)

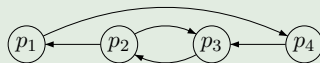The communication graph of the MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, <)$ is the directed graph $(V, \rightarrow)$ with:

- $V = \mathcal{P} \setminus \{\, p \in \mathcal{P} \mid E_p = \varnothing \,\}$, the set of active processes
- $(p, q) \in \rightarrow$ if and only if $l(e) = !(p, q, a)$ for some $e \in E$ and $a \in \mathcal{C}$

Example

an example MSC

its communication graph

Strongly connected components

Let $G = (V, \rightarrow)$ be a directed graph.

Strongly connected component

- $T \subseteq V$ is strongly connected if for any $v, w \in T$, vertices v and w are mutually reachable (via $\rightarrow$) from each other.
- T is a strongly connected component (SCC) of G it T is strongly connected and T is not properly contained in another SCC.

Determining the SCCs of a digraph can be done in linear time in the size of V and $\rightarrow$.

Communication closedness

Definition (Communication closedness)

MSG G is communication-closed if for any loop $\pi = v_1 v_2 \ldots v_n$ (with $v_1 = v_n$) in G, the MSC $M(\pi) = \lambda(v_1) \bullet \lambda(v_2) \bullet \ldots \bullet \lambda(v_n)$ has a strongly connected communication graph.

Example

On the black board.

Communication-closed vs. regularity

Theorem:
Any communication-closed MSG G is regular.
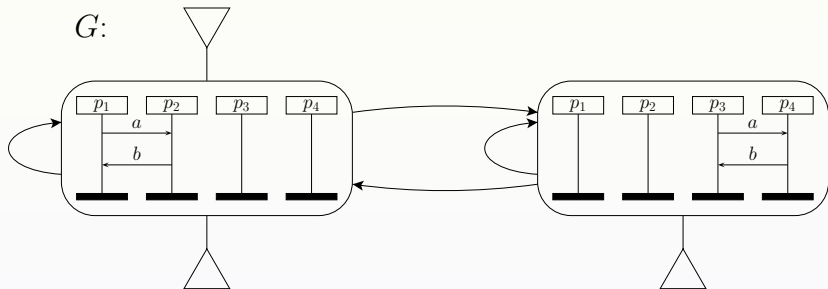
Example
Example on the black board.

Note:
The converse does not hold (cf. next slide).

Communication-closedness is not a necessary condition for regularity:

G:

MSG G is not communication-closed, but $Lin(G)$ is regular.

Checking communication-closedness

> **Theorem:**
>
> The decision problem "is MSG G communication closed?" is co-NP complete.

> **Proof**
>
> 1. Membership in co-NP can be proven in a standard way: guess a subgraph of G, check in polynomial time whether this subgraph has a loop passing through all its vertices, and check whether its communication graph is not strongly connected.
>
> 2. It can be shown that the problem is co-NP hard by a reduction from the 3-SAT problem.

Communication-closed vs. regularity

Definition (Asynchronous iteration)

For $\mathcal{M}_1, \mathcal{M}_2 \subseteq \mathbb{M}$ sets of MSCs, let:

$$\mathcal{M}_1 \bullet \mathcal{M}_2 = \{ M_1 \bullet M_2 \mid M_1 \in \mathcal{M}_1, M_2 \in \mathcal{M}_2 \}$$

For $\mathcal{M} \subseteq \mathbb{M}$ let

$$\mathcal{M}^i = \begin{cases} \{M_\epsilon\} & \text{if } i=0, \text{ where } M_\epsilon \text{ denotes the empty MSC} \\ \mathcal{M} \bullet \mathcal{M}^{i-1} & \text{if } i > 0 \end{cases}$$

The asynchronous iteration of $\mathcal{M}$ is now defined by:

$$\mathcal{M}^* = \bigcup_{i \geqslant 0} \mathcal{M}^i.$$

Finitely generated

Definition (Finitely generated)

Set of MSCs $\mathcal{M} \subseteq \mathbb{M}$ is finitely generated if there is a finite set of MSCs $\widehat{\mathcal{M}} \subseteq \mathbb{M}$ such that $\mathcal{M} \subseteq \widehat{\mathcal{M}}^*$.

Remarks:

1. Each set of MSCs defined by an MSG G is finitely generated.
2. Not every regular well-formed language is finitely generated.
3. Not every finitely generated set of MSCs is regular.
4. It is decidable to check whether a set of MSCs is finitely generated.
   - details of the last decidability results are outside the scope of this lecture.

Characterisation of communication-closedness

Theorem:

Let $\mathcal{M}$ be a set of MSCs. Then:

$$\mathcal{M} \text{ is finitely generated and regular}$$

$$\text{iff}$$

$$\mathcal{M} = L(G) \text{ for communication-closed MSG } G.$$

Local communication-closedness

Definition (Local communication-closedness)

MSG G is locally communication-closed if for each vertex (v, v') in G, the MSCs $\lambda(v)$, $\lambda(v')$, and $\lambda(v) \bullet \lambda(v')$ all have weakly connected communication graphs.

Notes:

1. A directed graph is weakly connected if its induced undirected graph (obtained by ignoring the directions of edges) is strongly connected.

2. Checking whether MSG G is locally communication-closed can be done in linear time.

Locally communication-closed MSGs are realisable

Theorem: [Genest et al., 2006]

For any locally communication-closed MSG G, there exists a CFM $\mathcal{A}$ with $L(\mathcal{A}) = L(G)$ of size $n^{\mathcal{O}(|\mathcal{P}|)}$ where n is the number of vertices in G.

Thus: every locally communication-closed MSG is realisable.

FIFO communication:

	finite MSGs*	communication-closed MSGs	general MSGs
realisability	co-NP complete	undecidable	undecidable
safe realisability	PTIME	EXPSPACE-complete	undecidable

* MSG G is finite if $L(G)$ is a finite set of MSCs.

non-FIFO communication[1]:

	finite MSGs	communication-closed MSGs	general MSGs
realisability	co-NP complete	PSPACE-hard	undecidable
safe realisability	PTIME	EXPSPACE-complete	undecidable

[1]Considered only for the sake of completeness.