Theoretical Foundations of the UML

Lecture 13: Verifying PDL Formulas

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Outline

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Overview

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A logic for MSCs

- This lecture will be devoted to a logic that is interpreted over MSCs
- The logic is used to umambigously express properties of MSCs
  - does a given MSC M satisfy the logical formula φ?
- And to characterise a set of MSCs by means of a logical formula
  - all MSCs that satisfy the formula φ
- Our logic is a variant of propositional dynamic logic (PDL) [Fischer & Ladner, 1979]
  - combines easy-to-grasp concepts such as regular expressions and Boolean operators
- We consider syntax, semantics, examples and the membership problem.

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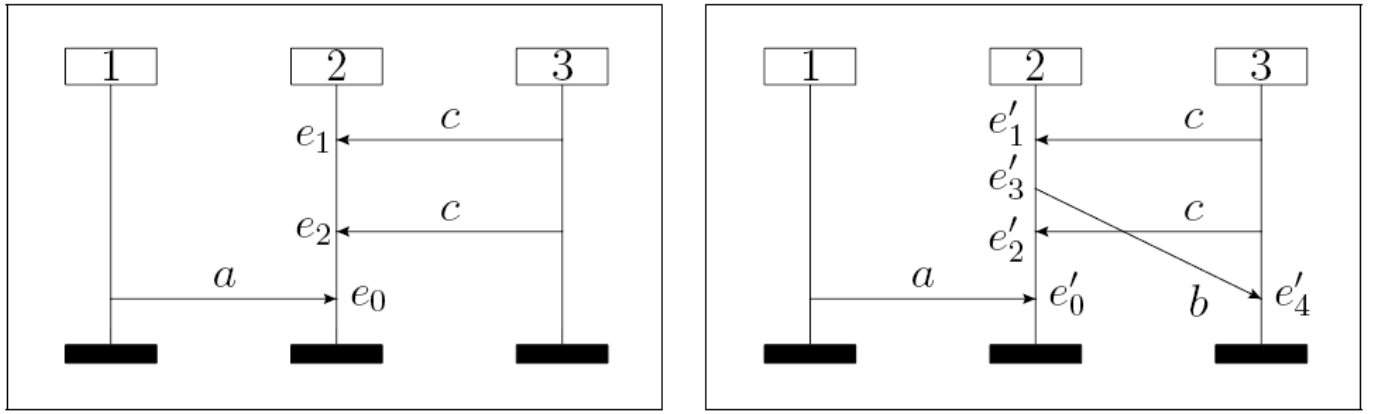
Some informal examples



1. The (unique) maximal event of M is labeled by $?(2, 1, a)$ Yes. No.
2. The maximal event on process 2 is labeled by $?(2, 1, a)$ Yes. Yes.
3. No two consecutive events are labeled with $?(2, 3, c)$ No. Yes.
4. The number of send events at process 3 is odd. No. No.

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Overview

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Local formulas

Local formulas

These are statements over single events in an MSC. That is, an event either satisfies or refutes such formula.

Example local formulas

- $!(1,2,a)$ The current event is labeled with $!(1,2,a)$
- $\langle \mathsf{proc} \rangle true$ There is a next event at the same process
- $\langle \mathsf{proc}; \mathsf{proc} \rangle true$ There are (at least) two next events at this process
- $[\mathsf{proc}]^{-1} false$ There is no preceding event at this process
- $\langle \mathsf{msg} \rangle true$ This send event matches a (next) receive event
- $\langle \{\, !(1,2,a) \,\}; \mathsf{proc} \rangle\, ?(1,2,b)$ Event $!(1,2,a)$ is followed by $?(1,2,b)$
- $[\mathsf{proc} + \mathsf{msg}] \neg \{\, !(1,2,a) \,\}$ Any next event differs from $!(1,2,a)$

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Local formulas

Definition (Syntax of local formulas)

For communication action $\sigma \in Act$ and path expression α, the grammar of local formulas is given by:

$$\varphi \ ::= \ true \ \mid \ \sigma \ \mid \ \neg\varphi \ \mid \ \varphi \vee \varphi \ \mid \ \langle\alpha\rangle\varphi \ \mid \ \langle\alpha\rangle^{-1}\varphi$$

Path expressions will be defined later on.

Definition (Derived operators)

$$\begin{aligned} false &= \neg true \\ \varphi_1 \wedge \varphi_2 &= \neg(\neg\varphi_1 \vee \neg\varphi_2) \\ \varphi_1 \rightarrow \varphi_2 &= \neg\varphi_1 \vee \varphi_2 \\ [\alpha]\varphi &= \neg\langle\alpha\rangle\neg\varphi \\ [\alpha]^{-1}\varphi &= \neg\langle\alpha\rangle^{-1}\neg\varphi \end{aligned}$$

Intuitive meaning of local formulas

$true$	Valid statement. Satisfied by every event.
σ	Current event is labelled with σ
$\neg\varphi$	Current event does not satisfy φ
$\varphi_1 \vee \varphi_2$	Current event satisfies φ_1 or φ_2
$\langle\alpha\rangle\varphi$	Some forward path satisfying α reaches an event satisfying φ
$\langle\alpha\rangle^{-1}\varphi$	Some backward path α reaches an event satisfying φ
$[\alpha]\varphi$	All forward paths satisfying α reach an event satisfying φ
$[\alpha]^{-1}\varphi$	All backward paths satisfying α reach an event satisfying φ

How are path expressions like α defined?

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Path expressions

Definition (Syntax of local formulas)

For communication action $\sigma \in Act$ and path expression α, the grammar of local formulas is given by:

$$\varphi \ ::= \ true \ \mid \ \sigma \ \mid \ \neg\varphi \ \mid \ \varphi \vee \varphi \ \mid \ \langle\alpha\rangle\varphi \ \mid \ \langle\alpha\rangle^{-1}\varphi$$

Definition (Syntax of path expressions)

For local formula φ, the grammar of path expressions is given by:

$$\alpha \ ::= \ \{\varphi\} \ \mid \ \mathsf{proc} \ \mid \ \mathsf{msg} \ \mid \ \alpha;\alpha \ \mid \ \alpha + \alpha \ \mid \ \alpha^*$$

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Intuitive meaning of path expressions

- $\{\,\varphi\,\}$ specifies an event that satisfies φ
- proc requires a (direct) successor relation between events at the same process
- msg requires a matching between current event and a receive event
- The composition $\alpha;\beta$ defines the set of pairs (e, e') for which there exist event e'' such that $(e, e'') \models \alpha$ and $(e'', e') \models \beta$
- $\alpha + \beta$ denotes the union of the relations α and β
- α^* denotes the reflexive and transitive closure of the relation α

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Intuitive meaning of local formulas

- Local formulas are interpreted over MSC events
- Event e satisfies $\underbrace{!(p,q,a)}_{\sigma}$ iff e is labelled with action $\underbrace{!(p,q,a)}_{\sigma}$
- Path expression α defines a binary relation between events:
  1. $\{\varphi\}$ is the set of pairs (e,e') such that e satisfies φ
  2. $(e,e') \models \mathsf{proc}$ iff e and e' reside at the same process and e' is a direct successor of e wrt. $<_p$
  3. $(e,e') \models \mathsf{msg}$ iff e' is the matching event of e, i.e., $e' = m(e)$

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Forward and backward local formulas

- Event e satisfies $\langle\alpha\rangle\varphi$ iff there is an event e' such that a path from e to e' satisfies α and e' satisfies φ

  Formula $\langle\alpha\rangle\varphi$ looks "forward" along the partial order of the MSC starting from the current event

- The interpretation of $\langle\alpha\rangle^{-1}\varphi$ is dual, i.e., e satisfies it iff there is an event e' such that some path from e' to e satisfies α and e' satisfies φ

  Formula $\langle\alpha\rangle^{-1}\varphi$ looks "backward" along the partial order of the MSC starting from the current event

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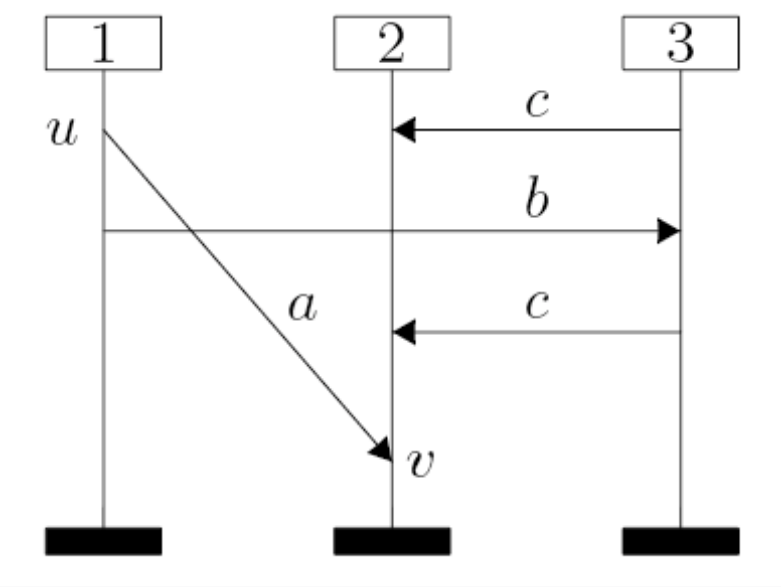
Example



1. $u \models !(1, 2, a)$ — u is labelled with the action $!(1, 2, a)$
2. $u \models [\mathsf{proc}]^{-1}\, \mathit{false}$ — u is the first event on the process line
3. $u \models \langle(\mathsf{proc} + \mathsf{msg})^*\rangle ?(2, 1, a)$ — event u happens before the event v

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Semantics of local formulas (1)

Definition (Syntax of local formulas)

For communication action $\sigma \in Act$ and path expression α:

$$\varphi \ ::= \ true \ \mid \ \sigma \ \mid \ \neg\varphi \ \mid \ \varphi \vee \varphi \ \mid \ \langle\alpha\rangle\varphi \ \mid \ \langle\alpha\rangle^{-1}\varphi$$

Definition (Semantics of base local formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC and $e \in E$.

Binary relation $\models$ is defined such that $((M, e), \varphi) \in \models$ iff event e of MSC M satisfies local formula φ. We write $M, e \models \varphi$ for $((M, e), \varphi) \in \models$.

$$\begin{aligned} M, e \models true & \qquad \text{for all } e \in E \\ M, e \models \sigma \quad \text{iff} & \quad l(e) = \sigma \\ M, e \models \neg\varphi \quad \text{iff} & \quad \text{not } M, e \models \varphi \\ M, e \models \varphi_1 \vee \varphi_2 \quad \text{iff} & \quad M, e \models \varphi_1 \text{ or } M, e \models \varphi_2 \end{aligned}$$

Semantics of local formulas (2)

Definition (Semantics of forward path formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC and $e \in E$.

$$
\begin{aligned}
e \models \langle \{\psi\} \rangle \varphi \quad &\text{iff} \quad e \models \psi \text{ and } e \models \varphi \\
e \models \langle \mathsf{proc} \rangle \varphi \quad &\text{iff} \quad \exists e' \in E.\, e \lessdot_p e' \text{ and } e' \models \varphi \\
e \models \langle \mathsf{msg} \rangle \varphi \quad &\text{iff} \quad \exists e' \in E.\, e' = m(e) \text{ and } e' \models \varphi \\
e \models \langle \alpha_1 ; \alpha_2 \rangle \varphi \quad &\text{iff} \quad e \models \langle \alpha_1 \rangle \langle \alpha_2 \rangle \varphi \\
e \models \langle \alpha_1 + \alpha_2 \rangle \varphi \quad &\text{iff} \quad e \models \langle \alpha_1 \rangle \varphi \text{ or } e \models \langle \alpha_2 \rangle \varphi \\
e \models \langle \alpha^* \rangle \varphi \quad &\text{iff} \quad \exists n \in \mathbb{N}.\, e \models (\langle \alpha \rangle)^n \varphi
\end{aligned}
$$

Where $e \lessdot_p e'$ iff $e <_p e'$ and $\neg(\exists e''.\, e <_p e'' <_p e')$, i.e., e' is a direct successor of e under $<_p$.

Semantics of local formulas (3)

Definition (Semantics of backward path formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC and $e \in E$.

$$
\begin{aligned}
e \models \langle \{\psi\} \rangle^{-1} \varphi \quad &\text{iff} \quad e \models \psi \text{ and } e \models \varphi \\
e \models \langle \mathsf{proc} \rangle^{-1} \varphi \quad &\text{iff} \quad \exists e' \in E.\, e' <_p e \text{ and } e' \models \varphi \\
e \models \langle \mathsf{msg} \rangle^{-1} \varphi \quad &\text{iff} \quad \exists e' \in E.\, e' = m^{-1}(e) \text{ and } e' \models \varphi \\
e \models \langle \alpha_1 ; \alpha_2 \rangle^{-1} \varphi \quad &\text{iff} \quad e \models \langle \alpha_1 \rangle^{-1} \langle \alpha_2 \rangle^{-1} \varphi \\
e \models \langle \alpha_1 + \alpha_2 \rangle^{-1} \varphi \quad &\text{iff} \quad e \models \langle \alpha_1 \rangle^{-1} \varphi \text{ or } e \models \langle \alpha_2 \rangle^{-1} \varphi \\
e \models \langle \alpha^* \rangle^{-1} \varphi \quad &\text{iff} \quad \exists n \in \mathbb{N}.\, e \models \left(\langle \alpha \rangle^{-1} \right)^n \varphi
\end{aligned}
$$

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Overview

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PDL formulas

Definition (Syntax of PDL formulas)

For local formula φ, the grammar of PDL formulas is given by:

$$\Phi \ ::= \ \exists\varphi \ \mid \ \forall\varphi \ \mid \ \Phi \wedge \Phi \ \mid \ \Phi \vee \Phi$$

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Intuitive meaning of PDL formulas

- MSC M satisfies $\exists\varphi$ if it has some event e satisfying φ
- MSC M satisfies $\exists\langle\alpha\rangle\varphi$ if from some event e in M, there exists an α-labelled path from e to an event e', say, satisfying φ
- MSC M satisfies $\exists[\alpha]\varphi$ if from some event e in M, any event that can be reached via an α-labelled path satisfies φ

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Semantics of PDL formulas

Definition (Semantics of PDL formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC.

$(M, \Phi) \in \models$ iff PDL formula Φ holds in MSC M.

$$
\begin{aligned}
M \models \exists\varphi \quad &\text{iff} \quad \exists e \in E.\, M, e \models \varphi \\
M \models \forall\varphi \quad &\text{iff} \quad \forall e \in E.\, M, e \models \varphi \\
M \models \Phi_1 \wedge \Phi_2 \quad &\text{iff} \quad M \models \Phi_1 \text{ and } M \models \Phi_2 \\
M \models \Phi_1 \vee \Phi_2 \quad &\text{iff} \quad M \models \Phi_1 \text{ or } M \models \Phi_2
\end{aligned}
$$

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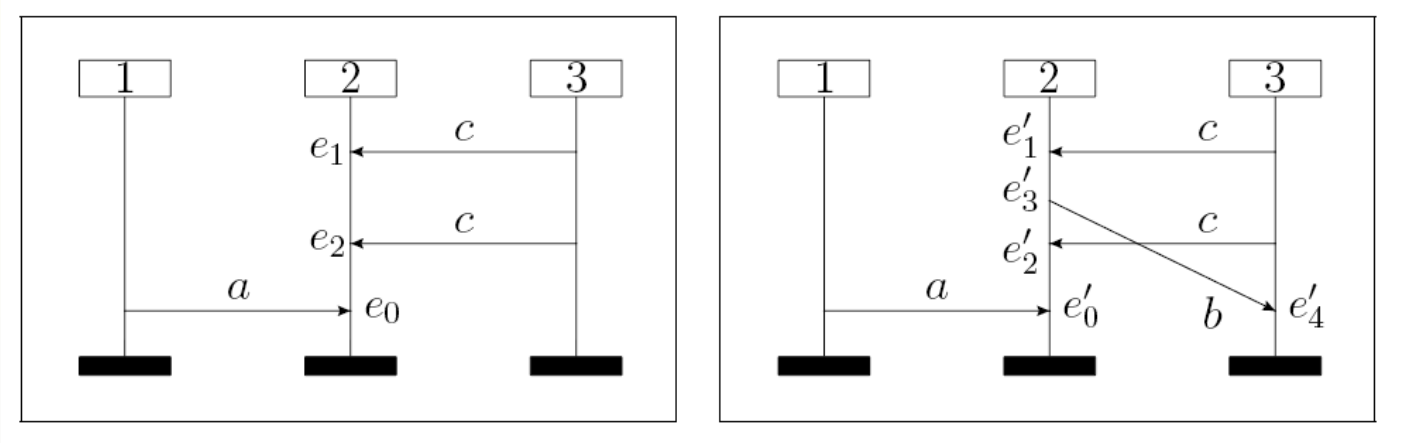
Example (1)



- The (unique) maximal event of M is labelled by $?(2, 1, a)$ Yes. No.
- $\forall\,(\langle(\mathsf{proc} + \mathsf{msg})^*\rangle([\mathsf{proc}]\,\mathit{false} \wedge ?(2, 1, a)))$ Yes. No.

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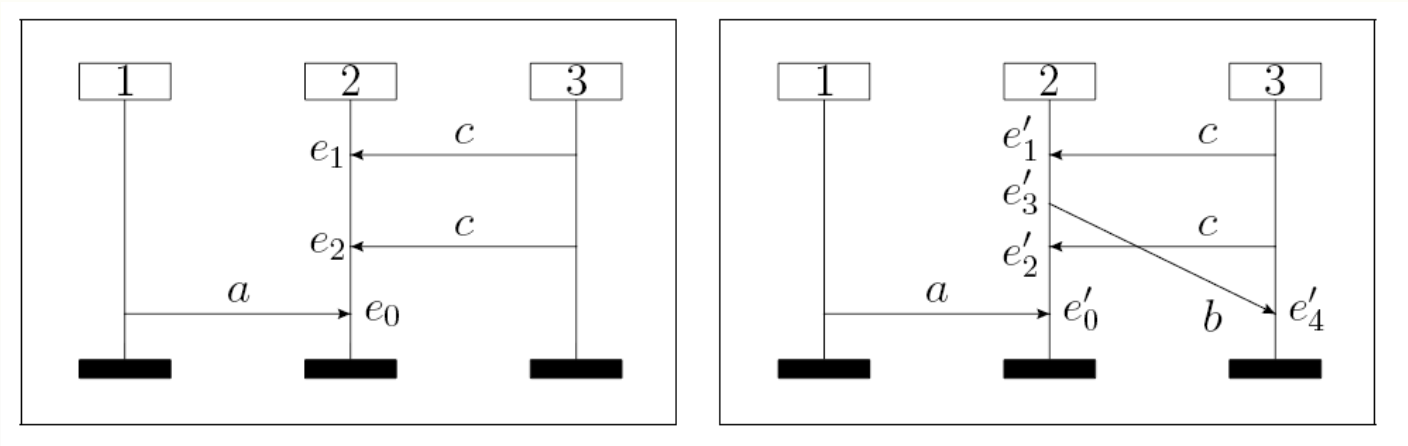
Example (2)



- The maximal event on process 2 is labelled by $?(2, 1, a)$ Yes. Yes.
- $\exists\,([\mathsf{proc}]\,\mathit{false} \wedge ?(2, 1, a))$ Yes. Yes.

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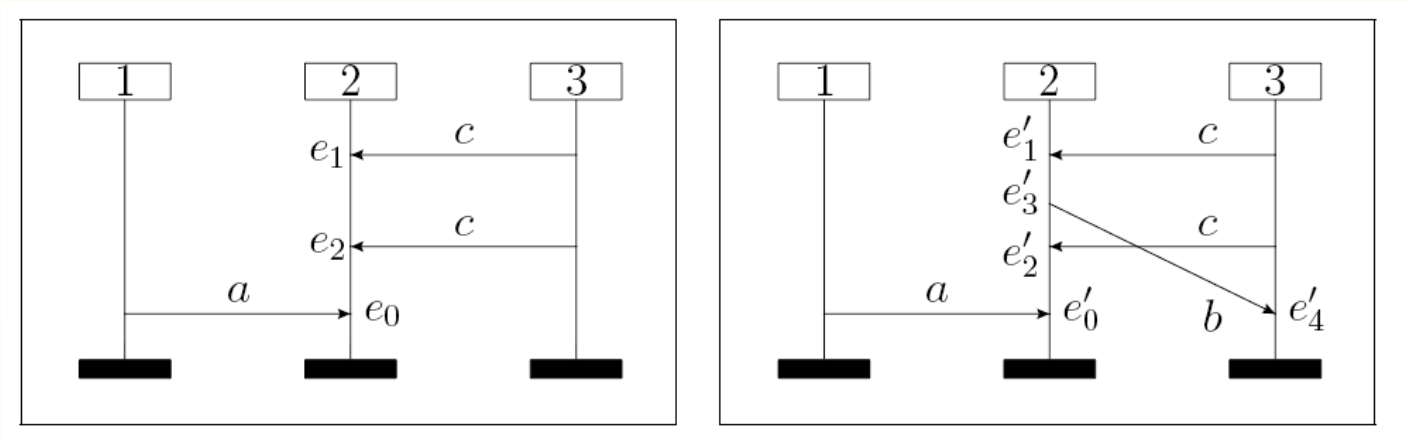
Example (3)



- No two consecutive events are labelled with $?(2, 3, c)$ No. Yes.
- $\forall\,([?(2, 3, c); \mathsf{proc}; ?(2, 3, c)]\, \mathit{false})$ No. Yes.

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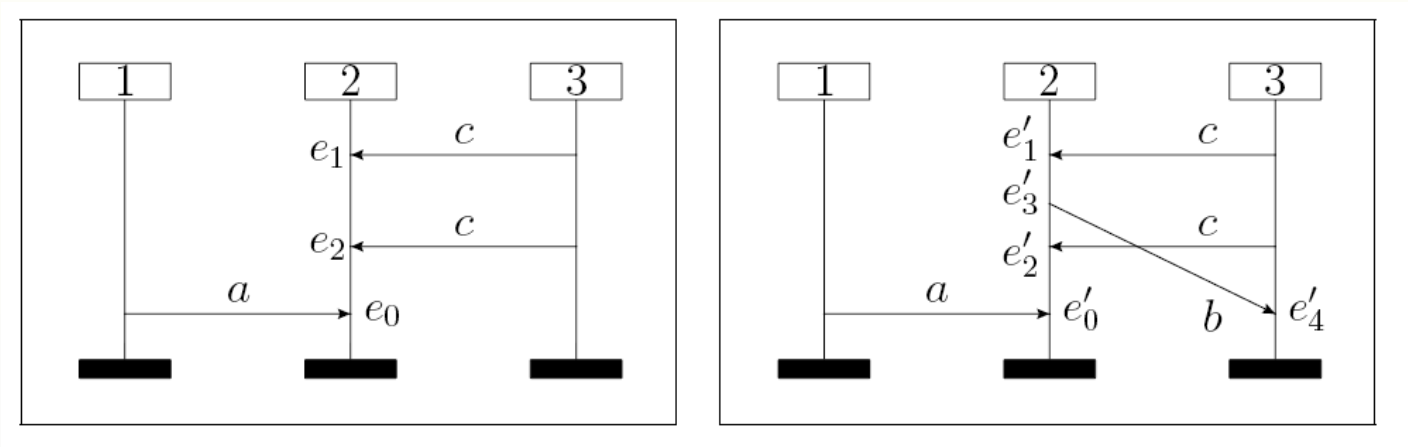
Example (4)



- The number of send events at process 3 is odd. No. No.
- See next slide

Example (slightly changed)

MSC M has an even number of messages sent from process 1 to 2:

$$\forall\Big(\quad \underbrace{[\mathsf{proc}]^{-1}\, \mathit{false} \wedge P_1}_{\text{minimal event on process 1}} \quad \rightarrow \quad \langle\alpha\rangle \quad \underbrace{[\mathsf{proc}]\, \mathit{false}}_{\text{maximal event on process}} \quad \Big)$$

where $P_1 = \bigvee_{j \in \mathcal{P}, j \neq 1} (!_{1,j} \vee ?_{1,j})$ with $!_{1,j} = \bigvee_{a \in \mathcal{C}} !(1, j, a)$ and $?_{1,j}$ is defined in a similar way, i.e., $e \models P_1$ iff e occurs at process 1.

Path expression α is defined by:

$$\alpha \;=\; ((\{\neg !_1\}; \mathsf{proc})^*; \{!_1\}; \mathsf{proc}; (\{\neg !_1\}; \mathsf{proc})^*; \{!_1\}; \mathsf{proc}; (\{\neg !_1\}; \mathsf{proc})^*)^*$$

and where $!_1$ abbreviates $\bigvee_{a \in \mathcal{C}} !(1, 2, a)$

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Overview

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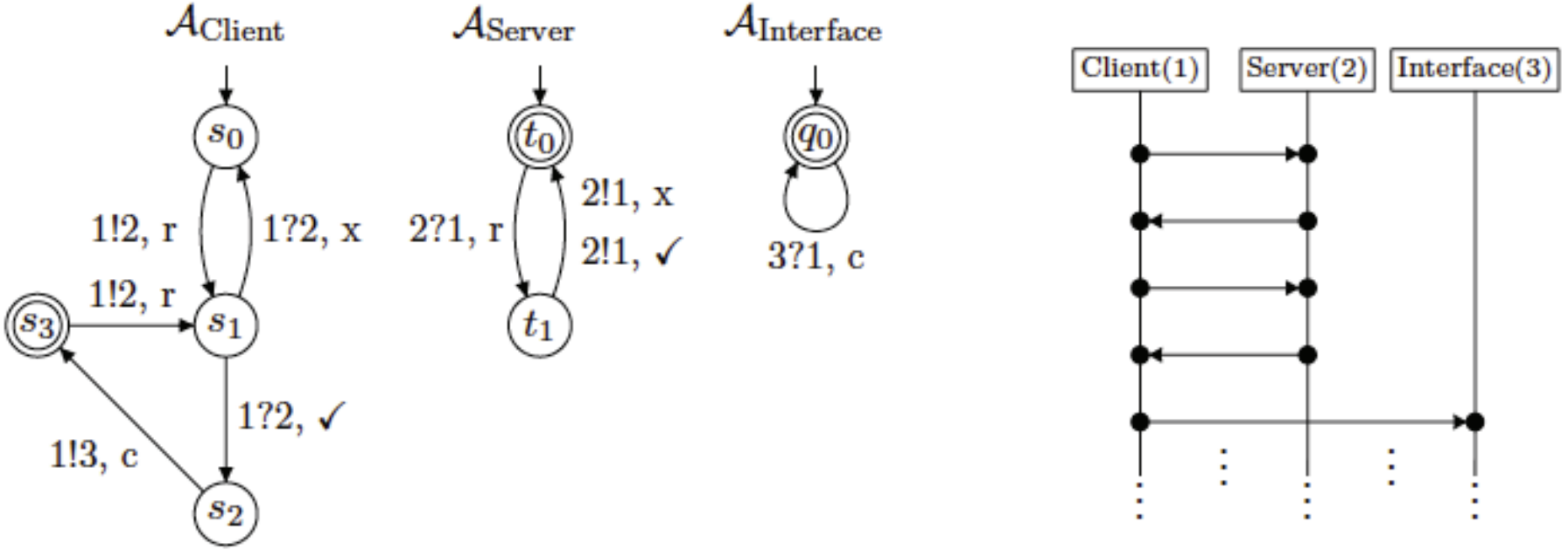
Communication finite-state machines

A CFM is accepting if all its processes have reached a local accepting state and either halt there or visit a local accepting state infinitely often.

An example CFM and an infinite MSC accepted by it



Client-server interaction to get access to an interface. Accepting state is (s_3, t_0, q_0).

PDL formulas on CFMs

A CFM is accepting if all its processes have reached a local accepting state and reside their ad infinitum.

The language $L(\mathcal{A})$ of CFM $\mathcal{A}$ is the set of MSCs that admit an accepting run.

CFM versus PDL

A CFM $\mathcal{A}$ satisfies PDL-formula Φ, denoted $\mathcal{A} \models \Phi$, whenever for all MSCs M it holds: $M \in L(\mathcal{A})$ if and only if $M \models \Phi$.

The example CFM satisfies $\forall\,(P_1 \rightarrow (\langle \mathsf{proc}^*; \mathsf{msg}; \mathsf{proc}^*; \mathsf{msg} \rangle\, P_3)$ where for $i \in \mathcal{P}$, formula $P_i = \bigvee_{j \in \mathcal{P}, j \neq i} (!_{i,j} \vee ?_{i,j})$, i.e., $M, e \models P_i$ iff e occurs at process i. The PDL formula asserts that process 3 (Interface) can be "reached" from 1 (Client) by exactly two messages using an intermediate process in between.

PDL model checking problem

Model checking CFMs versus PDL

The following model-checking problem is undecidable:

INPUT: a CFM $\mathcal{A}$, PDL-formula Φ

OUTPUT: is there an MSC $M \in L(\mathcal{A})$ with $M \models \Phi$?

Proof.

Follows immediately from the fact that the emptiness problem for CFMs is undecidable. By using the formula *true*, the above problem encodes the emptiness problem. □

To obtain decidability of the model-checking problem, we restrict ourselves to B-bounded MSCs.

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PDL model checking problem

Model checking CFMs versus PDL [Bollig et. al, 2011]

The following model-checking problem is PSPACE-complete:

INPUT: a CFM $\mathcal{A}$ and $B \in \mathbb{N}_{>0}$, PDL-formula Φ

OUTPUT: is there an $\exists B$-bounded MSC $M \in L(\mathcal{A})$ with $M \models \Phi$?

Proof.

(Sketch). Every PDL formula Φ can effectively be translated into a CFM $\mathcal{A}_\Phi$ such that $\mathcal{A}_\Phi \models \Phi$. The details are out of the scope of this lecture. This synthesis step is independent of the channel bound size B (if any). The size of $\mathcal{A}_\Phi$ is exponential in the length of Φ and the number of processes in $\mathcal{P}$. Then construct a CFM accepting $L(\mathcal{A}) \cap L(\mathcal{A}_\Phi)$. Decide whether the resulting CFM accepts some $\exists B$-bounded MSC. This can all be done in polynomial space. The PSPACE-hardness follows from the hardness of LTL model checking. □

Model checking an MSG versus PDL

Model checking MSGs versus PDL [Bollig et. al, 2011]

The following model-checking problem is PSPACE-complete:

INPUT: a MSG G and PDL-formula Φ

OUTPUT: is there an MSC $M \in L(G)$ with $M \models \Phi$?

Proof.

(Sketch.) For every vertex v, we can determine a linearization of the MSC $\lambda(v)$. Construct a finite automaton $\mathcal{A}_G$ that accepts a linearization for every $M \in L(G)$, and vice versa, each word accepted by $\mathcal{A}_G$ is a linearization of some $M \in L(G)$. The size of $\mathcal{A}_G$ is linear in the size of G. Construct a CFM $\mathcal{A}_\Phi$ for PDL-formula Φ with $M \in L(\mathcal{A}_\Phi)$ iff $M \models \Phi$. Construct a transition system by running $\mathcal{A}_G$ and $\mathcal{A}_\Phi$ simultaneously. This construction terminates as $\mathcal{A}_G$ only accepts linearizations that are B-bounded (as every linearization of MSG G is $\exists B$-bounded by definition). Deciding whether some simultaneous run is accepting can be done in polynomial space. The PSPACE-hardness follows from the hardness of LTL model checking. □